Logic Programming 2013–14
Assignment 2: Theory

This is the second coursework assignment for Logic Programming. It contributes 10% towards your grade for the course.

Your solutions must be handed in, on paper, at the ITO by **3pm** on **Monday 11th November**.

Marked and commented scripts will be available for collection from the ITO from 3pm on Monday 25th November.

1. (a) Consider the following Prolog program and query.

```
r(X, l(X)).
r(X, t(Y,_)) :- r(X,Y).
r(X, t(_,Z)) :- r(X,Z).

?- r(b, t(l(a),l(Z))), r(Z, t(l(c),l(W))).
```

   Rewrite both program and query in the notation of first-order logic, writing all quantifiers explicitly.

   [6 marks]

   (b) Draw the full Prolog search tree for the above program and query, and say what response Prolog gives to the query.

   [10 marks]

   (c) The following questions concern the status of negated query $\neg\mathtt{r}(\mathtt{a},\mathtt{l}(\mathtt{b}))$ relative to the program above. In each case, justify your answer.

   i. What response does Prolog give to the negated query `\+ r(a,l(b))` ?

   [2 marks]

   ii. Is $\neg\mathtt{r}(\mathtt{a},\mathtt{l}(\mathtt{b}))$ a logical consequence of the program?

   [4 marks]

   iii. Is $\neg\mathtt{r}(\mathtt{a},\mathtt{l}(\mathtt{b}))$ true in the minimum Herbrand model of the program?

   [3 marks]

2. (a) Let A be the set $\{0,1,2\}$. Consider the following functions from the power set of A to itself.

$$f_1, f_2, f_3, f_4 \colon \mathcal{P}(A) \to \mathcal{P}(A)$$

$$\begin{aligned}
f_1(Y) &= \{x \mid x \in \{0,1,2\} \text{ and } x \notin Y\} \\
f_2(Y) &= \{\text{ the sum, modulo 3, of elements in } Y\} \\
f_3(Y) &= \{0 \mid 0 \notin Y \text{ and } 2 \in Y\} \cup \{1\} \cup \{2 \mid 2 \notin Y \text{ and } 0 \in Y\} \\
f_4(Y) &= \{2 - x \mid x \in Y\}
\end{aligned}$$

   For each of these four functions answer the following 3 questions.

i. Is it monotone?

ii. What are its fixed points, if any?

iii. What is its least fixed point, if any?

[12 marks]

(b) Consider the following propositional Prolog program.

```
a.
b :- a,c.
c :- b,d.
d :- a,e.
e :- a.
j :- b,c.
```

The meaning of the program is defined by the least fixed point of a function $f\colon \mathcal{P}(\{\mathtt{a},\mathtt{b},\mathtt{c},\mathtt{d},\mathtt{e},\mathtt{j}\}) \to \mathcal{P}(\{\mathtt{a},\mathtt{b},\mathtt{c},\mathtt{d},\mathtt{e},\mathtt{j}\})$. Give a precise definition of the function f, and calculate sufficiently many iterated applications of f to the empty set to find its least fixed point. Show your workings.

[9 marks]

(c) Briefly explain how the calculation of a least-fixed-point can be used to implement a decision procedure for definite clause propositional logic.

[3 marks]

(d) Give one reason that it would not be appropriate to for Prolog to replace its proof-search-based strategy with a fixed-point-based decision procedure.

[1 marks]