Logic Programming

Lecture 2: Unification and proof search

Outline for today

- Quick review
- Equality and unification
- How Prolog searches for answers

Quick review

- Atoms `bart` `'Mr. Burns'`
- Variables `X` `Y` `Z`
- Predicates `p(`t_1`,...,`t_n`)`
- Terms
- Facts `father(homer, bart).`
- Goals `p(`t_1`,...,`t_n`), ..., q(`t_1`',...,`t_n`').`
- Rules `p(ts) :- q(ts'), ..., r(ts'').`

Infix operators

- Prolog has built-in *constants* and *infix operators*
- Examples:
  - Equality: `t = u` (or `=(t,u)`)
  - Pairing: `(t,u)` (or `,(t,u)`)
  - Empty list: `[]`
  - List concatenation: `[X|Y]` (or `.(X,Y)`)
- You can also define your own infix operators!

General observations

- Prolog is **untyped**
  - everything is a "term"
- Prolog is **declarative**
  - "predicates" with side effects, such as print, are the exception, not the rule
- Prolog does **not** have explicit control flow constructs (while, do)
  - the search strategy allows us to simulate iteration
  - but this is not usually the best way to program
- Therefore, try to **forget what you already know** from other languages

Terms

- Also have...
  - **Numbers**: `1 2 3 42 -0.12345`
  - Additional **constants** and **infix operators**
- More on these later.

Unification (I)

- The equation `t = u` is a basic goal
  - with a special meaning
- What happens if we ask:

```
?- X = c
?- f(X,g(Y,Z)) = f(c,g(X,Y))
?- f(X,g(Y,f(X))) = f(c,g(X,Y))
```

- *And how does it do that?*

Unification (II)

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```
?- X = c.
```


Unification (II)

```
?- X = c.
X=c
yes
```


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X=c
Y=c
Z=c
yes
```

Unification (II)

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no
```
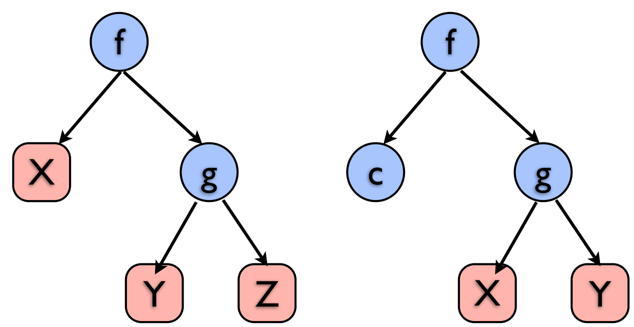


Unification (III)

- A *substitution* is a mapping from variables to terms
  - $X_1=t_1,\ldots,X_n=t_n$
- Given two terms `t` and `u`
  - with free variables $X_1 \ldots X_n$
- a *unifier* is a substitution that makes `t` and `u` equal

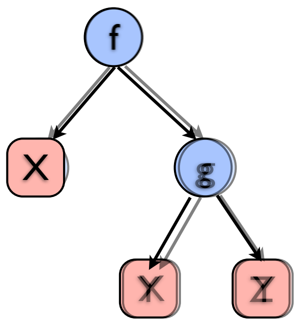
Example (I)

`f(X,g(Y,Z)) = f(c,g(X,Y))`



Example (I)

$f(X,g(Y,Z)) = f(c,g(X,Y))$



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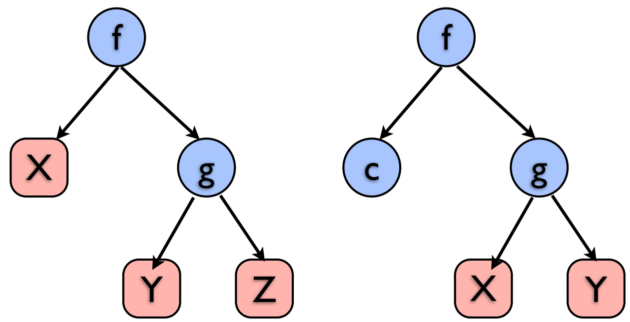

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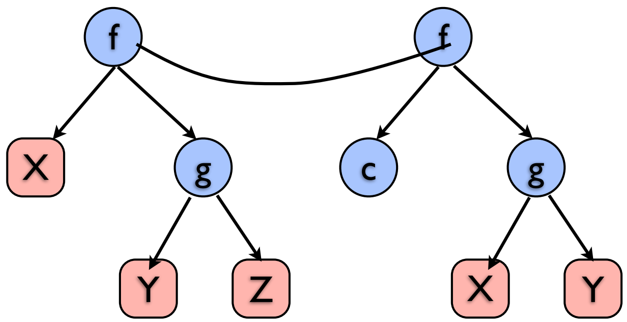

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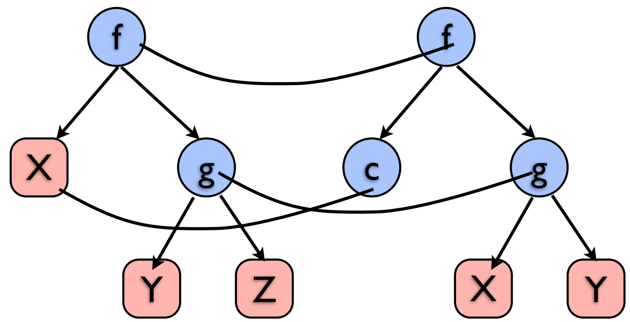

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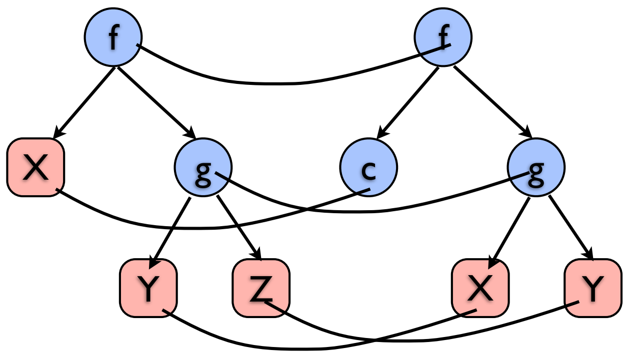

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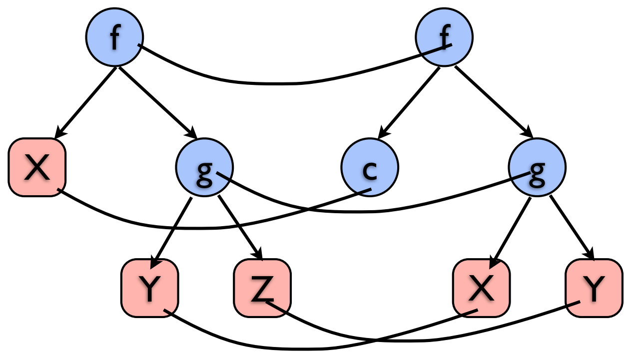
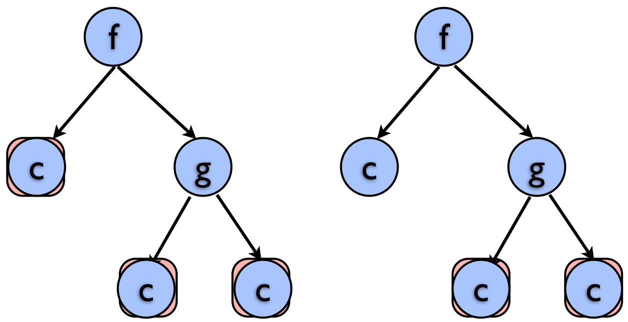



```
X = c
Y = X
Z = Y
```


Example (I)

`f(X,g(Y,Z)) = f(c,g(X,Y))`

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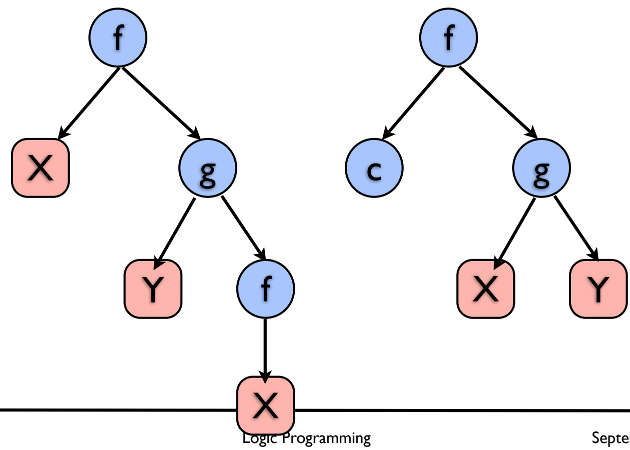
Example (II)

`f(X,g(Y,f(X))) = f(c,g(X,Y))`



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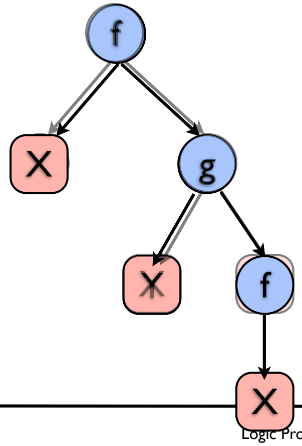

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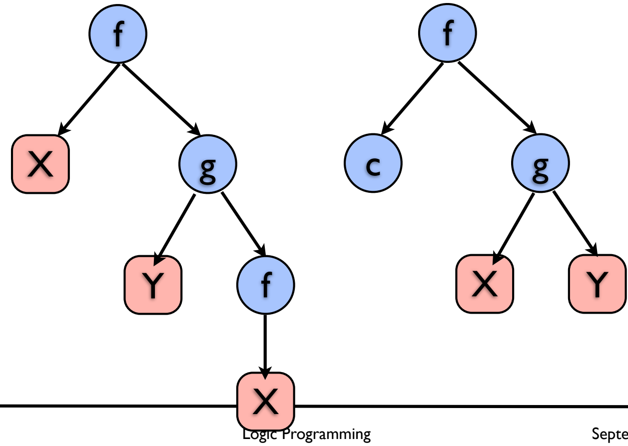

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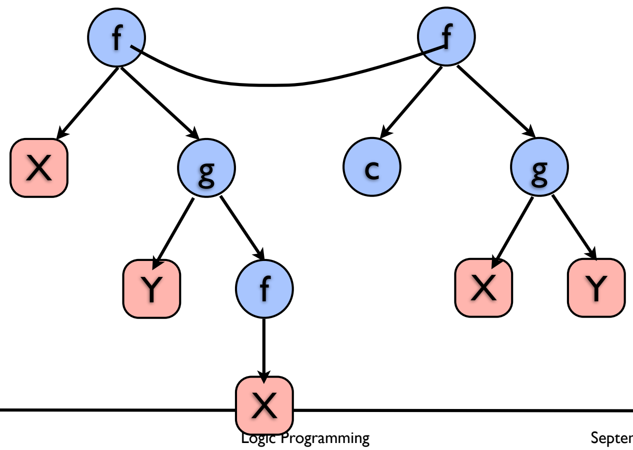

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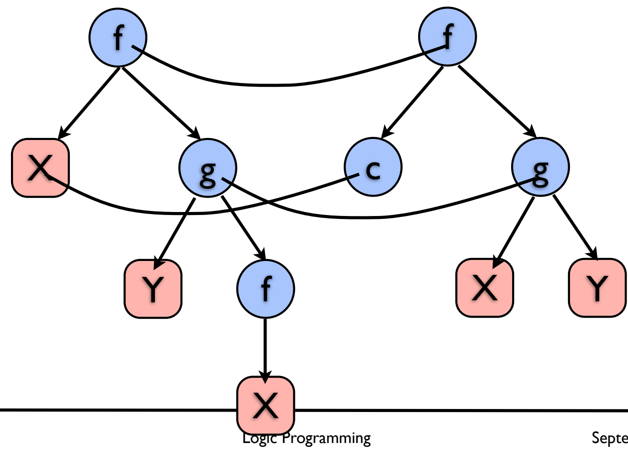

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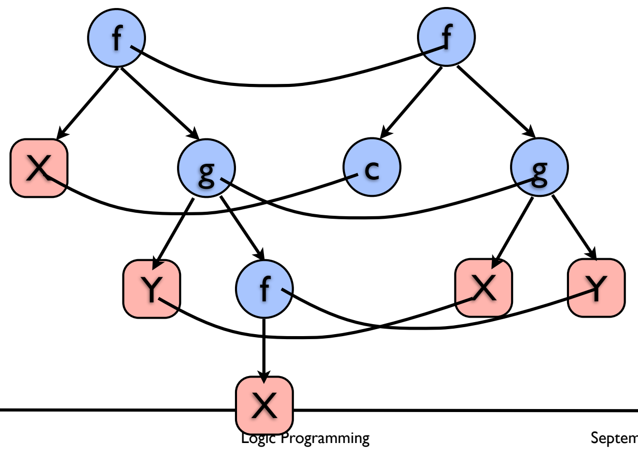
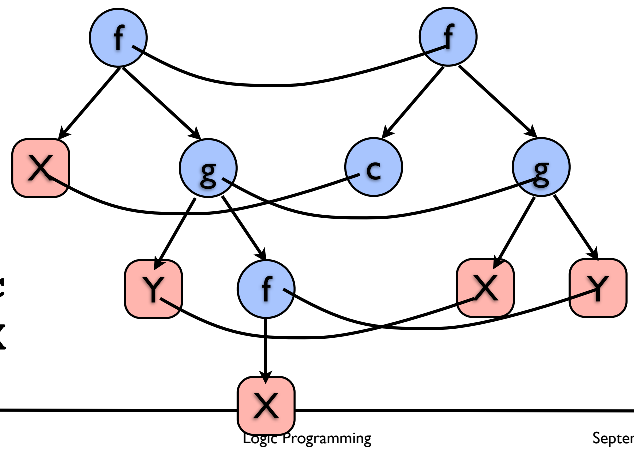


Example (II)

`f(X,g(Y,f(X))) = f(c,g(X,Y))`

```
X = c
Y = X
```



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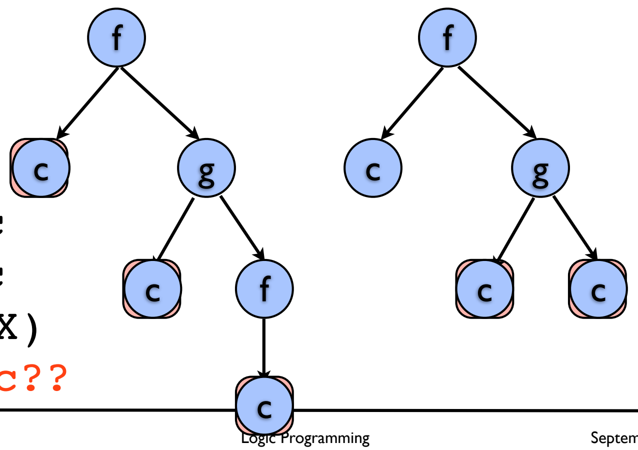


```
X = c
Y = c
Y = f(X)
f(X) = c??
```


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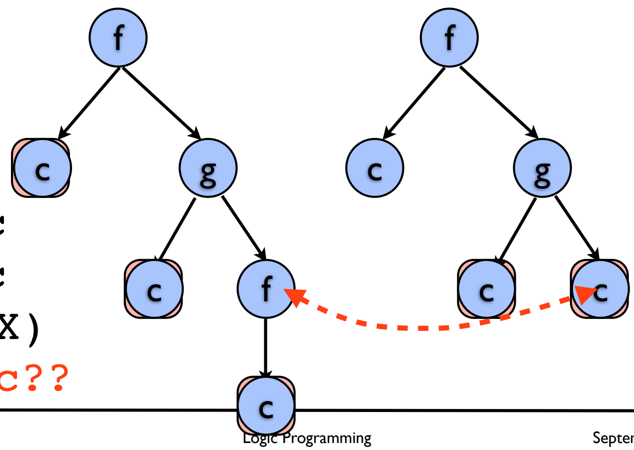


```
X = c
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```

Robinson's Algorithm (I)

- Consider a general unification problem

  $t_1 = u_1,\ t_2 = u_2,\ \ldots,\ t_n = u_n$

- Reduce the problem by decomposing each equation into one or more "smaller" equations
- Succeed if we reduce to a "solved form", otherwise fail

Robinson's Algorithm (II)

- Two constants unify if they are equal.

$$c = c,\ P \rightarrow P$$

$$c = d,\ P \rightarrow \text{fail.}$$

Robinson's Algorithm (III)

- Two function applications unify if the head symbols are equal, and the corresponding arguments unify.

$$f(t_1,\ldots,t_n) = f(u_1,\ldots,u_n),\ P \rightarrow$$
$$t_1 = u_1\ ,\ldots\ t_n = u_n\ ,\ P$$

- Must have equal numbers of arguments

$$f(\ldots) = g(\ldots), P \rightarrow \text{fail}$$

$$f(\ldots) = c,\ P \rightarrow \text{fail.}$$

Robinson's Algorithm (IV)

- Otherwise, a variable X unifies with a term t provided X does not occur in t.

$$X = t,\ P \rightarrow P[t/X]$$

(**occurs-check**: X must not be in Vars(t))

- Proceed by substituting t for X in P.

Occurs check

- What happens if we try to unify `X` with something that *contains* `X`?

  ```
  ?- X = f(X).
  ```

- Logically, this should fail
  - there is no (finite) unifier!
- Most Prolog implementations skip this check for efficiency reasons
  - can use `unify_with_occurs_check/2`

Execution model

- The query is run by trying to find a solution to the goal using the clauses
  - Unification is used to match goals and clauses
  - There may be zero, one, or many solutions
  - Execution may backtrack
- Formal model called **SLD** resolution
  - which you'll see in the theory lectures

Depth-first search (I)

- Idea: To solve atomic goal *A*,
  - *If B* is a **fact** in the program, *and* $\theta(A) = \theta(B)$, *then* return answer θ
  - Else, *if* B :- $G_1,\ldots,G_n$ is a **clause** in the program, *and* θ unifies *A* with *B*, *then* solve $\theta(G_1)$ ... $\theta(G_n)$
  - Else, give up on this goal.
    - **Backtrack** to last choice point
- Clauses are tried **in declaration order**
- Compound goals are tried **in left-right order**

Depth-first search (II)

- Prolog normally tries clauses in order of appearance in program.
- Assume: `foo(a). foo(b). foo(c).`
- Then:

  ```
  ?- foo(X).
  ```

  foo(X)

Depth-first search (II)

- Prolog normally searches for clauses in order of appearance in database.
- Assume: foo(a). foo(b). foo(c).
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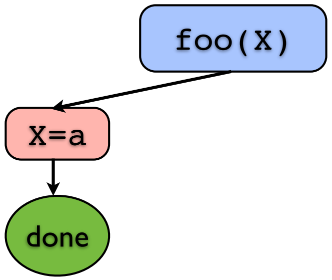
X = a



Depth-first search (II)

- Prolog normally searches for clauses in order of appearance in database.
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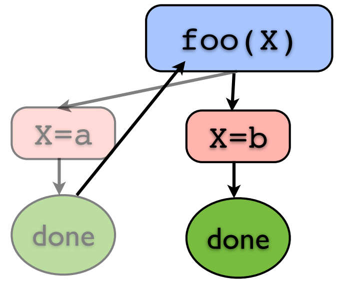
X = a;

X = b



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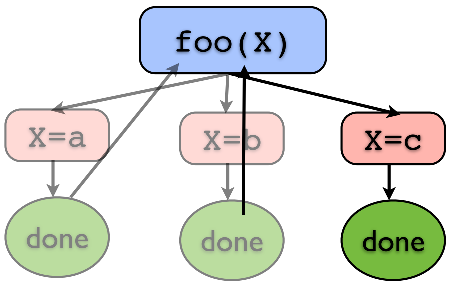

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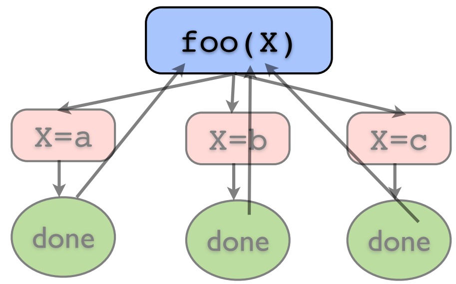

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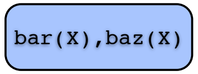
- Assume: `bar(b). bar(c). baz(c).`
- Then:

  `?- bar(X),baz(X).`



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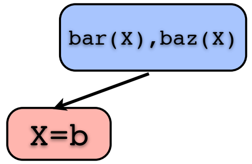

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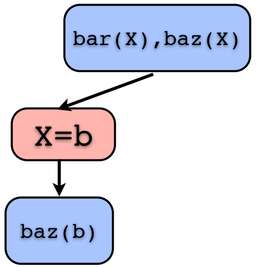

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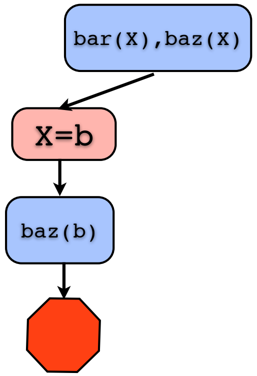

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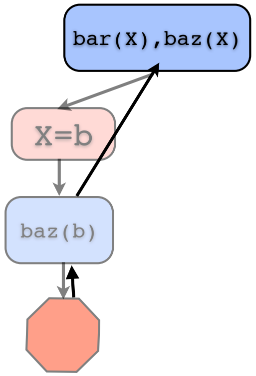

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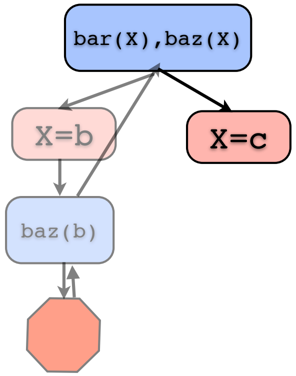

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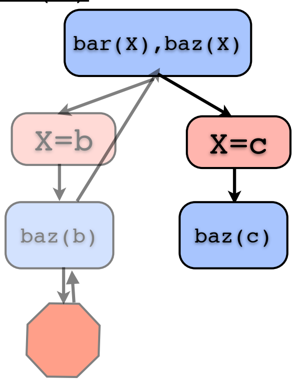

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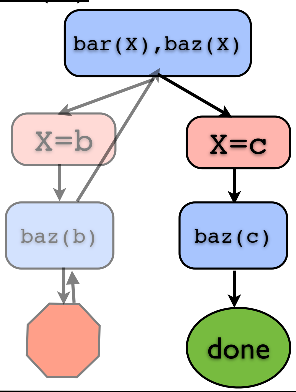
`X = c`



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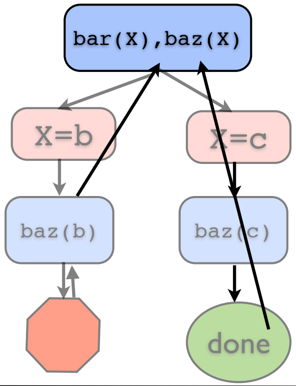
- Then:

```
?- bar(X),baz(X).
X = c;
no
```



"Generate and test"

- Common Prolog programming idiom:

```
find(X) :- generate(X), test(X).
```

- where test(X) **checks if X is a solution**
- generate(X) **searches for solutions**
  - Can use to constrain (infinite) search space
  - Can use different generators to get different search strategies besides depth-first

Limitations of depth-first search

- Recursion needs to be handled carefully to avoid loops
  - Rule order and goal order matter
  - More in next lecture
- Not complete "in practice"
  - legitimate answers may be missed due to loops

Other search strategies

- Breadth-first search / iterative deepening
  - Explore all alternatives, interleaved
  - Price: memory overhead
- Bottom-up (forward chaining)
  - Compute all possible answers derivable from facts & rules
  - Only viable for "Datalog" programs with "flat" data (only constants and variables)
  - Supported by commercial tools for big data (LogicBlox, DLV)

Next time

- Recursion
- Lists, trees, data structures
- Further reading: LPN ch. 2
- Tutorial #1 will be up soon