Information Theory — Tutorial 2

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October 8, 2010

1. **Entropy separability:** MacKay's book Exercise 4.2, p68.

2. **Entropy decomposition:** MacKay's book Exercise 2.28, p38.

3. **Symbol code statistics:** MacKay's book Exercise 5.31, p104.
   (The ensemble X and code C_3 were defined on pp. 92–93: symbols with probabilities $\mathcal{P}_X = \{1/2, 1/4, 1/8, 1/8\}$ are encoded with codewords $C_3 = \{\texttt{0}, \texttt{10}, \texttt{110}, \texttt{111}\}$.)

4. **Three questions on Huffman coding:**

   MacKay's book Exercise 5.21, p102.
   Notation: "codes for X^n" means codes for blocks of n concatenated symbols.
   For example, when coding X^2 the *source* alphabet is $\mathcal{A}_{X^2} = \{\texttt{00}, \texttt{01}, \texttt{10}, \texttt{11}\}$.

   MacKay Exercise 5.26, p103.

   MacKay Exercise 5.29, p103.

5. **Real-valued variables *(Optional)*:** We have focussed on discrete random variables, X, taking on values $\{x_i\}_{i=1}^{I}$. When compressing many observations of such a variable, the number of bits/symbol required is given by the entropy:

$$H(X) = \sum_i P(x_i) \log \frac{1}{P(x_i)}.$$

   How many bits/outcome are required to encode draws from a unit Gaussian distributed variable, which has probability density $p(x) = \exp(-x^2/2)/\sqrt{2\pi}$? How many bits on average would be required to store the answers to 3 decimal places?