KMM Tutorial 3

1. The purchasing manager for a restaurant performs an analysis of the chef's favourite dishes and ingredients. She uncovers the following: *Lamb*, *Beef*, *Tofu*, *Squid* and *Prawn* are the most common dishes, cooked in a range of sauces that contain *Cream*, *CoconutMilk*, *Chilli*, *LemonJuice* and *SoySauce*. Letting *cookedIn* and *contains* be binary relations that relate dishes to sauces, and sauces to their ingredients respectively, write necessary and sufficient definitions in the Description Logic ALC for the following:

i. SweetSauce is a Sauce that contains some *Cream*, and either contains only *Cream*, or contains some *CoconutMilk* and does not contain *Chilli*.

ii. SourSweetSauce is a Sauce that contains some *CoconutMilk* and contains some *LemonJuice* and contains some *Chilli*.

iii. HotSauce is a Sauce that contains some *Chilli* and contains only *Chilli* or *SoySauce*.

iv. RedMeat is *Lamb* or *Beef*.

v. A WinterDish is RedMeat that is cookedIn only a Sauce that contains some *Chilli*, and is cookedIn some Sauce.

vi. A SummerDish is *Tofu* or *Squid* that is cookedIn only a Sauce that contains some *CoconutMilk*, and is cookedIn some Sauce.

vii. Is *Tofu* with SourSweetSauce necessarily a WinterDish? Construct a FACT tableaux to decide whether or not this subsumption relationship holds.

viii. Is *Tofu* with SourSweetSauce necessarily a SummerDish? Construct a FACT tableaux to decide whether or not this subsumption relationship holds.

Answers

1. (a) Answer to part A of question 1

i. SweetSauce ≡ Sauce ⊓ ∃contains.Cream ⊓ (∀contains.Cream ⊔ (∃contains.CoconutMilk ⊓ ¬∃ contains.Chilli))

ii. SourSweetSauce ≡ Sauce ⊓ ∃contains.CoconutMilk ⊓ ∃contains.LemonJuice ⊓ ∃contains.Chilli

iii. HotSauce ≡ Sauce ⊓ ∃contains.Chilli ⊓ ∀contains. (Chilli ⊔ SoySauce)

iv. RedMeat ≡ (Beef ⊔ Lamb)

v. WinterDish ≡ RedMeat ⊓ ∀cookedIn.(Sauce ⊓ ∃contains.Chilli) ⊓ ∃cookedIn.Sauce

vi. SummerDish ≡ (Tofu ⊔ Squid) ⊓ ∀cookedIn.(Sauce ⊓ ∃contains.CoconutMilk) ⊓ ∃cookedIn.Sauce

vii. Tofu with SourSweetSauce is a SummerDish as the sauce contains some CoconutMilk. Assuming Tofu and RedMeat are disjoint, this dish cannot be a WinterDish despite the sauce containing Chilli.

vii. Is *Tofu* with SourSweetSauce necessarily a WinterDish? Construct a FACT tableaux to decide whether or not this subsumption relationship holds. Formally, using FACT:

Goal:	$(Tofu \sqcap \forall cookedIn.(Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli) \sqcap \exists cookedIn.Sauce)$ $\sqsubseteq WinterDish$
	$(Tofu \sqcap \forall cookedIn.(Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli) \sqcap \exists cookedIn.Sauce)$ $\sqcap \neg(RedMeat \sqcap \forall cookedIn.(Sauce \sqcap \exists contains.Chilli) \sqcap \exists cookedIn.Sauce)$
NNF: Node(a0)	$\{(Tofu \sqcap \forall cookedIn.(Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli) \sqcap \exists cookedIn.Sauce)$ $\sqcap (\neg RedMeat \sqcup \exists cookedIn.(\neg Sauce \sqcup \forall contains.\neg Chilli) \sqcup \forall cookedIn.\neg Sauce)\}$
Node(a0) ⊓ elim.	$\{Tofu \sqcap \forall cookedIn.(Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli) \sqcap \exists cookedIn.Sauce,$ $\neg RedMeat \sqcup \exists cookedIn.(\neg Sauce \sqcup \forall contains.\neg Chilli) \sqcup \forall cookedIn.\neg Sauce\}$
Node(a0) ⊓ elim. $*2$	$\{Tofu, \forall cookedIn.(Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli),$ $\exists cookedIn.Sauce,$ $\neg RedMeat \sqcup \exists cookedIn.(\neg Sauce \sqcup \forall contains.\neg Chilli) \sqcup \forall cookedIn.\neg Sauce\}$
Node(a0) disjunct1 ⊔ elim.	$\{Tofu, \forall cookedIn.(Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli),$ $\exists cookedIn.Sauce,$ $\neg RedMeat\}$
	unless we can show a contradiction between $Tofu$ and $\neg RedMeat$, disjunct1 remains open Continue with the rest of the FACT procedure, although we could stop here.
Node(a0) disjunct2 ⊔ elim.	$\{Tofu, \forall cookedIn.(Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli),$ $\exists cookedIn.Sauce,$ $\exists cookedIn.(\neg Sauce \sqcup \forall contains.\neg Chilli)\}$
	add edge labelled $cookedIn$ from a0 to a1
Node(a1) ∃ elim. ∀ elim.	$\{\neg Sauce \sqcup \forall contains.\neg Chilli,$ $Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli\}$
Node(a1) disjunct2a Node(a1) ⊓ elim.	$\{\neg Sauce,$ $Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli\}$ $\{\neg Sauce,$ $Sauce, \exists contains.CoconutMilk, \exists contains.LemonJuice, \exists contains.Chilli\}$ Clash ($Sauce$)
Node(a1) disjunct2b Node(a1) ⊓ elim.	$\{\forall contains.\neg Chilli,$ $Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli\}$ $\{\forall contains.\neg Chilli,$ $Sauce, \exists contains.CoconutMilk, \exists contains.LemonJuice, \exists contains.Chilli\}$
	add edge labelled $contains$ from a1 to a2
Node(a2) ∃ elim. ∀ elim.	$\{Chilli,$ $\neg Chilli\}$ Clash ($Chilli$)
	disjunct2 shows a clash as disjunct2a and disjunct2b clash

Node(a0) disjunct3 $\sqcup$ elim.	$\{Tofu, \forall cookedIn.(Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli),$ $\exists cookedIn.Sauce,$ $\forall cookedIn.\neg Sauce\}$
	add edge labelled *cookedIn* from a0 to a3
Node(a3) $\exists$ elim. $\forall$ elim.	 $\{Sauce,$ $\neg Sauce\}$ Clash ($Sauce$)
	disjunct3 shows a clash

Returning to the open disjunct (disjunct1), there ought to be a way to include the fact that nothing is both Tofu and RedMeat and develop the proof further. Note that the four FACT rules covered in lectures do not allow this goal to be proven.

Node(a0) disjunct1	$\{Tofu, \forall cookedIn.(Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli),$ $\exists cookedIn.Sauce,$ $\neg RedMeat\}$

You should be aware that disjointness means: $Tofu \sqcap RedMeat \sqsubseteq \bot$.
Disjointness can also be written: $Tofu \sqsubseteq \neg RedMeat$
and can be expressed as a constraint: $\top \sqsubseteq \neg Tofu \sqcup \neg RedMeat$ and so $\{\neg Tofu \sqcup \neg RedMeat\}$ holds universally.

However, this does not lead to a contradiction, just the opposite as if instance a0 instantiates the class $Tofu$, it necessarily instantiates $\neg RedMeat$ and so disjunct1 cannot show a clash. Adding $\neg Tofu \sqcup \neg RedMeat$ to the label set for a0 just returns the proof to the same state, as you can verify.

viii. Is *Tofu* with SourSweetSauce necessarily a SummerDish? Construct a FACT tableaux to decide whether or not this subsumption relationship holds.

Goal:	$(Tofu \sqcap \forall cookedIn.(Sauce \sqcap \exists contains.CoconutMilk \sqcap \exists contains.LemonJuice \sqcap \exists contains.Chilli)$ $\sqcap \exists cookedIn.Sauce)$ $\sqsubseteq SummerDish$
NNF:	
Node(a0)	

NNF: negation normal form
Node: node in FACT graph