Object Recognition General Overview

Several approaches to classification/recognition. Choose the same class as objects with:

- **Properties** - similar properties
- **Appearance** - similar pixel values
- **Geometric** - similar structures in similar places with similar parameters
- **Graph** - similar part relationships
- **Bag of Words** - enough similar descriptions

Object recognition key points

- Classification by comparing the relative probability of a shape belonging to different classes.
- Use Bayes rule to calculate the class probabilities.
- Class model is multivariate Gaussian distribution.
- Estimate the distribution parameters from the data.

The story so far...

Preprocessing:

1. Capture image
2. Threshold to isolate object
3. Locate binary region
4. Measure properties
   $\vec{x} = (compactness, ci_1, ci_2, ci_3, ci_4, ci_5, ci_6)'$

Probabilistic Object Recognition

$p(c|d)$ is the probability that c was the class given that we observed evidence d

We select most probable class c (*i.e.* $p(c|d)$ is the highest) or perhaps none if the probability for all classes is too low.

Computing $prob(c|d)$? Bayes Classifier

$p(c)$ is the *a priori* (before any observations) probability of observing class c

$p(d|c)$ is the probability that evidence d would have been observed if c was the class

Bayes rule:

$$p(c|d) = \frac{p(d|c)p(c)}{p(d)} = \frac{p(d|c)p(c)}{\sum_k p(d|k)p(k)}$$

Advantage: we learn $p(c)$ and $p(d|c)$ from examples

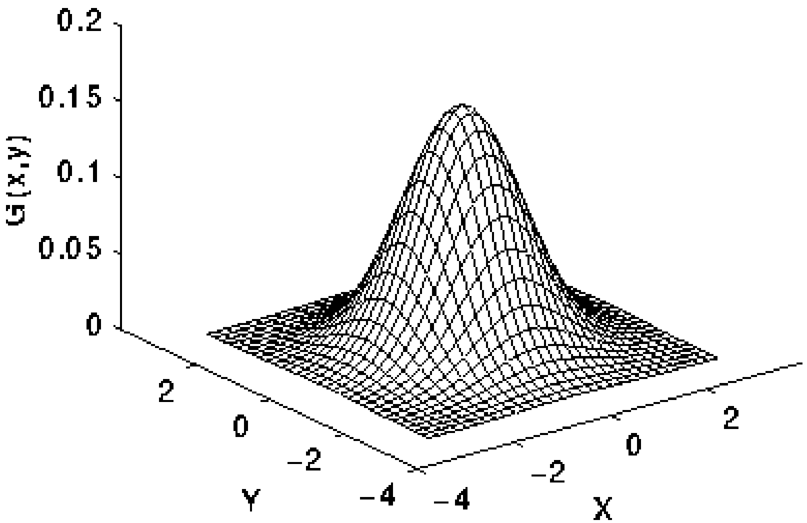
$prob(d|c)$? **Gaussian Distribution**

Data d is feature vector $\vec{x} = (f_1, f_2, \ldots, f_n)'$.

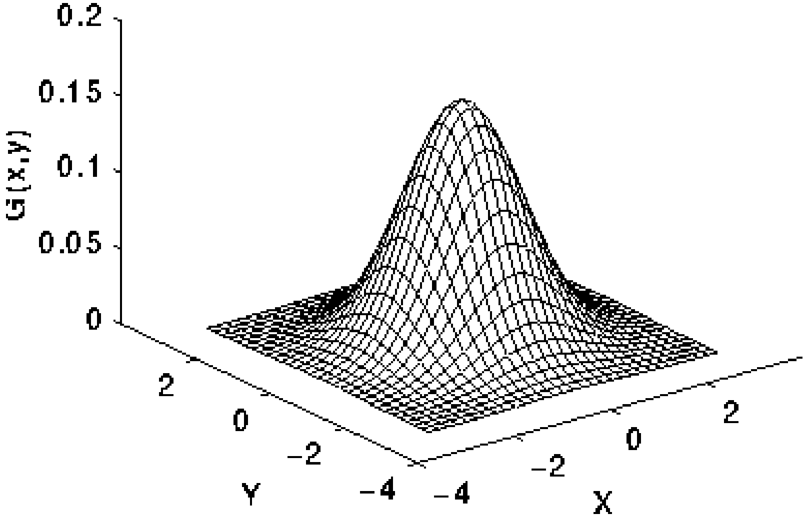
Expect some variation in property values, perhaps not independent between variables.

Commonly joint probability distribution of d is Multivariate Normal/Gaussian Distribution

For 2 properties, $\vec{x} = (f_1, f_2)'$ we have:



2D Gaussian Distribution



Characterised by mean $(m_1, m_2)'$ and covariance matrix

$$\begin{bmatrix} (\sigma_1)^2 & \rho_{ij}\sigma_1\sigma_2 \\ \rho_{ij}\sigma_1\sigma_2 & (\sigma_2)^2 \end{bmatrix}$$

σ_i - standard deviation of i^{th} property

ρ_{ij} - cross correlation coefficient between i and j

Multivariate Normal/Gaussian Distribution

For each class c need:

- Mean vector $\vec{m}_c$ of dimension n - the average value of the n properties for class c
- Covariance matrix $\mathcal{A}_c$ - the $n \times n$ matrix of joint variation between each pair of components of the vector.

Then, the probability of observing feature vector $\vec{x}$ is:

$$p(\vec{x}|c) = \frac{1}{(2\pi)^{\frac{n}{2}}} \frac{1}{det(\mathcal{A}_c)^{\frac{1}{2}}} e^{-\frac{1}{2}[(\vec{x}-\vec{m}_c)' \mathcal{A}_c^{-1} (\vec{x}-\vec{m}_c)]}$$

Estimating the Distribution Parameters - the Class Model

Given $k > n$ known instances of class c with properties $\{\vec{x}_i\}$.

Estimated Mean vector: $\vec{m}_c = \frac{1}{k} \sum_i \vec{x}_i$

Estimated Covariance matrix:

$$\mathcal{A}_c = \frac{1}{k-1} \sum_i (\vec{x}_i - \vec{m}_c)(\vec{x}_i - \vec{m}_c)'$$

Estimate $p(c)$ from the distribution of known samples

Probability Example

$n = 2$ classes. *a priori* probabilities p(1) = 0.6, p(2) = 0.4

Cls 1: $\vec{m}_1 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ $\mathcal{A}_1 = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ $\mathcal{A}_1^{-1} = \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

Cls 2: $\vec{m}_2 = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$ $\mathcal{A}_2 = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ $\mathcal{A}_2^{-1} = \frac{1}{5}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

$det(\mathcal{A}_1) = det(\mathcal{A}_2) = 5$

Data $\vec{x} = \begin{bmatrix} area \\ perimeter \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \end{bmatrix}$

Which is the most probable class?

Example continued

Class 1 if $p(1|\vec{x}) > p(2|\vec{x})$

Bayes rule:

$$p(1|\vec{x}) = \frac{p(\vec{x}|1)p(1)}{p(\vec{x})} > \frac{p(\vec{x}|2)p(2)}{p(\vec{x})} = p(2|\vec{x})$$

$$0.6 \times p(\vec{x}|1) > 0.4 \times p(\vec{x}|2)$$

$$p(\vec{x}|1) = \frac{1}{2\pi} \frac{1}{det(\mathcal{A}_1)^{\frac{1}{2}}} e^{-\frac{1}{2}[(\vec{x}-\vec{m}_1)' \mathcal{A}_1^{-1} (\vec{x}-\vec{m}_1)]}$$

$$= \frac{1}{2\pi}\frac{1}{5^{\frac{1}{2}}} e^{-\frac{1}{2}[(\begin{bmatrix} 3 \\ 10 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \end{bmatrix})' \mathcal{A}_1^{-1} (\begin{bmatrix} 3 \\ 10 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \end{bmatrix})]}$$

$$= \frac{1}{2\pi}\frac{1}{\sqrt{5}} e^{-\frac{1}{2}(\begin{bmatrix} 1 \\ 4 \end{bmatrix}' \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix})}$$

$$p(\vec{x}|1) = \frac{1}{2\pi}\frac{1}{\sqrt{5}} e^{-\frac{27}{10}} = 4.78 \times 10^{-3}$$

Similarly,

$$p(\vec{x}|2) = \frac{1}{2\pi}\frac{1}{\sqrt{5}}e^{-\frac{23}{10}} = 7.15 \times 10^{-3}$$

So, class 1 if

$$0.6 \times p(\vec{x}|1) > 0.4 \times p(\vec{x}|2)$$

$$2.87 \times 10^{-3} > 2.85 \times 10^{-3}$$

Thus most likely (barely) to be class 1.

Midlecture Problem

The data has a probability of 0.1 for class 1 and a probability of 0.2 for class 2. But, class 1 is 3 times as likely to be observed *a priori* as class 2.

Based on the two pieces of information, which is the most likely class to explain the observations?

Matlab code for probability calculation

```
function prob = multivariate(Vec,Mean,...
                             Invcor,apriori)
 diff = Vec-Mean;
 n = length(Vec);
 wgt = sqrt(det(Invcor));
 dist = diff*Invcor*diff';
 prob = apriori * ( 1 / (2*pi)^(n/2)) ...
         * wgt * exp(-0.5*dist);
```

Training

Calculating classifier parameters $p(c), \vec{m}_c, \mathcal{A}_c$

Should split data into:

- Training set - used to estimate parameters (eg. 50% of data)
- Validation set - used to tell when to stop training (eg. 25% of data)
- Test set - used to test performance (eg. 25% of data)

Must have more samples than properties!

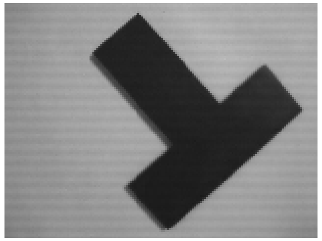
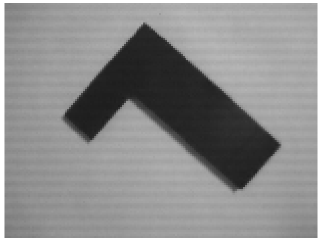
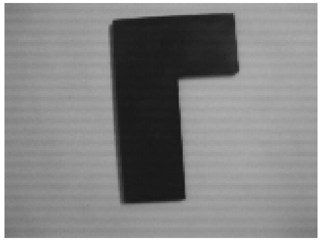
Training Example

For the batch-mode training used here, can merge Training and Validation sets.

In the example below, since we have only 4 samples of each class, we'll be naughty and use all 4 samples for both the Training and Test sets.

Can use at most 3 properties.

Test Objects

A:  B:  C: 

4 instances of each

a priori probability = 0.33

Training Code

```
Dim = 3;        % number of feature properties
modelfilename =
     input('Model file name (filename)\n?','s');
maxclasses = input('Number of classes (int)\n?');
trainfilestem = input('Training image file stem
                          (filestem)\n?','s');
N = input('Number of training images (int)\n?');
for imagenum = 1 : N
  currentimagergb = imread([trainfilestem,
              int2str(imagenum),'.jpg'],'jpg');
  currentimage = rgb2gray(currentimagergb);
```

```
  vec(imagenum,:) =
          extractprops(currentimage,0,0,0,0,0);
  trueclasses(imagenum) = input(['Train image ',
        int2str(imagenum),' true class (1..',
        int2str(maxclasses),')\n?']);
end
[Means,Invcors,Aprioris] = buildmodel(Dim,vec,N,
                         maxclasses,trueclasses);
eval(['save ',modelfilename,' maxclasses ...
                       Means Invcors Aprioris'])
```

Building statistical model

```
function [Means,Invcors,Aprioris] = ...
      buildmodel(Dim,Vecs,N,Numclass,Classes)
  for i = 1 : Numclass
    samples = find(Classes == i); % locate cls i
    M = length(samples); % num in class
    classvecs = Vecs(samples,:); % get members
    mn = mean(classvecs);
    Means(i,:) = mn;
    diffs = classvecs - ones(M,1)*mn;
    Invcors(i,:,:) = inv(diffs'*diffs/(M-1));
    Aprioris(i) = M/N;
  end
```

Training Log

```
doall(2)
Model file name (filename)
?blocks
Number of classes (int)
?3
Training image file stem (filestem)
?TESTDATA1/f
Number of training images (int)
?12
Train image 1 true class (1..3)
?1
***
```

Test Code

```
eval(['load ',modelfilename,...
        ' maxclasses Means Invcors Aprioris'])
imagestem = input('Test image file stem ...
                    (filestem)\n?','s');
run=1;
imagenum=0;
while ~(run == 0)
  imagenum = imagenum + 1;
  currentimagergb = imread([imagestem, ...
            int2str(imagenum),'.jpg'],'jpg');
  currentimage = rgb2gray(currentimagergb);
  vec = extractprops(currentimage,0,0,0,0,0);
```

```
  class=classify(vec,maxclasses,Means,Invcors,...
       Dim,Aprioris)
  run = input(['Want to process another image ',
               int2str(imagenum+1),' (0,1)\n?']);
end
```

Recognition Performance

Confusion matrix:

		Computed Class		
		A	B	C
True Class	A	4	0	0
	B	0	3	1
	C	0	0	4

Discussion

- Simple process but well founded
- Works well because reliable feature extraction and features easily discriminate
- Choosing good features usually hardest part.

What We Have Learned

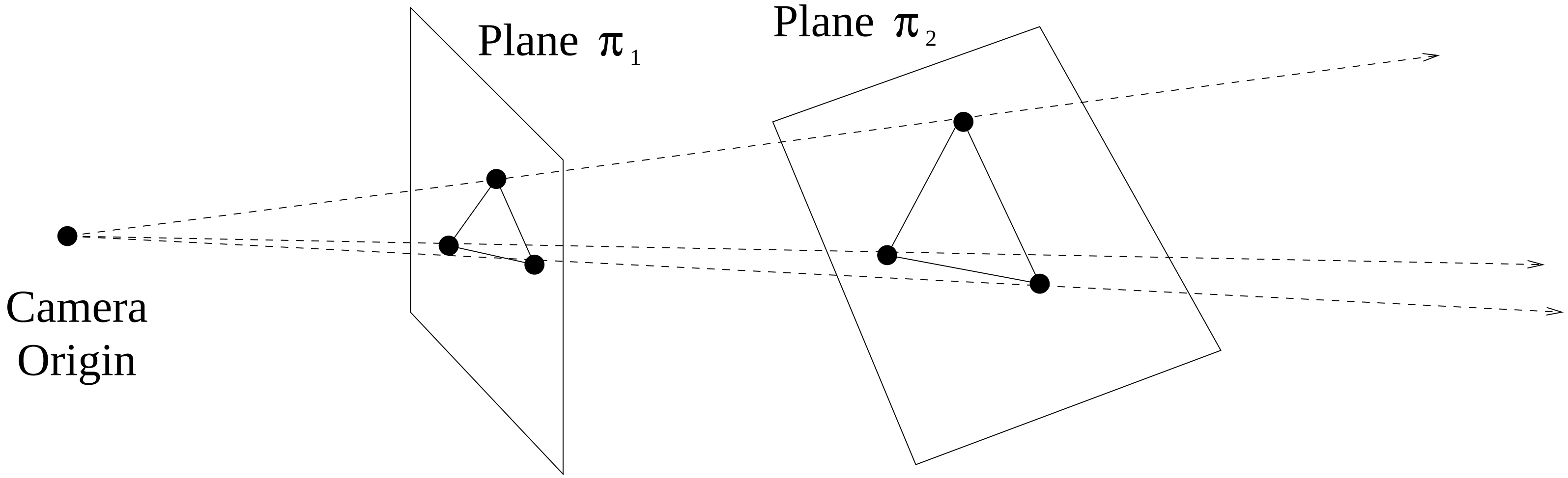
1. Bayes classifier

2. Multi-variate Gaussian distributions

3. Property-based recognition

Projective Geometry I

Projection: Any non-singular (ie. invertable) linear transformation $\mathbf{P}$ that maps points from one position to another



Plane π_2 shapes observed in a 3D position project onto image plane π_1 (2D $\rightarrow$ 2D)

$\mathbf{P}$: $\pi_2 \rightarrow \pi_1$

Projective Geometry II

3D→3D, 1D→1D also possible

Accounts for rotation, translation, scale, shear.

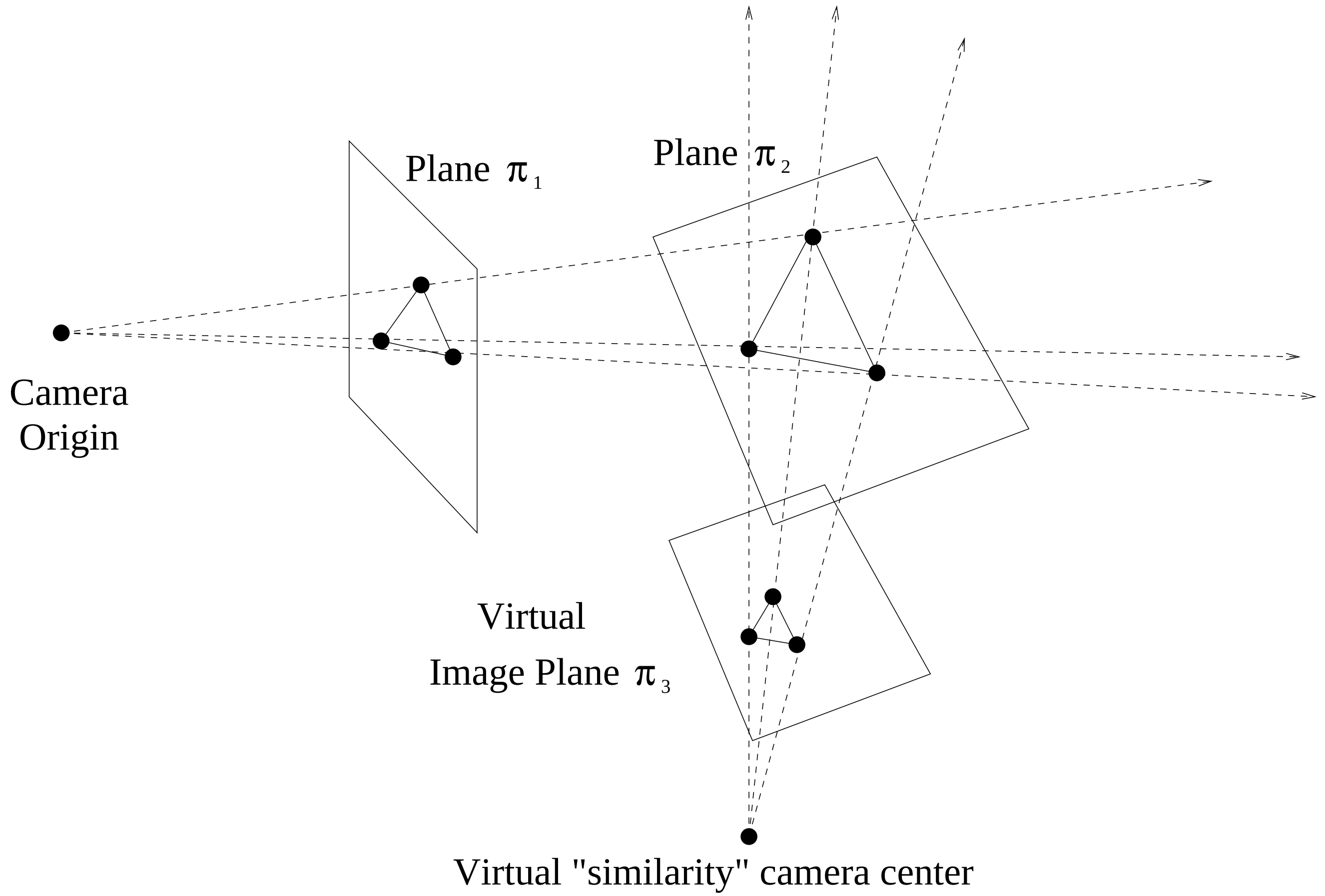
To do properly, use homogeneous coordinates

Augment point positions $(x, y)'$ to $(x, y, 1)'$

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Projective Transfer

Can use projective transfer to map observed projective image to another view



Use homogeneous coordinates

$\mathbf{P}_1$: $\pi_2 \rightarrow \pi_1$ (ie. copy scene plane into image plane)

$\mathbf{P}_2$: $\pi_2 \rightarrow \pi_3$ (ie. copy scene plane into new plane)

Therefore $\mathbf{P}_3 = \mathbf{P}_2\ (\mathbf{P}_1)^{-1}$: $\pi_1 \rightarrow \pi_3$

Projection matrix $\mathbf{P}_i$:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

Remapping Algorithm

Input image $I(x, y)$

Remapped image $R(u, v)$

If projective relation $\mathbf{P}$ between planes known, then can map (u, v) onto (x, y) using:

$$\begin{pmatrix} \lambda x \\ \lambda y \\ \lambda \end{pmatrix} = \mathbf{P} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

for each (u, v)

get (x, y) from projection divided by λ

$R(u, v) = I(x, y)$

See `remap.m`

Estimating P:$(u,v) \rightarrow (x,y)$

Direct Linear Transform Method

$N \geq 4$ matched points: $\{((x_i, y_i), (u_i, v_i))\}_{i=1}^{N}$

Let $\vec{p}' = (p_{11}, p_{12}, p_{13}, p_{21}, p_{22}, p_{23}, p_{31}, p_{32}, p_{33})'$

Let $\mathbf{A}_i =$

$$\begin{bmatrix} 0 & 0 & 0 & -u_i & -v_i & -1 & y_i u_i & y_i v_i & y_i \\ u_i & v_i & 1 & 0 & 0 & 0 & -x_i u_i & -x_i v_i & -x_i \end{bmatrix}$$

Estimating P cont.

Construct $\mathbf{A} = \begin{bmatrix} A_1 \\ A_2 \\ \ldots \\ A_N \end{bmatrix}$

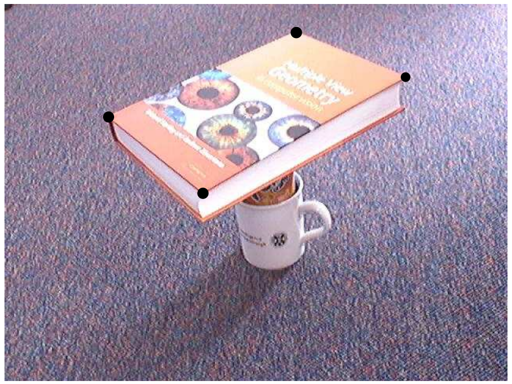
Compute SVD($\mathbf{A}$) $= \mathbf{UDV'}$

$\vec{p}$ is last column of $\mathbf{V}$ (eigenvector of smallest eigenvalue of $\mathbf{A}$)

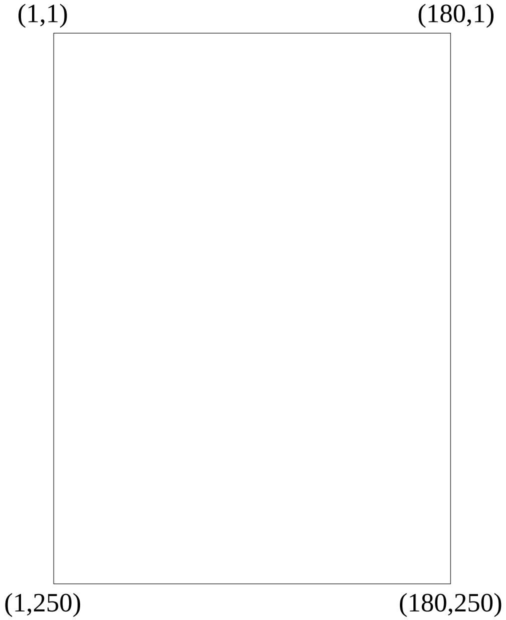
Repack $\vec{p}$ back into matrix $\mathbf{P}$

See `esthomog.m`

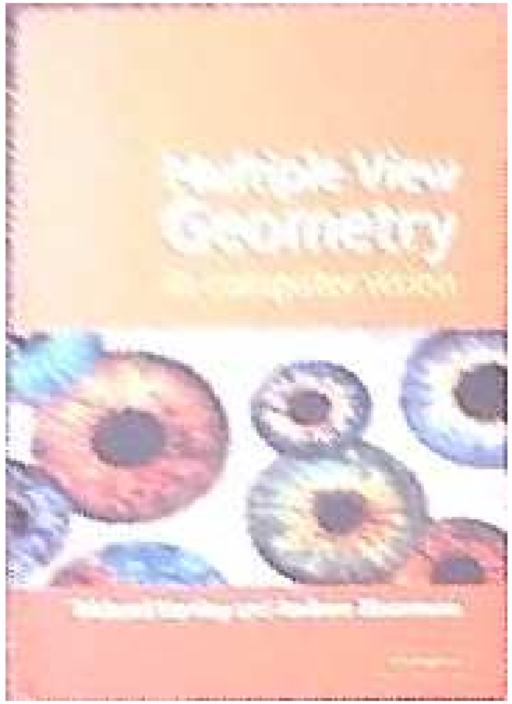
Projective Transfer Example



ORIGINAL



TARGET



REMAPPED