Learning from Data, Tutorial Sheet Number 1

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1. Let A and $\boldsymbol{v}$ be defined as

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ 3 & -3 & -3 \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

Calculate $A\boldsymbol{v}$. Is $\boldsymbol{v}$ an eigenvector of A, and if so what is the corresponding eigenvalue?

2. A random vector $\mathbf{x}$ has zero mean a diagonal covariance

$$E(\mathbf{x}\mathbf{x}^T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where E stands for expectation (or mean average) of a random variable. If $\mathbf{y} = A^T\mathbf{x}$ (using A from Q1) what is the covariance of the resulting random vector $\mathbf{y}$: $E(\mathbf{y}\mathbf{y}^T)$? You may use the fact that expectation is linear: $E(R\mathbf{x}\mathbf{x}^T S) = RE(\mathbf{x}\mathbf{x}^T)S$. This shows how covariances change under linear transformations.

3. Find the partial derivatives of the function $f(x, y, z) = (x + 2y)^2 \sin(xy)$.

4. Label the following problem classes as either supervised or unsupervised, and the solutions as either generative or conditional.

- PROBLEM: Using training data of film ratings for various users to help predict future ratings for particular users. SOLUTION Build a representation to group users and films into similar groups. Predict rating on the basis of which films liked by the group for which the user is a member.
- PROBLEM: Take past stock market data to predict future stock performance give recent history. SOLUTION: Model how the current state depends on the history and use that relationship for prediction.
- PROBLEM: Use past stock market data to predict stock prices relationships. SOLUTION: Compute covariance between time-derivatives of stock prices, and use the covariance to predict current relationships.

5. Probability. Lois knows that on average radio station RANDOM-FM plays 1 out of 4 of her requests. If she makes 3 requests, what is the probability that at least one request is played?

6. Let X be distributed according to a uniform distribution, i.e. $f(x) = 1$ for $0 \leq x \leq 1$, and 0 otherwise. Show that X has mean $\frac{1}{2}$ and variance $\frac{1}{12}$.

7. Find the (unconstrained) minimum of the function f where A is a positive definite symmetric matrix.

$$f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T A\boldsymbol{x} - \boldsymbol{b}^T\boldsymbol{x}$$

Find the minimum of $f(\boldsymbol{x})$ along the line $\boldsymbol{x} = \boldsymbol{a} + t\boldsymbol{v}$ where t is a real parameter.