Logic Programming 2013–14
Tutorial 3: QUIZ solutions

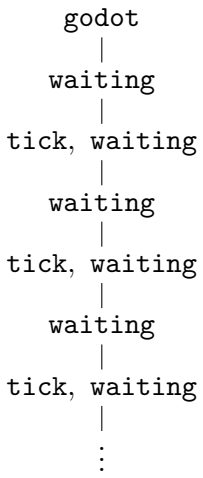
1 mark per part-question, leading to a maximum of 10 marks.

1. (a)

$$\begin{array}{c} \texttt{waiting} \rightarrow \texttt{godot} \\ \texttt{tick} \wedge \texttt{waiting} \rightarrow \texttt{waiting} \\ \texttt{tick} \end{array}$$

   Award mark only if everything is correct.

   (b) The search tree is an infinite linear branch:



   Award mark if tree is correctly drawn at least as far as a repeated node, and it is observed that the resulting branch then continues and is infinite.

   (c) Prolog goes into a loop and so does not return an answer.

   (d) A decision procedure would answer that the query `godot` is not a logical consequence of the theory. (Also award mark for answers such as "no" and "false".)

2. (a) Program in logical notation:

$$\begin{array}{c} \texttt{even(z)} \\ \forall\, \texttt{X}.\ \texttt{even(X)} \rightarrow \texttt{odd(s(X))} \\ \forall\, \texttt{X}.\ \texttt{odd(X)} \rightarrow \texttt{even(s(X))} \end{array}$$

   Award mark only if completely correct.

   (b) Query in logical notation:

$$\exists\, \texttt{X}.\ \texttt{odd(X)} \wedge \texttt{even(X)}$$

   Again, award mark only if completely correct.

(c) The models are $\mathcal{S}_1$ and $\mathcal{S}_3$. Award mark for identifying exactly these two models.

For the record, $\mathcal{S}_2$ is not a model because $\texttt{even}(\texttt{s}(\texttt{s}(\texttt{z})))$ holds in $\mathcal{S}_2$ but $\texttt{odd}(\texttt{s}(\texttt{s}(\texttt{s}(\texttt{z}))))$ does not hold, since the interpretation of $\texttt{s}(\texttt{s}(\texttt{z}))$ in $\mathcal{S}_2$ is 0. This means that the second axiom in the program is false in $\mathcal{S}_2$.

(d) The query formula, $\exists\, \texttt{X}.\, \texttt{odd}(\texttt{X}) \wedge \texttt{even}(\texttt{X})$, is not true in $\mathcal{S}_1$, which is a model of the program.

Award mark for correctly identifying a model of the program in which the query formula is false.

(e) It is not a logical consequence. So award mark for the answer "no".

To see it is not a logical consequence note that $\exists\, \texttt{X}.\, \texttt{odd}(\texttt{X}) \wedge \texttt{even}(\texttt{X})$ is true in $\mathcal{S}_3$ since $\texttt{odd}(\texttt{z})$ and $\texttt{even}(\texttt{z})$ both hold in $\mathcal{S}_3$. Thus $\neg\, \exists\, \texttt{X}.\, \texttt{odd}(\texttt{X}) \wedge \texttt{even}(\texttt{X})$ is false in $\mathcal{S}_3$, which is a model of the program.

(f) It is a logical consequence. Award mark simply for the answer "yes".

One justification that $\exists \texttt{X}.\, \texttt{odd}(\texttt{X})$ is a logical consequence consequence runs as follows. Prolog search succeeds for the query $\texttt{odd}(\texttt{X})$; indeed prolog will return $\texttt{X} = \texttt{s}(\texttt{z})$ as its first answer. And for programs and queries in definite clause predicate logic, Prolog search is sound. In fact this explanation anticipates material still to come in Theory Lecture 4. Nonetheless, many students should already have enough intuition to guess the answer to this question.