LFD Problem Set for Week 8
Solutions

Question 1

The regression model is:

$$
\begin{aligned}
y_1 &= mx_1 + c + \epsilon_1 \\
y_2 &= mx_2 + c + \epsilon_2 \\
&\cdots \\
y_P &= mx_P + c + \epsilon_P
\end{aligned} \tag{1.1}
$$

We need to choose values for m and c that minimize:

$$
Q = \sum_{i=1}^{P} \epsilon_i^2 = \sum_{i=1}^{P} [y_i - (mx_i + c)]^2 \tag{1.2}
$$

The two first order conditions are:

$$
\frac{\partial Q}{\partial m} = -2 \sum_{i=1}^{P} (y_i - mx_i - c)\, x_i = 0 \tag{1.3}
$$

$$
\frac{\partial Q}{\partial c} = -2 \sum_{i=1}^{P} (y_i - mx_i - c) = 0 \tag{1.4}
$$

These equations can be solved for m and c using algebraic substitution.

By moving in the $\sum$ and dividing through by $\frac{1}{P}$, 1.4 can be rewritten as:

$$
c = \bar{y} - m\bar{x} \tag{1.5}
$$

Plugging (1.5) into (1.3) and rearranging terms yields:

$$
m = \frac{\sum x_i y_i - P\bar{x}\bar{y}}{\sum x_i^2 - P\bar{x}^2} \tag{1.6}
$$

Question 4

Logistic Regression is a specific case of the classifier described in this question (i.e. when $\phi(x) = x$). The log likelihood is:

$$\ell(\mathbf{w}) = \sum_{\mu=1}^{P} c^{\mu} \ln\sigma\left(\mathbf{w}'\phi(\mathbf{x})\right) + (1 - c^{\mu}) \ln\left(1 - \sigma\left(\mathbf{w}'\phi(\mathbf{x})\right)\right)$$

and

$$\nabla_{w}\ell = \sum_{\mu=1}^{P} \left(c^{\mu} - \sigma\left(\mathbf{w}'\phi(\mathbf{x})\right)\right)\phi^{\mu}$$

An approach for calculating $\ell(\mathbf{w})$ and $\nabla_{w}\ell$ was outlined in Problem Set 6. The decision boundary can be found by solving $w'\phi(x) = 0$ since $\sigma(0) = .5$. Note that, unlike logistic regression, the decision boundary specified by this model can be nonlinear.