

Parts and Wholes



Parts and wholes

- A conceptual analysis of the part-of relation based on meta-properties
 - 6 different senses of part-of, including:
 - » Part of a system
 - » Constituent of a mixture
- A formal analysis
 - Axioms for part-of
 - Axioms for connected-to



Part-whole relations



- The part-whole relation '*part-of*' is fundamental to representing many domains
 - Parts of a system, e.g. Pump part-of CoolingSystem
 - Parts of the body, e.g. Cardiac Chamber part-of Heart
 - Parts of an organisation, e.g. Treasury part-of Government
 - Parts of a process, e.g. Reading part-of Learning
- The theory of parts and wholes is called mereology (meronymic relations) meros: part [in Greek]
 - Different senses of part-of can be distinguished
 - » Winston, M.E. Chaffin, R., Herrmann, D. (1987) A taxonomy of part-whole relations. Cognitive Science 11 : 417-444
 - Part-of can be axiomatised, usually assumed to be transitive
 - It is a theory that applies to instances, not classes



Part-whole relations



- Winston's approach is linguistic, consider:
 - X is part of Y, e.g. "the head is part of the body"
 - Y is partly X, e.g. "bicycles are partly aluminium"
 - Attempting to answer questions such as why transitivity holds in some cases and not others:
 - John's finger is part of John's hand
 - John's hand is part of John's body
 - John's finger is part of John's body
- But:
- John's finger is part of John
 - John is part of the Philosophy Department
 - *John's finger is part of the Philosophy Department (*seems incorrect)



Part-whole relations



- Part-of is clearly different from subClassOf (the class inclusion relation)
 - Wing--Canary, cf. Canary--Bird
 - Both are needed
- Also differs from attribution
 - Yellow--Canary
- Winston's proposal:
 - A taxonomy of part-of relations
 - Properties that distinguish between the different senses of part-of
 - Analysis confirms that part-of is transitive



Part-whole relations



- Meronymic relations can be distinguished from other semantic relations by the 'common argument' criterion where the subject is the same but different questions are answered:

"pedals are parts of bicycles" &

"bicycles are partly aluminium"

(Contrast with "bicycles are vehicles")

can be re-phrased:

"bicycles have pedals" &

"bicycles are made of aluminium"

where the two statements describe a common subject, bicycles, giving two senses:

- Component-object (bicycle—pedal)
- Stuff-object (bicycle—aluminium)



Part-whole relations



- Considering: *objects, collections, masses, activities and areas* we get 6 senses:

Component - Integral Object	Handle - Cup Punchline - Joke
Member - Collection	Tree - Forest Card - Deck
Portion - Mass	Slice - Pie Grain - Salt
Stuff - Object	Gin - Martini Steel - Bicycle
Feature - Activity	Paying - Shopping Dating - Adolescence
Place - Area	Everglades - Florida Oasis - Desert



Part-whole relations



Three distinguishing properties of part-whole relations

- Functional**
 - The part bears a functional relation to the whole
 - Parts are restricted, by their function, in their spatial or temporal location
 - E.g. the handle of a cup can only be placed in a limited number of positions to function as a handle
 - (not functional in the mathematical sense)
- Homeomerous** [homos:same; meros:part]
 - Homeomerous parts are the same kind of thing as their wholes, e.g. slice-pie
 - Non-homeomerous parts differ from their wholes, e.g. tree-forest
- Separable**
 - Separable parts can, in principle, be separated from the whole, e.g. handle-cup
 - Inseparable parts cannot, e.g. aluminium-bicycle



Part-whole relations



Relation	Examples	Functional	Homeomerous	Separable
Component - Integral Object	Handle - Cup Punchline - Joke	Yes	No	Yes
Member - Collection	Tree - Forest Card - Deck	No	No	Yes
Portion - Mass	Slice - Pie Grain - Salt	No	Yes	Yes
Stuff - Object	Gin - Martini Steel - Bicycle	No	No	No
Feature - Activity	Paying - Shopping Dating - Adolescence	Yes	No	No
Place - Area	Everglades - Florida Oasis - Desert	No	Yes	No



Component - Integral



Component - Integral Object

- Integral objects show some kind of patterned organisation or structure. E.g. wheel-car, chapter-book, fridge-kitchen.
- Their components are also patterned and usually bear specific structural and functional relationships to each other, and to the wholes which they compose.
- Concrete objects can have components in this sense, e.g. cup, bicycle, as can
 - representational objects, e.g. books, plays, symphonies, and
 - abstract objects, e.g. linguistics, meaning, and
 - organisations, e.g. IBM, NATO.
- Physical objects are extensive (occupy a physical volume) and the components are included in the spatial volume of the whole.
- The parts of abstract and organisations are not extensively included in the whole, but belong in a non-physical sense. (grouped here because of their patterned organisation)

Functional=yes; Homeomerous=no; Separable=yes

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Portion - Mass



- Portions of masses, extensive objects, and physical dimensions are different from components and members in being homeomerous - having parts similar to each other and to the whole they comprise. E.g. slice-pie, yard-mile, hunk-clay
 - Every portion of a pie is 'pie', and is similar to other portions. In contrast, components and members may be dissimilar to each other and different from the whole.
- This sense can be indicated by "some of" test, replace "part of" with "some of"
 - "She asked me for part of my orange."
 - "She asked me for some of my orange."Compare with (component sense):
 - "The engine of part of the car."
 - **"The engine is some of the car."
- The "one of" test may be needed to differentiate portion-mass from collection:
 - Portions cannot be individuated without a unit of measure
 - **"...one of water/beer" (**incorrect)
 - test works, we need to say: "Give me a glass of water"

but

**"Some of the students are seniors."

"One of the students is a senior." (collection/count sense, sounds OK)

Functional=no; Homeomerous=yes; Separable=yes



Member - Collection



Member - Collection

- Membership does not require members to perform a specific function, or possess a structural arrangement to each other or to their wholes. E.g. tree-forest, juror-jury, ship-fleet.
- Collections are not classes
 - Class membership is determined by similarity
 - » In English: "is a" not "part of"
 - Membership of a collection is determined by spatial proximity (tree-forest) or social connection. These may be referred to as groups.

Functional=no; Homeomerous=no; Separable=yes

KMM ontology Lecture 8



Stuff-Object



Stuff-Object

- This sense is most often expressed by the 'is partly' frame (or phrase), e.g. "water is partly hydrogen".
- It can be distinguished from the component sense by the common argument criterion:
 - Bicycle - Aluminium ("is partly")
 - Wheel - Bicycle ("is part of")
 - What is it made of? Contrast with: What are its parts?
- This sense answers questions about constituency, for things with multiple constituents
 - "The lens is made of glass."
 - **"The lens is partly glass" (*incorrect, glass is the only part)
- The stuff sense can be difficult to distinguish, it can be argued that if a part cannot be separated without altering the identity (e.g. water-hydrogen) then this sense is indicated. In contrast, a bicycle without a wheel is still a bicycle.

Functional=no; Homeomerous=no; Separable=no

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Feature - Activity



Feature - Activity

- In this sense, part designates features or phases of activities or processes, e.g. paying-shopping, bidding-bridge.
- Cannot be expressed by “X has Y”
*“Shopping has paying.”
(compare: “Books have chapters.”)
- Otherwise, feature-activity is like component-integral in that complex activities are structured by means of scripts.

Functional=yes; Homeomerous=no; Separable=no



Place - Area



Place - Area

- This sense connects areas to special places and locations within them,
 - The baseline is part of the tennis court
 - Morningside is part of Edinburgh
- Like members, there is no functional contribution to the whole.
- Like masses, place-area is homeomerous (every place is like every other place, and like the whole).
- Unlike portions, places cannot be separated from the area they are part of.

Functional=no; Homeomerous=yes; Separable=no



Part-whole relations



Non-meronymic relations

- **Spatial inclusion:** The relationship between a container/area/ temporal duration and the thing contained in it
“The wine is in the cooler”
“The prisoner is in the cell”
- **Meronymic relations have additional elements**
 - The heart is surrounded by the body, but is part-of (component-integral) the cardiovascular system
 - Prior to 1989:
 - » West Berlin was surrounded by, but not co-extensive with East Germany: spatial inclusion not part-of
 - Now:
 - » Berlin is part-of (place-area) Germany
- **Class inclusion**
 - Membership of a class is determined by similarity on some intrinsic property: taxonomic (bird, flower); functional (toy); heterogeneous (vegetable, medicine)
 - “is a” “kind of”



Part-whole relations



Explaining transitivity - 2 syllogisms

John’s finger is part-of John’s hand
John’s hand is part-of John’s body
John’s finger is part-of John’s body
– Follows as the same sense of part-of is used throughout (component-integral)

But:

John’s finger is part-of John (component-integral)
John is part-of the Philosophy Department (member-collection)
*John’s finger is part-of the Philosophy Department

Mixing the senses of part-of means John’s finger does not belong to the Philosophy Department either as a component or as a member

John’s finger is component of John
John is a member of the Philosophy Department
*John’s finger is component/member of the Philosophy Department



Part-whole relations



More difficult cases:

- The refrigerator is part of the kitchen.
- The kitchen is part of the house.
- (?) The refrigerator is part of the house.

This does not sound quite right, again two senses are being used, firstly, component-object, secondly, place-area.

Consider:

1. Wings are parts of birds. (component-integral)
2. Birds are creatures. (class inclusion)

Two conclusions:

3. Wings are parts of creatures.
- *4. Wings are creatures.

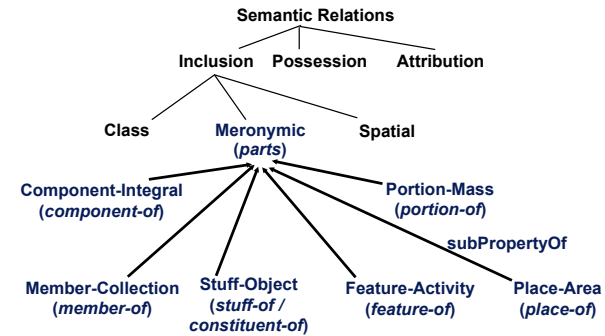
In mixed syllogisms, the conclusion is valid if it expresses the part-of relation (in a part-of/subclass syllogism), or the spatial inclusion relation (in a inclusion/subclass syllogism).



Part-whole relations



Winston's taxonomy of relations



parts cannot be transitive in this system of relations, subPropertyOf allows the inferences: component-of(a, b) & member-of(b, c) implies parts(a, b) & parts(b, c) Therefore if parts is transitive: parts(a,c) *incorrect



Part-whole relations



● Part-of and class membership

- Properly speaking, part-whole relations hold of instances
- “This wing is part of this canary.”
 - » Cf. Wing part of Canary
- “This wing is a component part of this canary.”

● Definitions of the Part class

- CarWheel:: Wheel and component-of only Car
- Heart:: Organ and component-of only CardiovascularSystem and component-of some CardiovascularSystem
- PizzaSlice:: portion-of only Pizza and portion-of some Pizza



Part-whole relations



● Definitions of the Whole Class

- Car:: Vehicle and has-component some Wheel and ...
- CardiovascularSystem:: OrganSystem and has-component some Heart and ...
- Pizza:: BakedProduct and has-portion only PizzaSlice
 - » This does not work!



Part-whole relations



- **Unity meta-property (from Dolce)**
 - Considers the way in which the parts relate to the whole
 - A functional relation indicates unity
 - A constituent-of/stuff-of relation indicates no unity

Consider Groups:

- **GroupOfPeople:: an assembly of 2 or more people**
 - Person member-of GroupOfPeople
 - » *person-instance-i...k member-of group-instance-1*
 - The class GroupOfPeople lacks unity as there is no functional link between its instances and their parts
 - Identity of the group-instance depends on its members
- **BoyBand:: an assembly of 2 or more young male musicians**
 - Person member-of BoyBand or
 - Person component-of BoyBand (a functional relation)
 - » The class BoyBand would have unity
 - » And the identity of its instances would survive the replacement of the components - i.e. the singers

Axioms for part-of



Axioms for parts and wholes

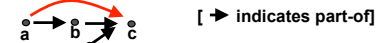
(Varzi, Data and Knowledge Engineering 20(3) :259-86 -read selectively)

Considering only a single sense of part-of P.

P(Part-instance, Whole-instance) Note the argument order

1. $\forall x \forall y \forall z P(x, y) \wedge P(y, z) \Rightarrow P(x, z)$ transitive
2. $\forall x P(x, x)$ reflexive
3. $\forall x \forall y P(x, y) \wedge P(y, x) \Rightarrow x=y$ antisymmetric

Part-of is a transitive relation (1) holding of instances (not classes).



Everything is part-of itself.



To say that a is part-of b and b is part-of a is to assert that a and b are the same thing.



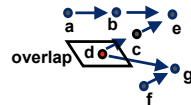
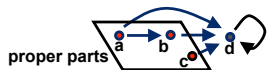
Axioms for part-of



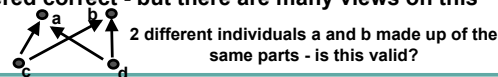
- D1. $\forall x \forall y PP(x, y) \equiv P(x, y) \wedge \neg P(y, x)$ PP :proper-part
- D2. $\forall x \forall y O(x, y) \equiv \exists z (P(z, x) \wedge P(z, y))$ O :overlap

Proper-parts are any sub-part excluding the whole

An overlap exists when two parts share a sub-part



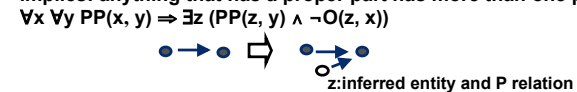
- A part-of structure cannot have cycles
- There need not be a single top node in the part-of structure
- Sums (x+y) and products (x•y) can be defined
- Mereology tries to constrain part-of structures to allow only those considered correct - but there are many views on this issue



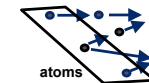
Axioms for part-of



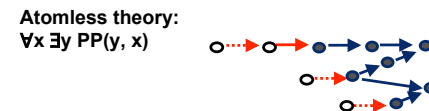
- 4. $\forall x \forall y \neg P(x, y) \Rightarrow \exists z (P(z, x) \wedge \neg O(z, y))$ Supplementation axiom
- Implies: anything that has a proper part has more than one proper part



Atomistic theory:
 $\forall x \exists y (P(y, x) \wedge \neg \exists z (PP(z, y)))$



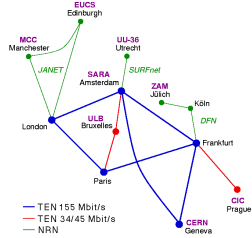
It follows (adding more axioms) that things built from the same atoms are identical...



Axioms for connected-to



- Wholeness is a global property, while parthood is a relational concept - every object may be part-of a greater whole.
- When a whole object is identified, its connectedness to other objects can be defined
- Topology
 - Derivation: topos: place [in Greek]
 - Represented by the relation: **C** connected-to



Computer networks have a topology:
 •With only a part-of relation, every node is part-of the 'network-as-a-whole'.
 •With connected-to, the nodes are independent wholes, with connections between some of them.



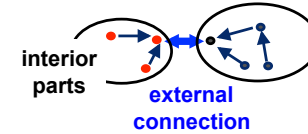
Axioms for connected-to



connected-to: **C**

- A. $\forall x C(x, x)$ reflexive
- B. $\forall x \forall y C(x, y) \Rightarrow C(y, x)$ symmetric
- C. $\forall x \forall y P(x, y) \Rightarrow \forall z (C(z, x) \Rightarrow C(z, y))$
 if x is a **part-of** y,
 then anything (z) connected-to x is connected-to y

- $\forall x \forall y EC(x,y) = C(x,y) \wedge \neg O(x,y)$ EC:external connection
- $\forall x \forall y TP(x,y) = P(x,y) \wedge \exists z (EC(z,x) \wedge EC(z,y))$ TP:tangential part
- $\forall x \forall y IP(x,y) = P(x,y) \wedge \neg TP(x,y)$ IP:internal part



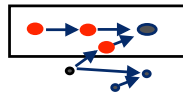
Mereotopology:
Axioms 1-3 + A-C



Parts and connections

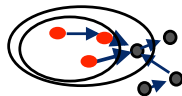


- Sums of parts that share a property can be defined



The sum of the red dots is the structure inside the box

- Interior parts and boundaries can be defined



- A complete theory of parts and connectedness can be constructed - mereotopology
 - Replace set theory as the basis of ontology!?



Summary



- Part-of has 6 senses
 - Distinguished by 3 meta-properties
- The formal theory of part-of is based on 3 axioms
 - Can be combined with 3 axioms for connected-to
- Thursday 10th Feb: Dr Jessica Chen-Burger

