Logic Programming

Lecture 4:
Nonlogical features:
arithmetic, I/O, cut

Quick note

- Several predicates discussed so far (or today) are built-in (sometimes with different names):
  - `append/3`
  - `mem` (`member/2`)
  - `len` (`length/2`)
- It is good to know how to define them from scratch, if necessary
- LPN text "predicate index" lists all built-in predicates you are expected to know (and more!)

Nonlogical features

- So far we've worked (mostly) in *pure Prolog*
  - Solid logical basis
  - Elegant solutions to symbolic problems
- But, many practical things become inconvenient
  - Arithmetic
  - I/O
- And standard proof search not always efficient
  - Can we control proof search better?

Outline for today

- Nonlogical features
  - Expression evaluation
  - I/O
  - "Cut" (pruning proof search)
  - Negation-as-failure
    - more in 2 weeks

Expression evaluation

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no.
?- X = 2+2.
X = 2+2
```

Expression evaluation

- Prolog has built-in syntax for arithmetic expressions
- But it is *uninterpreted*

```
?- 2 + 2 = 4.
no.
?- X = 2+2.
X = 2+2
?- display(2+2).
+(2,2)
```


Expression evaluation

- We *could* define unary arithmetic operations

```
add(M,N,P)
```

- and interpret expressions ourselves

```
eval(+(X,Y),V) :-
  eval(X,N), eval(Y,M), add(M,N,V).
```

- But this is **slooooooow**
  - (and floating-point would be even worse...)

The evaluation predicate "is"

- Prolog provides a built-in predicate "is"

```
?- X is 2+2.
X = 4
?- X is 6*7.
X = 42
```


Machine arithmetic with "is"

- addition (+), subtraction (–)

```
?- X is 2+(3-1).
X=4
```

- multiplication (*), division (/), "mod"

```
?- X is 42 mod 5, Y is 42 / 5.
X = 2,
Y = 8.4
```

Warning

- Unlike "=", "`is`" is **asymmetric**
- **only** has mode `(?,+)`

```
?- 2+(3-1) is X.
! Instantiation error...
```

- requires RHS to be a **ground expression**

```
?- X is foo(bar).
! Domain error...
```


Lists and arithmetic

- Length of a list

```
len([],0).
len([_|L], N) :-
    len(L,M), N is M+1.
```

- Only works in mode `(+,?)`.
- Built-in `length/2`
  - (works in both directions)

Building a list of length n

- Similar to list length...

```
build([],0).
build([_|L], N) :-
    M is N-1, build(L,M).
```

- Only works in mode `(?,+)`.
  - (But see built-in `length(?L,?N)`)

Lists and arithmetic

- Summing elements of a list

```
sumall([],0).
sumall([X|L],S) :-
    sumall(L,M), S is M+X.
```

- What mode can this have?

Arithmetic comparisons

- Binary relations also built-in as goals:
  - less than (<), greater than (>)
  - less/equal (=<), greater/equal (>=)
  - equality (=:=), inequality (=/=)
- All have mode (+,+)
  - both arguments must be ground!

Example

- "Maximum" predicate

```
max(X,Y,Y) :- X =< Y.
max(X,Y,X) :- X > Y.
```

- Works in mode `(+X,+Y,?M).`

Basic Input/Output

- `read(?X)` reads in a term (followed by ".")
- `write(+X)` prints out its argument as a term.
- `nl/0` prints a newline
- Simple expression calculator:

```
calc :- read(X),
        Y is X,
        write(X = Y), nl,
        calc.
```


Backtracking through I/O

- Short answer: can backtrack, but **can't undo I/O**

```
?- write(foo),fail; write(bar).
foobar
```

- Any **bindings** will be **undone**

```
?- read(X), fail; X = 1.
|: foo.
X = 1
```

Cut

- Sometimes we **know** we've made the right choice
  - No backtracking needed
- In Pure Prolog we can't take advantage of this
- Introducing "cut" (`!`), the **proof-search pruning** operator

Example

- The "member of a list" predicate

```
member(X,[X|L]).
member(X,[Y|L]) :- member(X,L).
```

- If `X` is ever found in `L`, it is pointless to backtrack and keep looking for solutions

Example

- The "member of a list" predicate

```
member(X,[X|L]) :- !.
member(X,[Y|L]) :- member(X,L).
```

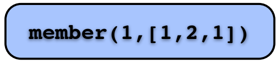
- If `X` is ever found in `L`, it is pointless to backtrack and keep looking for solutions
- Insert a cut **in first rule** to cut off search

How it works

- Remember choice points (places where we could have tried a different rule or branch).
- When we encounter a cut, "prune" all pending alternatives **since cut was introduced**

How it works

Without cut




```
R1: member(X,[X|_]).
R2: member(X,[_|L]) :- member(X,L)
```


How it works

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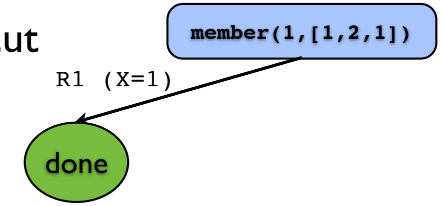


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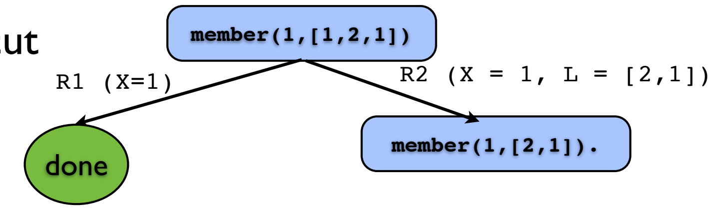


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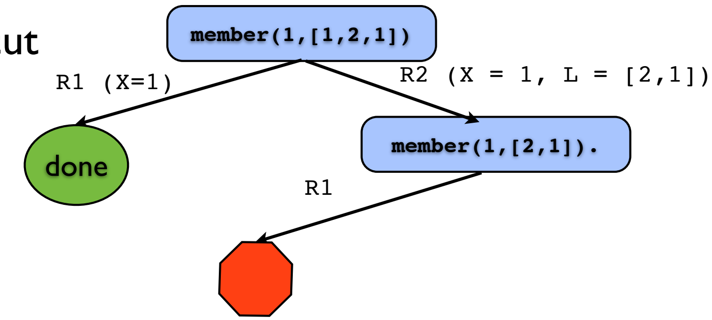


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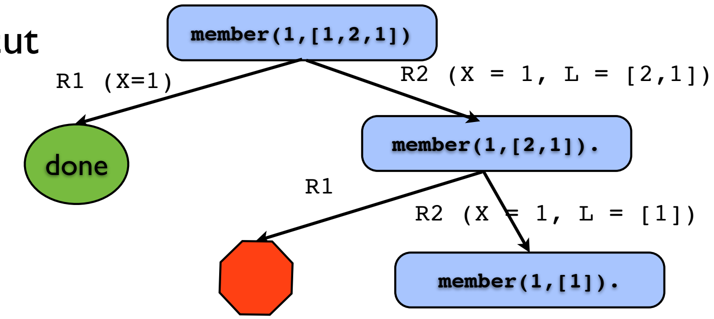


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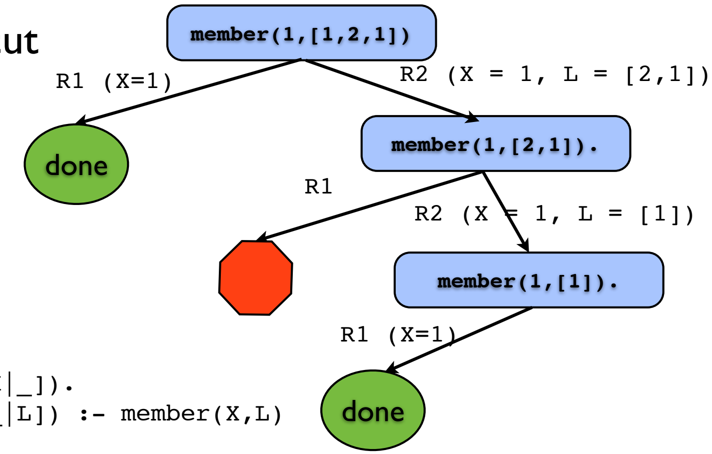


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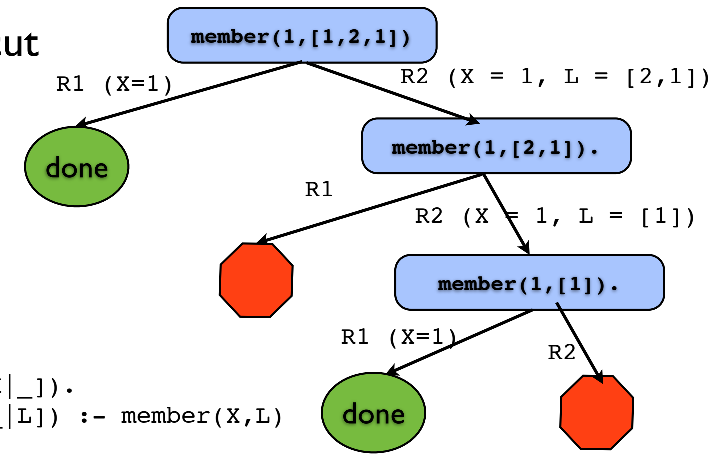


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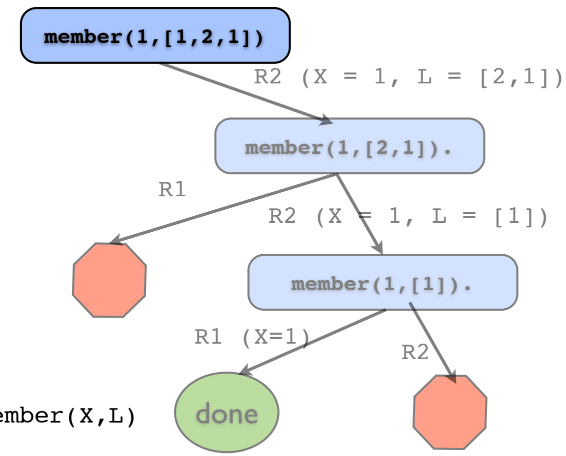


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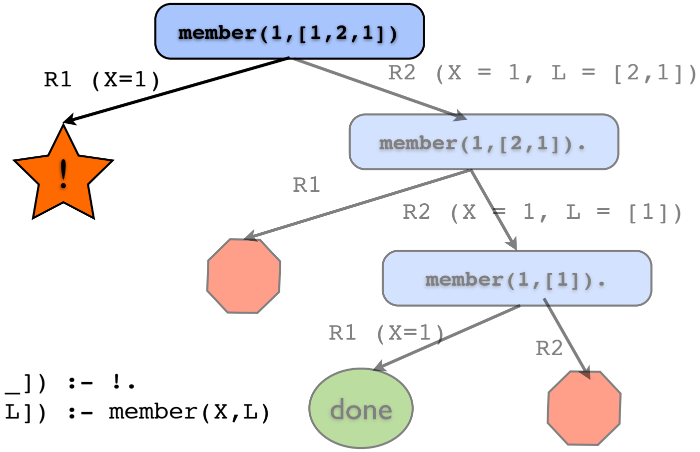


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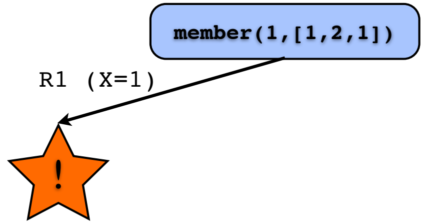


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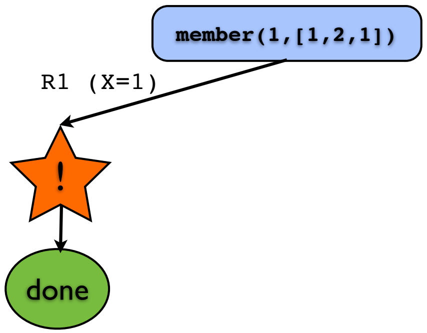


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R1: member(X,[X|_]) :- !.
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```
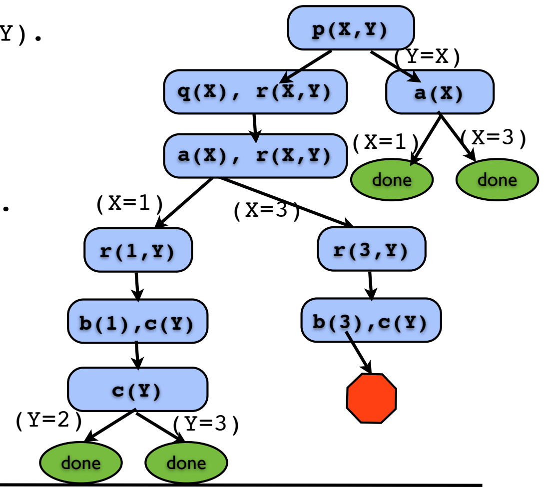


Another example

```
p(X,Y) :- q(X), r(X,Y).
p(X,X) :- a(X).
q(X) :- a(X).
r(X,Y) :- b(X), c(Y).
a(1).       a(3).
b(1). b(2).
      c(2). c(3).
```



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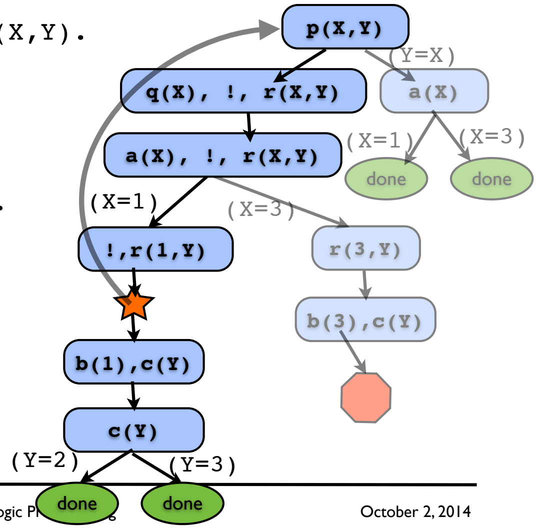

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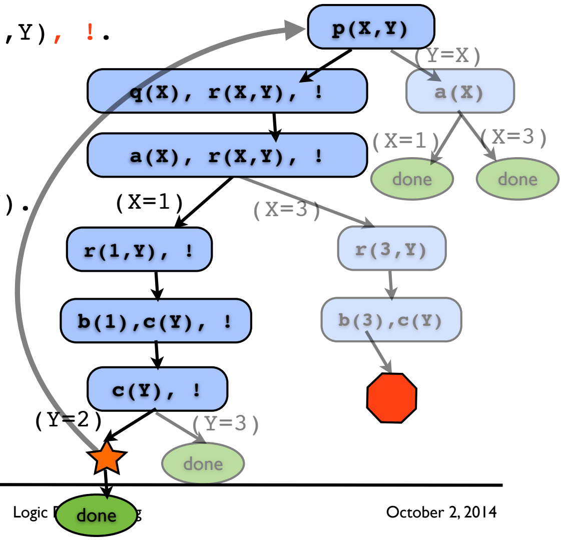

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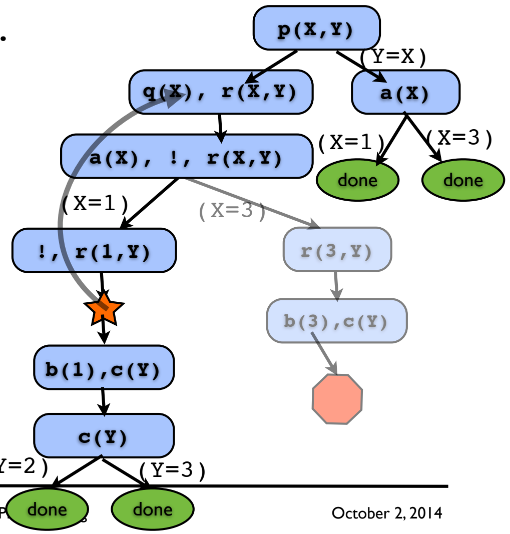

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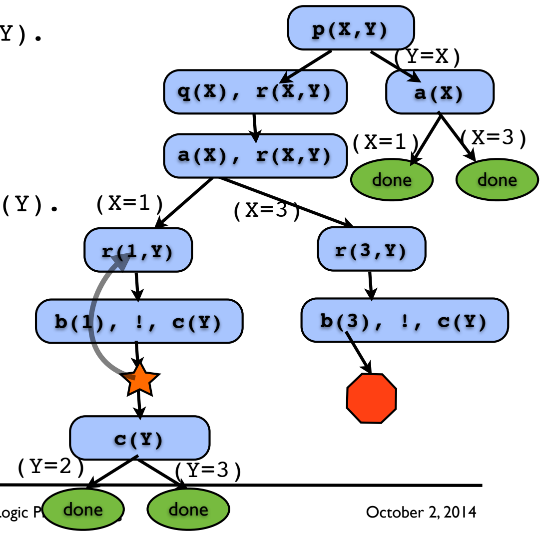

Max

```
max(X,Y,Y) :- X <= Y.
max(X,Y,X) :- X > Y.
```

- Pointless to try to backtrack
  - if the first goal succeeds, then the second won't!

Max using cut

```
max(X,Y,Y) :- X <= Y, !.
max(X,Y,X) :- X > Y.
```

- Pointless to try to backtrack
  - if the first goal succeeds, then the second won't!

But what about...

- Isn't it silly to test `X < Y` in second rule?

```
max(X,Y,Y) :- X <= Y, !.
max(X,Y,X).
```

- Maybe (slightly) more efficient to skip it
  - But damages transparency
  - `max(1,2,1)` and `max(1,2,2)` both succeed!
  - Rule order matters!

Safe use of cut

- Cut can make program more efficient
  - by avoiding pointless backtracking
- But as shown with "`max`", cuts can change meaning of program (not just efficiency)
  - "Green" cut - preserves meaning of program
  - "Red" cut - doesn't.

Next time

- More about cut & negation
- Further reading:
  - LPN, ch. 5 & 10