Introduction to Theoretical Computer Science

Tutorial Sheet 2

The following questions/comments are intended as prompts for discussion. Of course, you can ask/discuss about anything. Some of these topics we’ve touched on in discussion in lectures – this is an opportunity to think about them a bit more.

(1) We remarked that (computable) predicates are (computable) functions. A function $f : \mathbb{N} \to \mathbb{N}$ is also a predicate: its characteristic predicate $\chi_f(x, y)$ is true iff $y = f(x)$. Show that $f$ is computable iff $\chi_f$ is computable.

Given a computable predicate $\psi(x, y)$, is it decidable whether $\psi$ is the characteristic predicate of some function $f$?

(2) Suppose that $Q_1$ and $Q_2$ are semidecidable queries over the same domain $D$. Show that $Q_1 \cup Q_2$ and $Q_1 \cap Q_2$ are semidecidable.

Suppose that $L$ is a decidable language over some alphabet $\Sigma$. Show that the language $L^*$ is decidable.

(3) (These are quite hard, even with the hints.)

Let $P$ and $Q$ be two disjoint co-semidecidable queries over $D$. We say that the query $R$ separates $P$ and $Q$ if $P \subseteq R$ and $Q \subseteq D \setminus R$. Show that there is a decidable query $R$ that separates $P$ and $Q$. (Hint: run two machines in parallel.)

If $P$ and $Q$ are disjoint semidecidable queries, they are not necessarily separable by a decidable set. Can you come up with an example? (Use diagonalization: consider predicates given themselves as input.)