Information Theory — Tutorial 6

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This is the last tutorial sheet. Officially, the course has six tutorials:
`http://www.drps.ed.ac.uk/13-14/dpt/cxinfr11087.htm`

If you'd like a question-answer session later in the semester, let me know. I won't be in Edinburgh, or have regular access to email from 9 December 2013. However, I am keen to help motivated, organized students who contact me in good time.

1. **Communication channels:** Find the mutual information for the binary symmetric (BSC), binary erasure (BEC), and Z channels as a function of the input distribution. Maximize the mutual information to find the capacity of each channel. This question can be seen as a combination of several exercises in MacKay chapter 9. If this question is not straightforward, work through §9.4–5, pp. 148–151.

2. **The erasure channel and feedback:** Assume we want to send a very large K-bit source file over a binary erasure channel. We would like to approach the theoretical minimum number of uses of the channel: $K/C = K/(1-f)$.

   Does feedback improve this bound? Assume that the K bits are passed unencoded into the channel. On average, how many bits will not be received? If the receiver uses a noiseless feedback channel to ask for retransmission of those bits, how many bits will still need retransmitting? If the receiver keeps asking for any unknown bits to be retransmitted, how many uses of the channel are required on average to receive the whole file?

3. ***Optional,* data processing inequality:** MacKay Ex. 8.9, p141. MacKay's answer, which is the standard approach also found in Cover and Thomas, uses technology we haven't covered in the lectures. It is possible to prove the inequality, albeit less neatly, using only methods we have covered. One strategy is given at the bottom of the page.

Hint for the optional question: one relatively brute force way of showing the data processing inequality is to average $D_{\mathrm{KL}}[P(w\,|\,d)\;||\;P(w\,|\,r)]$ under $P(d,r)$.