Introduction to Theoretical Computer Science

Tutorial Sheet – Week 5 tutorials

Here are some fairly easy and routine questions:

(1) Let *Even* be the decision problem $(\mathbb{N}, \{n : n \text{ is even}\})$ and *Odd* be $(\mathbb{N}, \{n : n \text{ is odd}\})$. Give m-reductions between the two problems.
(2) We remarked that (computable) predicates are (computable) functions. A function $f : \mathbb{N} \to \mathbb{N}$ is also a predicate: its *characteristic predicate* $\chi_f(x, y)$ is true iff $y = f(x)$. Show that f is computable iff χ_f is computable.
(3) Suppose that Q_1 and Q_2 are semidecidable queries over the same domain D. Show that $Q_1 \cup Q_2$ and $Q_1 \cap Q_2$ are semidecidable.
(4) Suppose that L is a decidable language over some alphabet Σ. Show that the language L^* is decidable.

And here are some more challenging questions:

(5) Following on from question 2: Given a computable predicate $\psi(x, y)$, is it decidable whether ψ is the characteristic predicate of some function f?
(6) (These are quite hard, even with the hints.)

Let P and Q be two disjoint co-semidecidable queries over D. We say that the query R *separates* P and Q if $P \subseteq R$ and $Q \subseteq D \setminus R$. Show that there is a decidable query R that separates P and Q. (Hint: run two machines in parallel.)

If P and Q are disjoint semidecidable queries, they are not necessarily separable by a decidable set. Can you come up with an example? (Use diagonalization: consider predicates given themselves as input.)