Ready Reckoners

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Multiplication is vital to the conduct of commerce but is time-consuming and error-prone. Among the many aids developed over two centuries to address these issues, the most widely used was the ready reckoner, a printed book containing tables of precalculated multiplication results that could be useful in business transactions.

Multiplication was for many centuries regarded as a laborious chore, albeit vital to conduct commerce. It is only in the past 35 years that we have had electronic calculators capable of multiplying numbers comprising eight or more significant figures, and even these cannot directly perform calculations in nondecimal units.

From the 17th century on, people applied all kinds of ingenuity to the creation of aids for multiplication. The best-remembered mechanical devices are slide rules, stepped drum machines, pinwheel machines, and keyboard machines such as the Comptometer that could multiply by repeated addition.

Almost forgotten now are the printed aids, the ready reckoners. These printed books contained tables of precalculated results of all kinds of multiplication useful for commerce, principally price per unit times units, or price per pound weight times pound weight. The multiplications were also used for calculating wages and interest.

All aids for commerce had to combine high precision, speed, and accuracy with the ability to handle nondecimal measures and currencies. Aids also had to be cheap enough to be bought for everyday use by clerks and shop assistants.

For commercial use, ready reckoners were far more important in terms of volume sales than all the mechanical devices put together. This was because ready reckoners outperformed the other devices on the key factors in the buying decision:

- price,
- number of significant figures,
- accuracy,
- ability to handle nondecimal multiplications,
- time to learn how use the machine, and
- time per calculation.

The term *ready reckoner*, according to the *Oxford English Dictionary*, was first used by Daniel Fenning in 1757 as the title of his publication *The Ready Reckoner; or Trader's Most Useful Assistant*."[1] However, there were a number of earlier, similar books such as William Leybourne's *Panarithmologia: or, the Trader's Sure Guide*,"[2] which in later editions adopted Fenning's term, becoming *The Ready Reckoner: or, the Trader's Sure Guide*. However, the earliest known such publication may be due to mathematician Simon Stevin who was born in Bruges, Flanders (now Belgium), in 1548. According to Oystein Ore, "… shortly before the appearance of *Le Disme* [The Decimal System] in 1585 Stevin had computed and published a set of interest tables."[3] An Englishman, William Webster, published a book of tables for simple and compound interest and whose third edition appeared in 1634.[4] Figure 1 shows the front page of—and Figure 2, a compound interest table from—his book. In France, François Barrême's published *Le Livre des Comptes Faits*, whose third edition appeared in 1673.[5] Figure 3 shows the charming frontispiece of a later edition of this book, which was clearly targeted at the retail trade. It was still being reprinted in 1862, making it among the longest-lasting reckoners.

Despite their long history, little was written about these aids until the recent 2003 book *History of Mathematical Tables,* edited by Martin Campbell-Kelly,[6] which devoted just two pages to ready reckoners. Earlier standard works such as the catalog by David Eugene Smith[7] are of antiquarian rather than commercial interest, but Smith does reprint the list of books from *Arithmetical Books from the Invention of Printing to the Present Time* by Augustus De Morgan, 1847,[8] which is of some relevance because it lists a wide range of different books including ready reckoners.

Other books and articles about mathematical tables exist, but most of them ignore ready reckoners as not being of interest to mathematicians working with logarithms, trigonometry, gamma

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WEBSTERS
TABLES,

For { Simple Intereſt direct, at 10, 8, 7½, 7, 6, and 5 l. *per Centum*, from moneth to moneth to twelve moneths.
Simple Intereſt to rebate, at 8 l. *per Centum*, from moneth to moneth to 36 moneths.
Simple Intereſt to rebate at 7 and 6 l. *per Centum*, from moneth to moneth to twelve moneths. }

ALSO,
His Tables for Compound Intereſt, with his true valuation of Annuities, Leaſes, Fines and Reverſions: With a neceſſary Addition concerning halfe-yearly and quarterly payments.

TOGETHER WITH
A neceſſary Table for the ſpeedy and exact ſumming up of the price of Commodities: ſerving alſo for a Table of Reduction, and for ſimple Intereſt direct at 5 *per Centum*.

The third Edition, with very large Additions,
BY
WILLIAM WEBSTER.

LONDON,
Printed by *M. Fleſher* for *Nicolas Bourne* at the South Entrance of the *Royall Exchange*. 1634.

Figure 1. Front page from Webster's reckoner, *Webster's Tables,* 1634. (Source: British Library.)

Compound Intereſt direct at 8 per Centum.

The yearly increaſe of 10 l, wherby may be found the like increaſe of any ſumme to 31 yeares, reckoning Intereſt upon Intereſt at 8 *per Centum*, neceſſary for al ſuch as put forth, or forbeare any ſum of monie, for any yeares aforeſaid. *Fractions are part of pence.*

yeares	l s d	yeares	l s d
1	11.13.0	17	36.19.11 $\frac{22}{25}$
2	11.13.3 $\frac{2}{25}$	18	39.19.2 $\frac{7}{25}$
3	12.11.11 $\frac{7}{25}$	19	43.03.1 $\frac{13}{25}$
4	13.12.1 $\frac{4}{25}$	20	46.12.2 $\frac{3}{25}$
5	14.13.10 $\frac{9}{25}$	21	50.06.9
6	15.17.4 $\frac{12}{25}$	22	54.07.3 $\frac{12}{25}$
7	17.02.9 $\frac{4}{25}$	23	58.14.3 $\frac{7}{25}$
8	18.10.2 $\frac{5}{25}$	24	63.08.2 $\frac{14}{25}$
9	19.19.9 $\frac{14}{25}$	25	68.09.8 $\frac{1}{25}$
10	21.11.9 $\frac{7}{25}$	26	73.19.2 $\frac{23}{25}$
11	23.06.3 $\frac{21}{25}$	27	79.17.7
12	25.03.7 $\frac{21}{25}$	28	86.05.4 $\frac{17}{25}$
13	27.03.11	29	93.03.5 $\frac{1}{25}$
14	29.07.5 $\frac{4}{25}$	30	100.12.5 $\frac{23}{25}$
15	31.14.05 $\frac{2}{25}$	31	108.13.5 $\frac{21}{25}$
16	34.05.2 $\frac{23}{25}$		

Figure 2. Compound interest table from *Webster's Tables,* 1634. (Source: British Library.)

Figure 3. Frontispiece of François Barrême's book, *Le Livre des Comptes Faits,* 1762. (Source: Authors' collections.)

functions, and the like. James Glaisher's well-known paper on mathematical tables is 175 pages long. It does include a handful of multiplication tables, but on page 2 he writes:

> Life-assurance and annuity tables will be excluded. With regard to these last however, although all tables such as ready reckoners and common interest tables will in general be omitted, any one that is of value in relation to mathematics as a science will be included.[9]

Table 1. Examples of nondecimal measures used in Great Britain and the US.

Unit of Measure	Country of Origin	Unit-of-Measure Details
Currency	Great Britain	4 farthings = 1 penny (d); 12d =1 shilling (s); 20s = 1 pound (£)
Length	Great Britain	12 inches = 1 ft.; 3 ft. = 1 yard; 22 yards =1 Gunters Chain; 80 chains = 1 mile = 1,760 yds.
Area	Great Britain	9 sq. ft. = 1 sq. yd.; 4,840 sq. yd. = 1 Acre
Area	United States	9 sq. ft = 1 sq. yd.; 4,840 sq. yd. = 1 acre; 640 acres = 1 sq. mile = 1 section; 36 sections = 1 township
Weight	Great Britain	16 ounces (oz.) = 1 pound (lb.); 14 pounds = 1 stone; 2 stones = 1 quarter (qtr.); 4 qtrs. = 1 hundredweight (cwt); 20 cwt = 1 ton = 2,240 lbs.
Weight	United States	16 ounces (oz.) = 1 pound (lb.); 100 pounds = 1 hundredweight (cwt); 20 cwt = 1 ton = 2000 lbs.

For a comprehensive historical review of nondecimal measures, see Klein.[12] *Also see Frank Tapson's dictionary of units.*[13]
Notes:
—US usage of cwt and ton differs from Great Britain.
—There were many nondecimal measures in British India, and a number of reckoners were published to handle these.

Many years later, the 1946 *Index of Mathematical Tables* by A. Fletcher, J.C.P. Miller, and L. Rosenhead[10] also omits mention of ready reckoners.

Ready reckoners, however, have a place in history, and we review that history in this article. We also position the reckoners in the context of other contemporary calculation aids available to businesspeople.

Complex mix of commercial requirements for multiplying aids

The preface to the first edition in 1874 of the *National Ready Reckoner*,[11] as reproduced in the 1959 edition, summarizes the compiler's priorities as "perfect accuracy in all calculations, simplicity of arrangement, and such fullness of detail as to meet the wants of all business requirements." To provide such calculations, a multiplying aid—or ready reckoner—required that it handle nondecimal units; precisely and accurately calculate up to six figures; be of a relatively low cost; and be easy to learn and use.

Ability to handle nondecimal units

As Table 1 shows, nondecimal systems were a fact of life for all businessmen in Great Britain and the US, since most American measures are still largely the old British predecimal ones. (We use "Great Britain" chiefly when referring to pre-1900 and "UK" for later years.)

Decimalization was achieved more slowly in the UK than in Europe. For example, France adopted metric units for length, weight, and liquid measures in 1801, while the UK did so only in 1964. The US has yet to adopt the metric system. In Europe, the UK, and the US, units of time are still computed sexagesimally.

It might seem that calculating aids would be most needed in countries with nondecimal systems. However, the calculation of both wages and interest payments involve nondecimal measures because of the use of years, months, days, hours, and minutes. So to a considerable extent, businessmen in the US had the same needs as those in Great Britain to handle many nondecimal calculations.

Many reckoners included conversion tables and sets of constants to help users convert between nondecimal measures such as weight, time, and distance into decimal fractions of a large unit such as a ton, hour, or mile as appropriate. This facilitated multiplication by converting all the amounts into decimal notation. A consequence of regular use of conversion tables was that clerks and staff came to carry relatively obscure decimal fractions and constants in their heads.

Six-figure precision and accuracy

Precision is the number of significant figures in the result of a calculation. Commercial aids had to be precise in their results to the lowest unit of currency (the amount of payment to settle a transaction): a farthing in Great Britain or a cent in the US.

One farthing in £1000 was one part in 1,000 $\times$ 240 $\times$ 4 $\approx$ 1,000,000 as is 1 cent in \$10,000. This is six significant figures. Ready reckoners delivered this precision, which is several orders of magnitude greater than anything needed in the calculations for 19th-century engineering.

Accuracy in this context meant that every

Table 2. Schematic part of a steel warehouse invoice with eight multiplications.

Item	Length	Width	Material	Area in Square Inches	Weight per Inch or Square Inch	Weight T C Q lbs*	Price per ton in £sd	Value £sd
	A	B	C	D = A × AB or π × A × A/4	G Data	H = D × G or A × G	I Data	J = H × I
1	3 ft. 6 in.	4 ft. 8 in.	1/4″ Mild steel plate	**Product 1**	W	**Product 3**	P	**Product 6**
2	8 ft. 6 in.		Rolled steel joist		W	**Product 4**	P	**Product 7**
3	5 ft.	Circle	1/4″ Mild steel plate	**Product 2**	W	**Product 5**	P	**Product 8**
						Sum of weights		Sum of lines

**TCQ lbs: Tons Hundredweights (abbrev. to Cwt) Quarters Pounds (lbs.) —28 lbs = 1 Qtr, 4 Qtrs = 1 Cwt, 20 Cwt = 1 ton.*

digit in the result had to be correct, but even this is a slight oversimplification. The businessman wanted to know that in calculating each line of an invoice, the right price had been multiplied by the right quantity, and that the correct answer had been entered in the right place. For a discussion of the use of check figures to ensure the accuracy of addition and multiplication in commerce and a classification of errors, see Williams.[14]

Low price

Most calculations in commerce were done by low-paid clerks and shop assistants. Any mechanical aid to help them had to be cost-effective. A US clerk's wages were about $800 a year in 1920[15] yet a Comptometer, for instance, cost $300 to $400.[16] Its annual cost would have been about one-fifth of the capital cost, say $80 a year, or roughly 10 percent of a clerk's wage. If a company employed only one or two clerks, there would be no opportunity to reduce staff and make savings until the company was big enough to employ a pool of, say, eight full-time Comptometer operators to do the work previously done by nine or ten clerks. This would have limited the sales of such equipment to larger companies. Worldwide sales of Comptometers between 1920 and 1926 were about 8,000 a year.[17] Ready reckoners, on the other hand, were inexpensive and affordable compared to mechanical devices, and they required no training.

Easy to learn and use

Businesses wanted aids that could be used by employees with minimal education and lacking special training, primarily because special training in the use of machines could be expensive, although details are sparse. An advertising feature for training Comptometer operators, included on a Web site devoted to the Sumlock Comptometer Company[18] says that in 1967 courses to train Comptometer operators lasted 15 weeks and cost 35 guineas (£36.75)—or approximately $101—in London and 30 guineas (£31.50) outside London. A course lasting more than three months and at such a high cost, in addition to the cost of the machine, would have been an intolerable burden for smaller companies. Unlike mechanical aids, which required at least some training, ready reckoners were immediately accessible to anyone who knew how to read and do simple calculations.

Multiplication's importance in business

Much attention has been given recently to the importance of adding machines in business.[16,19] However, before listing sums of money owed on an invoice, for example, or creating columns of debits and credits on a financial report, an employee would have to multiply a price by a quantity to obtain the amount of a transaction.

Consider what is involved in a typical single transaction, which Table 2 represents diagrammatically. This figure shows part of an invoice from a steel warehouse regarding the sale of a square of steel plate, a length of steel joist, and a circle of steel plate. First, the area of the pieces of plate had to be worked out, and from this the weight could be derived. The weight of the joist was looked up in a table per linear foot. All these weights then had to be multiplied by the price per ton to give the line value. Consequently, this one invoice involved eight nondecimal multiplications, averaging two to three per invoice line. The total weight was later reconciled with the weigh ticket for

Table 3. Comparison of adding versus multiplication time.

Function	Sterling	Decimal
Add column of 150 entries	152 seconds (secs)	118 seconds (secs)
Average time per item	1 sec	0.80 sec
Add 200 stock sheet entries	29 minutes, 37 secs	16 minutes, 50 secs
Average time per item	88 secs	55 secs
Ratio multiply/add	88 secs	69 secs

the total truckload. Note that Item 3 involves not one multiplication but the squaring of a number, its multiplication by π, with the result being divided by 4. In addition, although the length is shown in feet and inches, someone would first have to have mentally converted this into inches.

Typically, an invoice averaging 10 lines might involve 30 multiplications and two additions. Multiplication is much more time-consuming than addition. Multiplying 4 tons, 8 hundredweight, 2 quarters, and 6 pounds by £25, 8 shillings, 6 pence (£25.8s 6d) per ton by hand would have taken minutes rather than seconds. However, clerks could add a column of, say, 20 pound-shilling-pence (£sd) amounts in a few seconds.

An anecdote will help illustrate. The 1,000-employee family business where author Williams worked in the 1950s employed between six and ten Comptometer operators extending invoices in £sd and tons, hundredweights, quarters, and pounds. The company employed about six bookkeepers for the ledgers. Another three to four people worked out wages and salaries, which was essentially a multiplication task. The company also employed half a dozen estimators working out prices, much of the work being multiplication. Roughly, then, there were 15 "multipliers" to 6 "adders." So it is not surprising that companies were willing to spend more money on multiplying aids than on simple adders.

A more scientific comparison of multiplication versus addition forms a part of a study that was made in 1960 in Australia by professional accounting bodies which timed additions and multiplications in sterling and decimal. These tests, reported in *Office Magazine,*[20] were made using "key-driven adding–calculating (non-listing) machines." Table 3 shows that they found that multiplication was 88 times longer than adding in sterling.

Range of aids available

Businesses could choose a calculation aid from a range of tools, both nonmechanical[21] and mechanical.[16]

Nonmechanical aids

The abacus was widely used in the Far East and in Russia until the 1990s. However, although widely used in the West until 250 years ago, its use is not even a folk memory, surprisingly even in countries using largely decimal units.[22] And yet for most of the 19th and 20th centuries it was by far the cheapest, fastest, and most precise device available for decimal calculations. On at least two occasions that have been documented, the abacus has beaten the performance of both an electrical calculator[23] and an electronic computer.[24]

Tables of logarithms[25] were produced from 1614 to the 1960s, when cheap electronic calculators became available. However, the commonly available four-figure tables lacked the precision needed for commerce, having only four significant figures. For nondecimal amounts, users had to convert them to decimal, carry out the multiplication using logarithms and anti-logarithms, then convert the answer back to nondecimal. The most practical way to do these conversions to and from decimals was to use a ready reckoner.

Tables of quarter squares were printed from 1817 to 1933. These little-known devices were used in business and were possibly more useful than log tables but still only worked for decimals. They work on the following principle:

$(a + b)^2 = a^2 + 2ab + b^2$ and
$(a - b)^2 = a^2 - 2ab + b^2$

By subtracting these two expressions, the product $ab = 1/4$ of $(a + b)^2 - 1/4$ of $(a - b)^2$. Thus, to multiply a by b, we'd look up the quarter square of $a + b$ and subtract the quarter square of $a - b$. The calculation is claimed to be simpler than using logarithms, because we need only look up two numbers.[26,27]

Slide rules, mass-produced and reasonably priced,[28] were available from about 1880. However, their precision was even less than log tables. There were all sorts of attempts to increase the precision with longer scales.[29] In the grid-iron types, the scale was cut up into a number of strips. In the helical devices, the scale was wrapped around a cylinder, to give a length up to 500 inches and a claimed accuracy of five significant figures. Finally, there were the bulky and expensive 500-inch Fuller machines,[30] although they could not readily handle nondecimal systems. In writing about the history of slide rules, Peter Hopp notes that "at approx. £5/10s/0d in 1938, it was very expensive, and it is doubtful whether many private individuals had the means to afford one."[31]

Table 4. **Characteristics of different classes of aids. Prices are converted into sterling at the contemporary exchange rate and the price index applied to convert to 1914 £. Price (inflation) indices began at the start of World War I to prevent profiteering.**

Class	Nondecimal Capability (Yes/No)	Significant Figures	Typical Number of Days to Learn the Device's Operation	Time to Perform Calculations (in seconds)	Value in 1914 pounds (£1914)
Abacus	No	12	Japanese children take several years to learn	10	£0.20
Pinwheel	No	10	2	12	£50.00
Step drum (mechanical multiplier)	No	16	2	20	£25.00
Repeat add (Comptometer type)	No	12	5	8	£150.00
Ready reckoner	**Yes**	**6–10**	**0.1**	**10**	**£0.10**
Slide rule	No	3-6	2	15	£0.75

Mechanical devices

Machines for multiplication were expensive, hard to use, and not easy to learn. The Britannic pinwheel machine cost £50 (approximately US$200) in 1928, and the Stanley Co. catalog claimed "even a junior can be instructed in its use in less than an hour",[32] but it may have well taken longer to attain full proficiency. A machine of this type was still in use at the Bank of England in 1965[33] and another at Shell Oil Co. until 1962 (communication from the archivist to the authors). The most elaborate multiplication machine was the Millionaire designed by Steiger and made by the Egli Company.[21] Capable of 20 significant figures, about 4,700 Millionaires were sold between 1897 and 1935 at a 1914 price of about US$500, according to Luc de Brabandere.[34] These machines produced no printed record of the calculation, so there was no check on the correct entry of multiplier or multiplicand, although these were visible on windows until the machine was reset for the next calculation.

Comptometers, originally adders, were useful for multiplying by repeated addition. However, as noted, they were not cheap and required specially trained operators. Comptometers differed from other machines used for multiplication in that there was no temporary record of the multiplicand or the multiplier. Businesses therefore often employed supervisors who only accepted a result when two operators independently arrived at the same answer. They came into use in the UK for multiplication in larger companies such as Shell in the 1920s.[35] In 1923, a service bureau was set up in London to provide computing services to smaller companies, based on Comptometers and a Hollerith installation.[36] Comptometers appeared in smaller firms after World War II. The operators still had to learn to decimalize money and measures in their heads. In 1909 Dorr E. Felt, the Comptometer's inventor and maker, took out two British patents for a Comptometer with an attached cylindrical tabular calculator in an unsuccessful attempt to overcome the £sd problem. For more about the history of Comptometers, see Boering.[17]

Punched card multipliers had little penetration in commerce before World War II, being used mainly in a few large companies for producing analyses of invoices or payroll sheets rather than extending the individual lines. Leslie Comrie's detailed 1930 article[37] stated that multiplication was not offered at that time by the Hollerith or Powers companies in the UK. However, the Science Museum in London has on display photocopies of an invoice dated 1930, with the prices extended, for the Ardath Tobacco Co. together with the punched card from Powers SAMAS (Société Anonyme des Machines à Statistiques), established in 1929 to market the product that generated the invoice lines. We have not found an explanation for this apparent contradiction. For a fuller discussion of punch card systems, see Arthur Norberg.[38]

Dominant position of ready reckoners

When deciding what type of calculation aids to purchase, businesses considered several key factors: price, number of significant figures, time to master the machine, and calculation time. Table 4, which lists different classes of aids, shows how ready reckoners surpassed other types in terms of key considerations.

As Table 4 shows, ready reckoners were cost-effective, speedy, easy to learn, and simple to use.

Comparative sales figures are hard to find. The best tentative estimates of annual world sales volume we have found for types of calculation aids are as follows:

- ready reckoners: 30,000;
- Comptometers (repeat add type): 8,000, based on data by Boering using serial numbers;[17]
- pinwheel multipliers: about 40,000,[39]
- Otis King helical slide rule: 5,000,[40] and,
- step drum: up to 5,000, based on data for the Thomas machine.[41]

A qualitative discussion of ready reckoners' dominant position in the late 19th and early 20th centuries begins with a look at historical sources. The Victorians, for example, were fascinated by ready reckoners. Looking back at that period, M. Norton Wise notes:

> By the 1860's, the favorite device for lessening the work of the computer (human), the mathematical table, had become an object whose dizzying rows of printed figures would fascinate the Victorian public. These tables displayed the limitless fecundity of numbers, and transformed them into a commodity that would bring the power of calculation within the reach of the ordinary citizen. The centrality of tables of numbers and calculation to mid-Victorian life was famously portrayed by Charles Dickens' character Thomas Gradgrind, who always had a rule and a pair of scales and the multiplication table in his pocket.[42]

In 1908, an anonymous writer in *The Organiser* magazine noted under the heading "Some Useful Ready Reckoners":

> We have lately had several queries from subscribers relating to reckonings and rules for reckoning, and we therefore think that it will be useful to many of our readers if we direct their attention to some books published on that subject. …[43]

A 1923 issue of *Office Magazine* noted that the Commercial Calculating Company "compiled original decimal equivalent tables and business ready reckoners for special trades and professions."[44]

In 1933, an article in the trade magazine *Office Management* noted: "Before the advent of counting machines, the ready reckoner was the only aid we had in the making of office calculations; many offices have still no other aid."[45]

A 1951 guide to using punch card office equipment declared that

> The {punch card} operator could read the hours and rate from the clock card or time sheet. She would have to obtain the value from a ready reckoner (unless somebody else had worked it all out previously on the clock cards) or alternatively she could leave the value column blank and later put all the cards through a Multiplying Punch, if one was available. And not only would she have to do all that, but all her work would have to be repeated by the verifying operator.[46]

Ready reckoners still had a place in the office in the late 1950s. A 1958 article noted under a review of a Unilever Co. paper titled "Good Ideas from Unilever Ready-Reckoners":

> The authors are firm advocates of ready-reckoners, specially prepared to meet the needs of each department. The weakness of the standard-book variety, they say, is that it attempts to cater for too many users. Several drawings are reproduced to show the different format that ready-reckoners can take and how they can be built into a general procedure.[47]

A 1959 article in *Office Magazine* on calculator terminology observed:

> When considering the use of a calculator in one's own office, the first thing to do is to write down in detail the number and the precise nature of the calculations that have to be done, at the same time ascertaining the maximum number of digits normally involved. If this investigation reveals that the bulk of the work consists of one type of calculation, the purchase of ready-reckoner tables may prove a better investment than a machine. For some types of calculations within limited ranges, such as barrelage conversion in the brewing industry, suitable tables can easily be prepared. The tables may be drawn up in colors to facilitate quick reference or, if the range of factors is small "blown-up" tables may be used. Ready-reckoner tables are speedy, accurate to the required degree and avoid decimalization in the case of sterling calculations.[48]

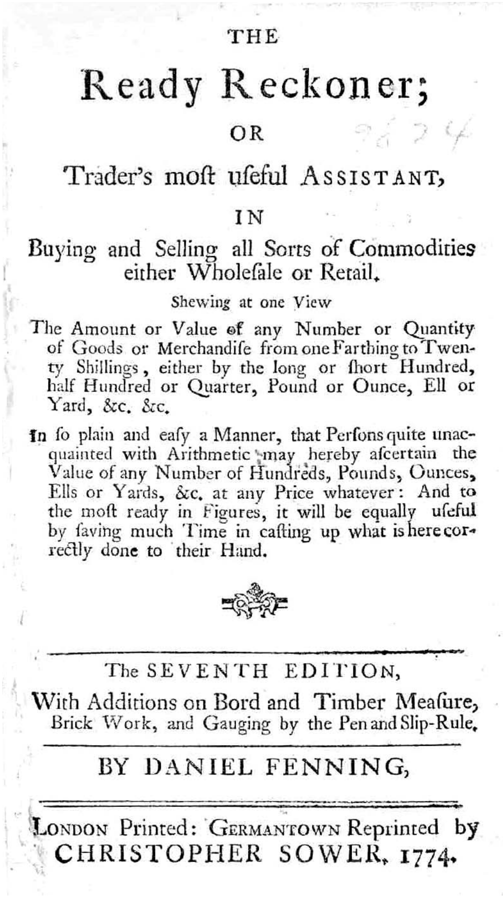
And even in 1960, an *Office Magazine* article by Louis M. Nation-Tellery mentions reckoners. In describing the installation of a Class 32 SM (Sterling Multiplier) machine at a British wholesale grocer, the article explained that previously:

Ready reckoners were used for extensions, with the additions done mentally. All the amounts and totals entered by the pricing clerk were handwritten. Because of the volume of work handled, two pricing clerks were necessary, but worked in tandem with two more clerks whose function was to check all the figure work.[49]

The grocer handled 2,500 extensions a day. Ready reckoners played an important role in many offices from their inception to the 1960s. Nonetheless, while clearly a widely used, cheap, labor-saving aid, the reckoner's book format had limited legibility and row/column identification. Perhaps recognizing the limitations of ready reckoners, Nation-Tellery took out a Great Britain patent for a tabular calculator in 1948.

Design of ready reckoners

The basic concept of a set of precalculated tables sounds simple enough. With other devices, such as mechanical devices, slide rules, and logarithm tables, users can do any calculation, albeit only to the level of precision built into the device and usually decimal only. But with a ready reckoner, users can do only those calculations selected by its compiler, the person who determined the calculations to include in the reckoner. So the compiler had to understand the likely user needs and to optimize the variables at his disposal: number and size of pages, font and type size, ranges of values, and the step sizes.


THE

Ready Reckoner;

OR

Trader's moſt uſeful ASSISTANT,

IN

Buying and Selling all Sorts of Commodities either Wholeſale or Retail.

Shewing at one View

The Amount or Value of any Number or Quantity of Goods or Merchandiſe from one Farthing to Twenty Shillings, either by the long or ſhort Hundred, half Hundred or Quarter, Pound or Ounce, Ell or Yard, &c. &c.

In ſo plain and eaſy a Manner, that Perſons quite unacquainted with Arithmetic may hereby aſcertain the Value of any Number of Hundreds, Pounds, Ounces, Ells or Yards, &c. at any Price whatever: And to the moſt ready in Figures, it will be equally uſeful by ſaving much Time in caſting up what is here correctly done to their Hand.

The SEVENTH EDITION,

With Additions on Bord and Timber Meaſure, Brick Work, and Gauging by the Pen and Slip-Rule.

BY DANIEL FENNING,

LONDON Printed: GERMANTOWN Reprinted by CHRISTOPHER SOWER, 1774.

Figure 4. Title page from first American ready reckoner, printed in 1774. (Source: Authors' collections.)

Pagination, language, and size

Ready reckoners came in many sizes ranging from pocket books to hefty desktop tomes, weighing several pounds yet still much lighter and potentially more portable than the alternative mechanical devices. Using a sample of 572 ready reckoners with known pagination, the average length is 160 pages, with a median value of 104 pages reflecting the vast diversity.

Reckoners came in many languages. There is evidence that—probably because of the extensive work involved in their initial preparation—reckoners were translated from one language to another. Fenning's ready reckoner[1] was originally printed in English in 1757 and went through numerous editions. The seventh edition was translated into German and published in Germantown, Pennsylvania, by the famous pioneer press of Christopher Sower in 1774 alongside an English language version (see Figure 4). Both versions were subsequently reprinted a number of times.

A major activity at the Annual International Book Fair at Frankfurt, Germany, is the buying and selling of foreign rights for material that is not language specific, such as tables and illustrations. However, we have not yet found any English versions of French or German material although J.C. Oehlschlager produced in Winnipeg, Canada, a German-English bilingual version of his earlier German ready reckoner, published in Philadelphia circa 1868.[50]

Layout

Figure 5 shows a page from William Leybourne's 1798 ready reckoner.[51] This shows how Leybourne displayed the products for one price, 20 pence and 3 farthings, on one page. This page gives results for 1 to 99 items, and then 100 to 900 in hundreds and 1,000 to 10,000 in thousands. The total number of the product cells is therefore (99 + 9 + 10) = 118. The main set of tables in this book required 223 pages like this. The number of calculations was thus 223 times 118 = 26,314. The little

At 20 *Pence* 3 *Farthings.* 83

N.	l.	s.	d.	q.	N.	l.	s.	d.	q.	N.	l.	s.	d.	q.
1	0	1	8	3	45	3	17	9	3	89	7	13	10	3
2	0	3	5	2	46	3	19	6	2	90	7	15	7	2
3	0	5	2	1	47	4	1	3	1	91	7	17	4	1
4	0	6	11	0	48	4	3	0	0	92	7	19	1	0
5	0	8	7	3	49	4	4	8	3	93	8	0	9	3
6	0	10	4	2	50	4	6	5	2	94	8	2	6	2
7	0	12	1	1	51	4	8	2	1	95	8	4	3	1
8	0	13	10	0	52	4	9	11	0	96	8	6	0	0
9	0	15	6	3	53	4	11	7	3	97	8	7	8	3
10	0	17	3	2	54	4	13	4	2	98	8	9	5	2
11	0	19	0	1	55	4	15	1	1	99	8	11	2	1
12	1	0	9	0	[56]	4	16	10	0	100	8	12	11	0
13	1	2	5	3	57	4	18	6	3	200	17	5	10	0
14	1	4	2	2	58	5	0	3	2	300	25	18	9	0
15	1	5	11	1	59	5	2	0	1	400	34	11	8	0
16	1	7	8	0	60	5	3	9	0	500	43	4	7	0
17	1	9	4	3	61	5	5	5	3	600	51	17	6	0
18	1	11	1	2	62	5	7	2	2	700	60	10	5	0
19	1	12	10	1	63	5	8	11	1	800	69	3	4	0
20	1	14	7	0	64	5	10	8	0	900	77	16	3	0
21	1	16	3	3	65	5	12	4	3	1000	86	9	2	0
22	1	18	0	2	66	5	14	1	2	2000	172	18	4	0
23	1	19	9	1	67	5	15	10	1	3000	259	7	6	0
24	2	1	6	0	68	5	17	7	0	4000	345	16	8	0
25	2	3	2	3	69	5	19	3	3	5000	432	5	10	0
26	2	4	11	2	70	6	1	0	2	6000	518	15	0	0
27	2	6	8	1	71	6	2	9	1	7000	605	4	2	0
[28]	2	8	5	0	72	6	4	6	0	8000	691	13	4	0
29	2	10	1	3	73	6	6	2	3	9000	778	2	6	0
30	2	11	10	2	74	6	7	11	2	10000	864	11	8	0
31	2	13	7	1	75	6	9	8	1					
32	2	15	4	0	76	6	11	5	0					
33	2	17	0	3	77	6	13	1	3	*Great Hundred*				
34	2	18	9	2	78	6	14	10	2	112	9	13	8	0
35	3	0	6	1	79	6	16	7	1	*Grofs*				
36	3	2	3	0	80	6	18	4	0	144	12	9	0	0
37	3	3	11	3	81	7	0	0	3	*Wey*				
38	3	5	8	2	82	7	1	9	2	256	22	2	8	0
39	3	7	5	1	83	7	3	6	1	*Days in a Year*				
40	3	9	2	0	[84]	7	5	3	0	365	31	11	1	3
41	3	10	10	3	85	7	6	11	3	*Feet in a Rod*				
42	3	12	7	2	86	7	8	8	2	272	23	10	4	0
43	3	14	4	1	87	7	10	5	1					
44	3	16	1	0	88	7	12	2	0					

Figure 5. Page from Leybourne's *Ready Reckoner* or *Trader's Sure Guide*, 1798. (Source: Authors' collections.)

square at bottom right added more value by giving

> the great or long hundred, viz. 112 lbs, by which most heavy goods are bought and sold; the gross viz. twelve dozen by which Gloves, Buttons, and c. are bought and sold; the Wey of cheese, Salt, and c. which is 256 lbs.; The number of days in a Year, by which the amount of Daily expenses, or Wages may be known; and likewise the number of feet in a solid Rod of Brick Wall, which are 272.[52]

Formatting of reckoners was always a challenge for compilers and printers. Suppose a ready reckoner were to provide prices, with a range from 6d per ton to £140 a ton, by steps of 6d, for weights from 1 pound to 2 tons, in 1-pound steps. This could easily take 5,600 pages, so drastic reduction was required. A compiler could do this by breaking up the table into ranges or brackets, for example, making the weight step larger, say, a quarter (28 pounds); and adding one small table for 1 to 27 pounds. This could cut the size down to a manageable 500 pages. To get the price for 3 quarters and 7 pounds, a user would look up the price for 3 quarters in the main table and then the smaller table used for 7 pounds, and finally add the partial products together.

The compiler could also break down the prices into brackets, say 1d to 11d, 1s to 19s, £1 to £10. The user then had to add up several subtotals, for example: number of pence times pounds weight, number of pence times quarters, number of shillings times pounds weight, number of shillings times quarters, number of pounds £ times pounds weight, number of pounds £ times quarters.

In fact, J. Gall Inglis of publishing firm Gall and Inglis addressed this problem by printing several books, stating "Owing to the enormous range required by the increased rates now current, it has not been possible to include them within the limits of a single volume."[53] So the publisher produced volumes for 1–56/–, 56–140/– and 140–224/– per cwt. (This was the contemporary notation for 1s[shilling] to 56s, 56s to 140s, and 140s to 224s.) As a consequence the user might have had to consult three volumes, each costing 2/6d. Each volume, with ancillary tables, is just under 200 pages.

There were some other tricks to cut space. In pure multiplication tables, say for 1 to 100 times 1 to 100, half of the cells are repeated (14 × 68 = 68 × 14). The duplication could be removed by omitting half the products and then rearranging the layout of the resulting triangle, as Figure 6 shows.

In 1862, *The Readiest Reckoner Ever Invented* claimed a unique approach to calculating price:

> The present work differs from every book of this kind: the reference being in the first instance to the number instead of the price: for example, if 103 be the quantity wanted, at 17s 11d per pound, yard, and c., turn to page 103 and opposite 17s11d will be found £92.5s.5d. The answer required.[54]

The pages went from 1 to 110 rising in steps of one at a time, then went 112, 144, 200, 250, 256, 272, 300, 365, 400, 500, 600, 700, 750, 800, 900, 1000, 1250, 1500, 2000, 3000, 4000, and by thousands to 10000. Others came up with different approaches, such as attempts to cut a page into strips. One such example is R. Dunlop's reckoner page-strip design, abridged patent 124 of 1862 and which survives in Bonn's Arithmeum Museum (http://www.arithmeum.uni-bonn.de/). Another new design was developed by Robert Anlezark.[55] He managed to make the left-hand column act both for days for interest calculations and for units for pricing. Figure 7 shows page 1.

An ingenious reckoner for simple and compound interest was devised by Remig Rees and called *Der stumme Diener* [mute servant] *Universal Schnellrechner*, as Figure 8 shows. It was published by Merkur Verlag in Wehingen, Germany, and is undated.

The *Schnellrechner* is an example of the need, with some designs of reckoner to add up partial products to arrive at the final answer. A logical idea was to attach a small adding device to the ready reckoner. Thus the Bergmann Universal calculator of 1920 added a stylus adder, the Correntator. This adder, made in Berlin by Firma Continentale Buro-Reform Jean Bergmann, was sold in the US by Universal Calculator Corporation, New York.

MULTIPLICATION TABLE [Look for larger

	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
1	11	12	13	14	15	16	17	18	19	20	21	22	23	24	1
2	22	24	26	28	30	32	34	36	38	40	42	44	46	48	2
3	33	36	39	42	45	48	51	54	57	60	63	66	69	72	3
4	44	48	52	56	60	64	68	72	76	80	84	88	92	96	4
5	55	60	65	70	75	80	85	90	95	100	105	110	115	120	5
6	66	72	78	84	90	96	102	108	114	120	126	132	138	144	6
7	77	84	91	98	105	112	119	126	133	140	147	154	161	168	7
8	88	96	104	112	120	128	136	144	152	160	168	176	184	192	8
9	99	108	117	126	135	144	153	162	171	180	189	198	207	216	9
10	110	120	130	140	150	160	170	180	190	200	210	220	230	240	10
11	121	132	143	154	165	176	187	198	209	220	231	242	253	264	11
12	..	144	156	168	180	192	204	216	228	240	252	264	276	288	12
13	..	..	169	182	195	208	221	234	247	260	273	286	299	312	13
14	..	..	..	196	210	224	238	252	266	280	294	308	322	336	14
15	..	..	..	..	225	240	255	270	285	300	315	330	345	360	15
..	..	..	..	..	..	256	272	288	304	320	336	352	368	384	16
..	..	..	..	..	..	..	289	306	323	340	357	374	391	408	17
								324	342	360	378	396	414	432	18
									361	380	399	418	437	456	19
										400	420	440	460	480	20
											441	462	483	504	21
												484	506	528	22
													529	552	23
														576	24

Fractional Mult. Table

	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	
¾	1	1⅛	1¼	1½	1¾	1⅞	2								
1	1¼	1½	1¾	2	2¼	2½	2¾	3							
¼	1½	1⅞	2¼	2½	2¾	3⅛	3½	3¾	4						
1½	1⅞	2¼	2⅝	3	3⅜	3¾	4⅛	4½	4⅞	5¼					
¾	2¼	2⅝	3	3½	4	4⅜	4¾	5¼	5¾	6⅛	6½				
2	2½	3	3½	4	4½	5	5½	6	6½	7	7½	8			
¼	2¾	3⅜	4	4½	5	5⅝	6¼	6¾	7¼	7⅞	8½	9	9½		
2½	3⅛	3¾	4⅜	5	5⅝	6¼	6⅞	7½	8⅛	8¾	9⅜	10	10⅝	11¼	2½
¾	3½	4⅛	4¾	5½	6¼	6⅞	7½	8¼	9	9⅝	10¼	11	11¾	12⅜	¾
3	3¾	4½	5¼	6	6¾	7½	8¼	9	9¾	10½	11¼	12	12¾	13½	3
¼	4	4⅞	5¾	6½	7¼	8⅛	9	9¾	10½	11⅜	12¼	13	13¾	14⅝	¼
3½	4⅜	5¼	6⅛	7	7⅞	8¾	9⅝	10½	11⅜	12¼	13⅛	14	14⅞	15¾	3½
¾	4¾	5⅝	6½	7½	8½	9⅜	10¼	11¼	12¼	13⅛	14	15	1C	16⅞	¾
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	4
¼	5¼	6⅜	7½	8½	9½	10⅝	11¾	12¾	13¾	14⅞	16	17	18	19⅛	¼
4½	5⅝	6¾	7⅞	9	10⅛	11¼	12⅜	13½	14⅝	15¾	16⅞	18	19⅛	20¼	4½
¾	6	7⅛	8¼	9½	10¾	11⅞	13	14¼	15½	16⅝	17¾	19	20¼	21⅜	¾
5	6¼	7½	8¾	10	11¼	12½	13¾	15	16¼	17½	18¾	20	21¼	22½	5
¼	6½	7⅞	9¼	10½	11¾	13⅛	14½	15¾	17	18⅜	19¾	21	22¼	23⅝	¼
5½	6⅞	8¼	9⅝	11	12⅜	13¾	15⅛	16½	17⅞	19¼	20⅝	22	23⅜	24¾	5½
¾	7¼	8⅝	10	11½	13	14⅜	15¾	17¼	18¾	20⅛	21½	23	24½	25⅞	¾
6	7½	9	10½	12	13½	15	16½	18	19½	21	22½	24	25½	27	6
¼	7¾	9⅜	11	12½	14	15⅝	17¼	18¾	20¼	21⅞	23½	25	26½	28⅛	¼
6½	8⅛	9¾	11⅜	13	14⅝	16¼	17⅞	19½	21⅛	22¾	24⅜	26	27⅝	29¼	6½
¾	8½	10⅛	11¾	13½	15¼	16⅞	18½	20¼	22	23⅝	25¼	27	28¾	30⅜	¾
7	8¾	10½	12¼	14	15¾	17½	19¼	21	22¾	24½	26¼	28	29¾	31½	7
¼	9	10⅞	12¾	14½	16¼	18⅛	20	21¾	23½	25⅜	27¼	29	30¾	32⅝	¼
7½	9⅜	11¼	13⅛	15	16⅞	18¾	20⅝	22½	24⅜	26¼	28⅛	30	31⅞	33¾	7½
¾	9¾	11⅝	13½	15½	17½	19⅜	21¼	23¼	25¼	27⅛	29	31	33	34⅞	¾
8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	8
¼	10¼	12⅜	14½	16½	18½	20⅝	22¾	24¾	26¾	28⅞	31	33	35	37⅛	¼
8½	10⅝	12¾	14⅞	17	19⅛	21¼	23⅜	25½	27⅝	29¾	31⅞	34	36⅛	38¼	8½
¾	11	13⅛	15¼	17½	19¾	21⅞	24	26¼	28½	30⅝	32¾	35	37¼	39⅜	¾
9	11¼	13½	15¾	18	20¼	22½	24¾	27	29¼	31½	33¾	36	38¼	40½	9
¼	11½	13⅞	16¼	18½	20¾	23⅛	25½	27¾	30	32⅜	34¾	37	39¼	41⅝	¼
9½	11⅞	14¼	16⅝	19	21⅜	23¾	26⅛	28½	30⅞	33¼	35⅝	38	40⅜	42¾	9½
¾	12¼	14⅝	17	19½	22	24⅜	26¾	29¼	31¾	34⅛	36½	39	41½	43⅞	¾
10	12½	15	17½	20	22½	25	27½	30	32½	35	37½	40	42½	45	10

Figure 6. Triangular multiplication table from *The Express Ready Reckoner* published by Gall and Inglis. No date. (Source: Authors' collections.)

Short cuts

There were some application-specific short cuts, such as this one for interest calculations from an American reckoner of 1911:

> The method of reckoning interest generally used by the best accountants and book-keepers is what is known as the sixty day method. By this method 360 days are reckoned as a year and 30 days as a month. Six p. c. for 12 months, or 1 year, is equivalent to 1 p. c. for 2 months, or 60 days, and 1 p. c. of any number is easily found by moving the decimal point two places to the left. Therefore, the interest on any amount at 6 p. c. per annum for 2 months, or 60 days, may be found by moving the decimal point two places to the left. Having found the interest for 60 days, in order to find it from

1

Days.	1	2 / 1	4 / 2 / 1		8 / 4 / 2 / 1			16 / 8 / 4 / 2			24 / 12 / 6 / 3			32 / 16 / 8 / 4			40 / 20 / 10 / 5			48 / 24 / 12 / 6		
	d.	d.	d.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
1	1/32	1/16	⅛	¼	0	0	0¼	0	0	0½	0	0	0¾	0	0	1	0	0	1¼	0	0	1½
2	1/16	⅛	¼	½	0	0	0½	0	0	1	0	0	1½	0	0	2	0	0	2½	0	0	3
3	3/32	3/16	⅜	¾	0	0	0¾	0	0	1½	0	0	2¼	0	0	3	0	0	3¾	0	0	4½
4	⅛	¼	½	1	0	0	1	0	0	2	0	0	3	0	0	4	0	0	5	0	0	6
5	5/32	5/16	⅝	1¼	0	0	1¼	0	0	2½	0	0	3¾	0	0	5	0	0	6¼	0	0	7½
6	3/16	⅜	¾	1½	0	0	1½	0	0	3	0	0	4½	0	0	6	0	0	7½	0	0	9
7	7/32	7/16	⅞	1¾	0	0	1¾	0	0	3½	0	0	5¼	0	0	7	0	0	8¾	0	0	10½
8	¼	½	1	2	0	0	2	0	0	4	0	0	6	0	0	8	0	0	10	0	1	0
9	9/32	9/16	1⅛	2¼	0	0	2¼	0	0	4½	0	0	6¾	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Figure 7. Anlezark's entirely new reckoner system, 1871. (Source: British Library.)

	C	B	A		C	B	A		C	B	A
1	0	01	1	2	0	02	1	3	0	03	1
	0	10	2		0	19	2		0	29	2
	0	97	3		1	94	3		2	92	3
	9	72	4		19	44	4		29	17	4
	97	22	5		194	44	5		291	66	5
	972	20	6		1944	39	6		2916	59	6
	9721	95	7		19443	90	7		29165	86	7
4	0	04	1	5	0	05	1	6	0	06	1
	0	39	2		0	49	2		0	58	2
	3	89	3		4	86	3		5	83	3
	38	89	4		48	61	4		58	33	4
	388	88	5		486	10	5		583	32	5
	3888	78	6		4860	98	6		5833	17	6
	38887	81	7		48609	76	7		58331	71	7
7	0	07	1	8	0	08	1	9	0	09	1
	0	68	2		0	78	2		0	87	2
	6	81	3		7	78	3		8	75	3
	68	05	4		77	78	4		87	50	4
	680	54	5		777	76	5		874	98	5
	6805	37	6		7777	56	6		8749	76	6
	68053	67	7		77775	62	7		87497	57	7

Directions for the use of the tables of interest drawn up by **Remig Rees, Wehingen,** Wurtemberg (Germany).

The calculation of the interest quota, which is found by multiplying the capital with the number of days, is always in the front.

The large printed numbers 1 to 9 indicate the single numerals of the interest quota.

A the numerals 1 to 7 indicate the value of their position from one to a million (1 - 7 places),
B shows the interest in hundreths of coinage and
C shows the interest in whole coins.

Example: How much interest does 866 bear in 273 days at 3½%?
866 times 273 is 236418
The interest quota is therefore . . . 236418
the sequence of the different numbers 654321
The interest is now calculated from the table as follows:
(see white figures in black squares)

Interest on 8 place 1	0,08
" " 1 " 2	0,10
" " 4 " 3	3,89
" " 6 " 4	58,33
" " 3 " 5	291,66
" " 2 " 6	1944,39
Total	2298,45

The total interest is therefore 2298 pfennigs, francs, hellers, oere, copecks, yen, dollars, cent.

These directions for use apply to the further division tables of the same kind in this work.

Figure 8. Universal Schnellrechner ["Universal speedy calculator"]. (Source: Arithmeum Bonn.)

> that amount for any number of days, simply divide them into aliquot parts of 60.[56]

In later years, devices for interest calculations were developed to handle not only 365 days in a year but also leap years, public holidays, and so on.

Arithmetic short cuts were a regular feature of introductory sections of ready reckoners. They also appear in books such as Pitman's *Office Desk Book,* which gives a range of "Arithmetic Short Cuts" devoted entirely to multiplication, division, and closely related calculations,[57] where they are supplemented with a basic set of ready reckoner tables and conversions.

Table preparation and accuracy

Eliminating all errors was always recognized as a major challenge. In the introduction to his early ready reckoner, its compiler, Daniel Fenning, states that

> 'Tis true that nothing is more liable to Errors than large Tables of any Sort; for though there requires no great Scholarship to make such a plain Book; yet the vast Multiplicity of Figures, and continual Series of different Numbers, render the work much more difficult than Persons in general are aware of; and though it is possible that such a Book may be correct, yet it may naturally be expected that Errors of some Sort or other may slip the Notice of the most attentive single Examiner. This being the real Case of Works of this Sort, I have taken all the Care that Time and Ability would allow of, to prevent it, and can assure the Public that every single and separate Sheet has been examined by three different Persons with all possible Caution and Attention, so that I am apt to think you may depend on the Exactness of the Calculations.[58]

Notwithstanding the former checking, the ninth edition goes on to record that it has been "carefully revised and corrected by Joseph Moon, Mathematician, Salisbury."[59]

Some ready reckoners claim to be the result of two independent sets of calculations. For example, the preface to *The Readiest Reckoner Ever Invented*, first published in 1811, notes:

> The correctness of the Tables is insured by the circumstance of two sets of them being separately calculated by different individuals, by each of which the other has been checked, and the printed sheets have been revised in the most careful manner from the copy not used in the printing office.[60]

In 1862, the preface to the 13th edition of the *Readiest Reckoner* states that it was recalculated by Charles Ody Rooks. The preface to the 13th edition says:

> The publishers deem it necessary to state that the greatest attention has been given to have the Work perfectly correct; and with this view, besides careful examination with the original (in the production of which the most indefatigable pains were bestowed to ensure its correctness), the whole has been submitted to a fresh calculation; it is therefore presumed that it may with safety be relied upon.[61]

Three principal classes of error exist in the preparation of ready reckoners:

- The multiplication is wrong because of either a computing error or a printer error.
- The multiplication is correct, but is placed in the wrong cell by either the human computer or the printer.
- The row or column label is incorrect due to printer error.

Some early examples of ready reckoners admit "defects and omissions" in previous editions, such as *The Moulders' Ready Reckoner*.[62] In a table from this reckoner, author Williams verified all the product cells using Microsoft Excel but found an example of the third type of error in the Diameter column, where there are two rows marked 6, but the second one should be 6 and 1/2.

An advertisement for *A Series of Metric Tables* compiled by C.H. Dowling, 1886, and published by Crosby Lockwood and Co., quotes from the *Builder* magazine:[63] "Their accuracy has been certified by Professor Airy." (Airy was professor of astronomy at Cambridge and later the Astronomer Royal.) Crosby Lockwood also published a discount guide and claimed that

> We have the high authority of Professor J R Young (Late Professor of Mathematics at Belfast College, Ten editions of whose Rudimentary Treatise on Arithmetic were published by Crosby, Lockwood,) that the tables throughout the work are constructed upon strictly accurate principles.[63]

Machine calculation did not eliminate error. There is at least one machine-calculated reckoner, Shorthose's *Commercial Ready Reckoner* (10th edition) of 1921, which says on the title page:

> Stating the value of weights ranging from 1/4 Cwt to 50 Tons at Prices extending between one Penny to £3 14/– per ton inclusive. The calculations of prices from £2 10s 3d to £3 14s 0d have been compiled on a Burroughs Calculator Machine by the Burroughs Company Ltd, Cannon St, London E.C.4.[64]

Despite the Burroughs involvement, there exist several erratum slips. These have survived intact: The copy went from the Standard Telephone and Cable Technical Library to the Science Museum Library in 1972. At some time the corrections have been inked in by hand, as Figure 9 shows.

Finally, we feel bound to observe that despite the care taken over the preparation of the ready reckoner, its users still had to note the price for tons and the cwts, then the fractional cwts, and finally add the three to get the total price. Inevitably, this must also have given rise to errors.

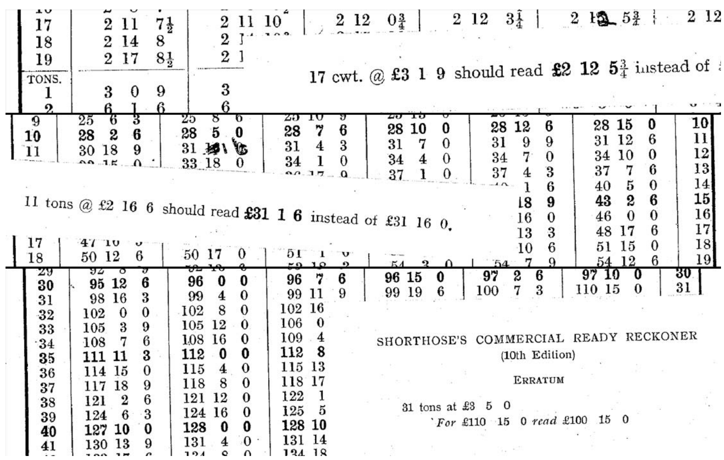


Figure 9. Three erratum slips—probably due to printer's error. (Source: Science Museum Library.)

Doron Swade in Campbell-Kelly[6] discusses calculating and checking. Given Swade's interest in Babbage, Swade's comments mainly relate to classes of table where the use of differences for initial calculation and then checking is particularly relevant. Some of these classes exclude the largely nondecimal products in many ready reckoners that often have less obviously regular differences between rows. Interestingly, in light of our earlier comments on the double checking of Comptometer results, Swade points out that "The technique of double computation was not foolproof and it was not unknown for computers who, despite insulation from each other, produced the same incorrect result."[65]

The archive of UK publishing house Routledge at University College, London, throws further light on accuracy and calculation. There are hundreds of handwritten ledgers called the *publishing journals* that record full details of every publication including details of print runs, prices, and so on. Tucked in one of the Routledge firm's publishing journals is a printed slip showing the corrigenda for the 1921 42nd reset edition of James Laurie's *High Interest Tables*. This shows more than 30 corrections, presumably due either to errors in resetting the type or errors in the previous edition. There is a note that a new table of 1/8 percent was inserted into this edition for the preparation of which the Commercial Calculating Co. Ltd was paid £2 2s, while W.J. Macdougal received £30 for "editorial work on new tables."

Ready reckoners compiled by electronic computer

The Rich Man's Ready Reckoner published in 1970 claims:

> Other reckoners leave off at a miserable 10,000. This reckoner is strictly for people who deal in real money—calculations go up to ten million in both sterling and decimals. … You don't have to be rich to find this book useful. You just have to be tired of working out complicated figures in a hurry. Especially when you're dealing with really large numbers. We were and that's why we make no excuse for putting yet another ready reckoner on the market. Ours include the present currency along with the decimal one, since we know that for a considerable period of time people will need to compare the two. It is also possibly the first book of this type to be compiled entirely by computer.[66]

Despite the computer, the publishers were taking no chances:

> While every effort has been made to ensure that the contents of this book are accurate and correct, the publishers do not hold themselves responsible for any loss that might arise through any inadvertent errors contained in the text.[66]

Also in 1970, there appeared a set of tables to calculate financial compound interest and annuities.[67] Author Williams used a copy for discounted cash flow calculations for many years, up to the advent of the scientific pocket calculator. The book does not say what computer was used. The pages are a mix of a main portion in sans-serif characters that look like a photocopy of a computer printout and elegant typeset borders.

Three years later in 1973, we find *Nuttall's Metric Conversion Tables,* which were computed by London University Computing Services Ltd.[68] It does not say how it was computed, although the company used the London University Ferranti Atlas computer at the time. It is a small booklet of conversion factors with no multiplications. The pages look like images of a computer printout.

Printing process

Swade[69] describes printing and the use of stereotypes. A stereotype is a printing plate (usually mounted as a block). Stereotypes were often mass-produced and sent to many printers and newspapers. The stereo was produced by a molding process. The advantage was that when another edition was needed the stereotype was ready to go, while conventional type might have been unset or knocked about. A stereotype avoided introducing new errors in a new typesetting, but preserved any errors from the previous edition. Swade concludes that stereotypes provided an economical way of preserving the investment in typesetting and proof reading, and extended confidence in highly reputable tables to subsequent editions. Historian Alexis Weedon[70] discusses stereotyping and electrotyping in some detail, but with no specific comments on reckoners.

The authors have found five ready reckoners published between 1814 and 1850 that explicitly state that they were printed from stereotypes. In addition to these early reckoners, nearly all of the 45 editions published by Routledge in the first half of the 20th century, which we discuss later, were printed from stereos, as shown by the frequent small charges for repairs listed in the publishing journals in the Routledge archive. Tucked into one publishing journal is the following letter to Routledge from its printer, Brendon and Son Ltd. of Plymouth, dated 19 February 1926:

> Master's Ready Reckoner—With regard to your suggestion to make a new set of moulds before the plates show signs of wear, we advise you that we already hold a set of moulds for this particular Ready Reckoner, complete with the additional tables at the foot of the pages. The moulds of the tables are separate, but are complete with the folio, and in the event of casting a set of plates for any future commission it would be necessary only to join on the stereo of the table to the top portion.[71]

Prices of ready reckoners

The main sources of data on prices are the *Reference Catalogue of Current Literature*[72] and the *Cumulative Book Index*.[73] In our research, we sought to relate price to usefulness. The number of calculations in the reckoner seemed a likely measure, and indeed publishers emphasized this measure. For example, the cover of William Chadwick's 420-page *Combined Number and Weight Calculator*[74] states it contains "250,000 direct calculations producing by a single addition to each a combination of over 20 million calculations!" The cover of Warne's *Model Ready Reckoner*, pocket size, 288 pages, announced 40,000 calculations.[75] We found no correlation between price and either the number of pages or of pages and number of calculations. Given the wide range in the number of calculations per page due to variations in page size, layout, and font size, this was to be expected.

We have also examined the relationship between the number of direct calculations and their price. Data is available for 12 ready reckoners published by Delbridge in the US from 1911 to 1920. Similarly, we have found data for 15 British ready reckoners from 1920 to 1959.[72,73] We analyzed the data statistically and found that the regressions are statistically significant with a cost of £0.0035 per calculation while the Delbridge data gives a cost of US$0.0006, both converted to 1914 values.

An important circumstance regarding pricing concerned the nature of the binding and paper. Reckoners had to withstand rough usage, so publishers offered a range of bindings, from paper through board to leather. In addition, different papers were used and India paper was sometimes offered, presumably to give strength and reduce bulk. Consequently, in 1915 the Gall and Inglis Gem waistcoat pocket reckoner cost 1s 6d in leather printed on India paper while an abridged plain version was only 1 shilling, or two-thirds that price. The leather-bound Crosby Lockwood reckoner[74] cost 30s, a great deal of money at the time, but author Williams personally has a leather-bound copy in perfect condition today apart from slight scuffing of the corners of the cover, attesting to its hard-working life.

Production volume

To estimate the number of reckoners that were produced over time, we searched several world-renowned libraries. We also reviewed publisher data.

Titles or "items"

Searching the British Library's online integrated catalog yields 633 items in a search for "ready reckoner" (see http://catalogue.bl.uk). France's Bibliothèque Nationale lists 730 titles from a search for *barreme*; see http://www.bnf.fr/pages/zNavigat/frame/catalog.htm. The word *barreme* is named after M. Barreme who created an early (presumably first) French ready reckoner. The US Library of Congress (see http://www.loc.gov/) has about 210 titles for "reckoner." It is possible that the Library of Congress holding is less representative than the British Library's. Some ready reckoners are listed under "Calculator" but this category covers much other material and we have not attempted to extract those that might be of interest to this study.

We merged the data from the British Library and the Library of Congress with price and publication data from the US *Cumulative Book Index,*[73] the Great Britain *Reference Catalogue of Current Literature,*[72] and publisher records. After removing duplications and items that are primarily tax rate tables, we are left with just under 1,300 different "items." Each item is a separate edition, but may be one of many editions of the same work. This constitutes the authors' database. No doubt there are more items to be found in catalogs or lists we have not examined, particularly for those in Germany. We have obviously not examined every item, but have categorized them on the basis of the title and publisher information.

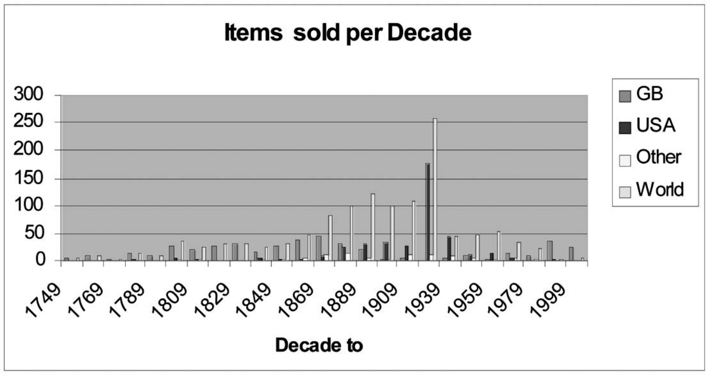


Figure 10. World sales by decade from 1749 to 1999.

Figure 10 shows that there was a peak in the number of items worldwide about 1890 of 100 per decade, 10 per year, and then a fairly steady rate of 5 per year until 1930. The publication of ready reckoners declined thereafter, but there were still some published between 1959 and 1969, then publication tailed off to one a year. The decimalization of the Sterling Currency Zone (sterling used in Australia, New Zealand, South Africa, West Indies, and so on) in the late 1960s and early 1970s created a final surge before pocket calculators took over.

There were some tens of specialist publishers in Great Britain. The top 13 publishers accounted for 175 items out of a total of 854 for Great Britain. Gall and Inglis were in Edinburgh, but the rest were mainly in London. Nearly all reckoners were published in many editions over many years, and provided a steady business. Collins[11] records in its 1959 edition, "Some 85 years ago the first edition of this book was published and it is now the oldest title on our list."

Using the Library of Congress catalog, we found fewer US publishers having produced reckoners, but the prolific Delbridge Publishing Co. produced more than 159 items from 1890 to 1947, almost as many as all the Great Britain publishers combined.

One may wonder how Charles L. Delbridge managed to create so many different items. The answer is that he identified different groups of users and within a group sliced the table into fine brackets. This is clearly reflected in the listing of his works in the Library of Congress catalog.

Print runs

Weedon,[70] who has made exhaustive analyses of the print runs of Victorian books in Great Britain based on individual publishers' records, points out that her data is skewed because publishers printed short initial runs of books to avoid heavy outlay and to test the market.

Accordingly, we analyzed direct data for two types of print runs from two sources: publishers' claims in advertising and publishers' records. Publishers' claims in advertising material are vague and may well be overstated:

- The *Reference Catalogue of Current Literature*[72] entry for Warne in 1913 advertises Nuttall's *Penny Table Book* and claimed that more than one million copies had been sold—but it is not clear over what period as the title may have changed. If it was 10 years, then this was 100,000 copies a year or per item.
- UK publisher Cassell's entry in the 1924 *Reference Catalogue of Current Literature* advertises its *Pocket Reference Library* and stated that more than half a million copies had been sold. The series comprised 12 titles, of which one was a ready reckoner, so an average would be 17,000 copies per item.

These advertising figures would thus suggest print runs of 2,000 to 100,000 copies per item.

Publisher's records are available for four publishers: Blackie, Gall and Inglis, Pitman, and Routledge. So far we have examined only the Routledge archive at University College, London. These record print runs from 1902 to 1941 and comprise details of both new items and reprints. These show first editions having small print runs of 500, presumably to test the market, and print runs growing over time until during the 1930s they reach 4,000 to 5,000 per print run. There appear to have been about 12 different titles and a total of 45 print runs. James Laurie's *High Interest Tables* reached their 14th (and last recorded) impression in 1933. The records show an average annual print quantity of slightly more than 3,000 throughout the 1930s.

We can therefore make an approximate estimate of world sales as follows. First, in Great Britain, the Routledge archive reveals they were printing around 3,000 copies per year. The total published in Great Britain could well have been at least five times that number when we consider Gall and Inglis, Warne, Crosby Lockwood, and so on.

Assuming a similar situation in the US, we estimate world sales of reckoners at around 30,000 a year. Although the UK and the US were the dominant economies in this period, plenty of examples of ready reckoners exist from Europe. In addition, others are known to exist from elsewhere around the globe. It is therefore clear that the total numbers printed and sold must have far exceeded any other calculating aid available at that time.

Conclusion

Given their general superiority concerning ease of use, it is not surprising that ready reckoners were the dominant aid used for multiplication in trade from 1800 to 1950. Throughout this period their sales far exceeded any other calculating aid used in trade to assist in making routine calculations.

However, despite all the ingenuity exercised, ready reckoners and the other kinds of tables were not in some respects as user friendly and useful as was hoped. They let users work with one page at a time, and they consisted of many pages with small print. Consequently, inventors and designers sought to make improvements in legibility and row/column selection by gluing the tables onto a variety of disks, cylinders, rolls, and cards. Such devices are known collectively as tabular calculators. These have been briefly described elsewhere,[76] and we are now preparing a more thorough exposition.

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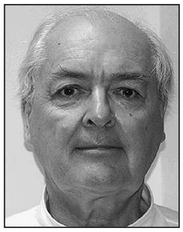

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