



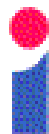
# Situation Calculus

R&N: § 10.3

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(slides adapted from Robert Wilensky)



# Using Logic to Plan

- We need ways of:
  - representing the world.
  - representing the goal.
  - representing how actions change the world.
- We haven't said much about the last.
  - **Difficulty**: After an action, new things are true, and some previously true facts are no longer true.

# Situations

## Idea:

- Introduce a notion of *situations*, which are logical terms
  - Consist of initial situation (usually called  $S_0$ ) and all situations generated by applying an action to a situation.
- State facts about situations.
  - By relativizing predications to situations.
  - E.g., instead of saying just **On(A,B)**, say (somehow) **On(A,B)** in situation  $S_0$
- Actions are thus
  - performed in a situation, and
  - produce new situations with new facts.
  - Examples: Forward and Turn(Right)

# Representing Predications Relative to a Situation

- Can add an argument for a situation to each predicate that can change.
  - E.g., instead of **On(A,B)**, write **On(A,B,S<sub>0</sub>)**
- Alternatively, introduce a predicate **Holds** and turn **On**, etc., into *functions*:
  - E.g., **Holds(On(A,B),S<sub>0</sub>)**
  - What do things like **On(A,B)** now mean?
    - Either a category of situations, in which **A** is on **B**, or a set of those situations.

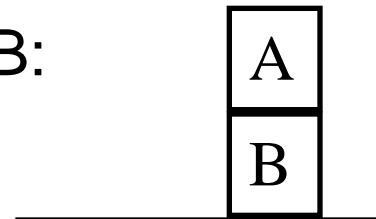
# How This Will Work

- Before some action, we might have in our KB:

**On(A,B,S<sub>0</sub>)**

**On(B,Table,S<sub>0</sub>)**

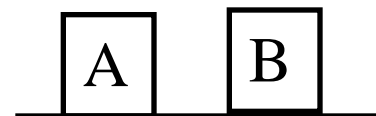
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- After an action that moves A to the table, say, we add

**Clear(B,S<sub>1</sub>)**

**On(A,Table,S<sub>1</sub>)**



- All these propositions are true. We have dealt with the issue of change, by keeping track of what is true when.

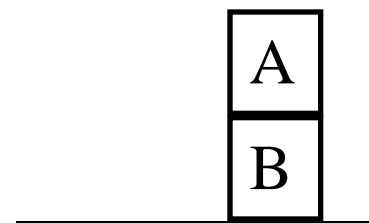
# Same Thing, Slightly Different Notation

. Before :

**Holds(On(A,B),S<sub>0</sub>)**

**Holds(On(B,Table),S<sub>0</sub>)**

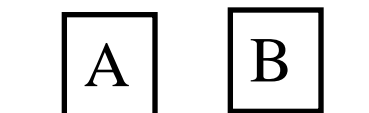
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. After, add

**Holds(Clear(B),S<sub>1</sub>)**

**Holds(On(A,Table),S<sub>1</sub>)**



# Representing Actions

- Need to represent:
  - Results of doing an action
  - Conditions that need to be in place to perform an action.
- For convenience, we will define *functions* to abbreviate actions:
  - E.g., **Move(A,B)** denotes the *action type* of moving **A** onto **B**.
  - These are *action types*, because actions themselves are specific to time, etc.
- Now, introduce a *function* **Result**, designating “the situation resulting from doing an action type in some situation”.
  - E.g., **Result(Move(A,B),S<sub>0</sub>)** means “the situation resulting from doing an action of type **Move(A,B)** in situation **S<sub>0</sub>**”.

# How This Works

- Keep in mind that things like **Result(Move(A,B),S<sub>0</sub>)** are *terms*, and denote *situations*. They can appear anywhere we would expect a situation.
- So we can say things like **S<sub>1</sub>=Result(Move(A,B),S<sub>0</sub>)**,  
**On(A,B,Result(Move(A,B),S<sub>0</sub>))**,  
**On(A,B,S<sub>1</sub>)**, etc.  
(Alternatively, **Holds(On(A,B),Result(Move(A,B),S<sub>0</sub>))**,  
etc.)

# Axiomatizing Actions

- Now, we can describe the results of actions, together with their preconditions.
- E.g., 'If nothing is on x and y, then one can move x to on top of y, in which case x will then be on y.'

$$\forall x,y,s \text{ Clear}(x,s) \wedge \text{Clear}(y,s) \\ \Rightarrow \text{On}(x,y, \text{Result}(\text{Move}(x,y),s))$$

- Alternatively:

$$\forall x,y,s \\ \text{Holds}(\text{Clear}(x),s) \wedge \text{Holds}(\text{Clear}(y),s) \\ \Rightarrow \text{Holds}(\text{On}(x,y), \text{Result}(\text{Move}(x,y),s))$$

- This is an *effect axiom*.
  - It includes a precondition as well.

# Situation Calculus

- This approach is called the *situation calculus*.
- We axiomatize all our actions, then use a general theorem prover to prove that a situation exists in which our goal is true.
- The actions in the proof would comprise our plan.

# A Very Simple Example

**KB:**

**On(A,Table,S<sub>0</sub>)**

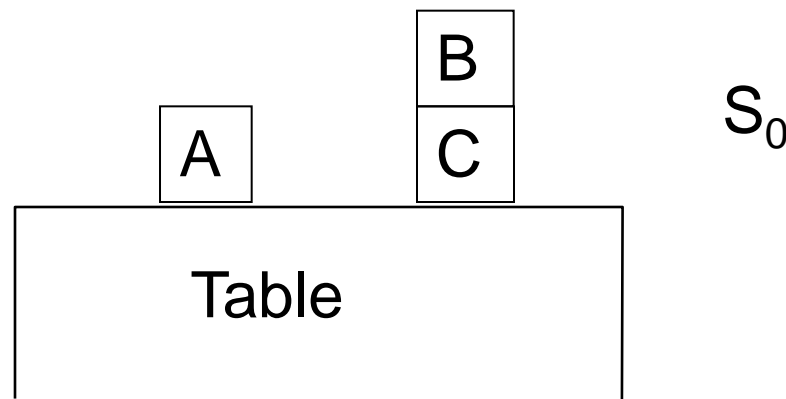
**On(B,C,S<sub>0</sub>)**

**On(C,Table,S<sub>0</sub>)**

**Clear(A,S<sub>0</sub>)**

**Clear(B,S<sub>0</sub>)**

and axioms about actions, etc.



**Goal:**

**∃s'. On(A,B,s')**

# What happens?

- We try to prove **On(A,B,s')** for some s'
  - Find axiom
$$\forall x,y,s. \text{Clear}(x,s) \wedge \text{Clear}(y,s) \Rightarrow \text{On}(x,y,\text{Result}(\text{Move}(x,y),s))$$
  - By chaining, e.g., goal would be true if we could prove **Clear(A,s)  $\wedge$  Clear(B,s)** by backward chaining.
  - But both are true in **S<sub>0</sub>**, so we can conclude **On(A,B,Result(Move(A,B),S<sub>0</sub>))**
- We are done!
- We look in the proof and see only one action, **Move(A,B)**, which is executed in situation **S<sub>0</sub>**, so this is our **plan**.

# Tougher Example: Same Initial World, Harder Goal

KB:

$\text{On}(A, \text{Table}, S_0)$

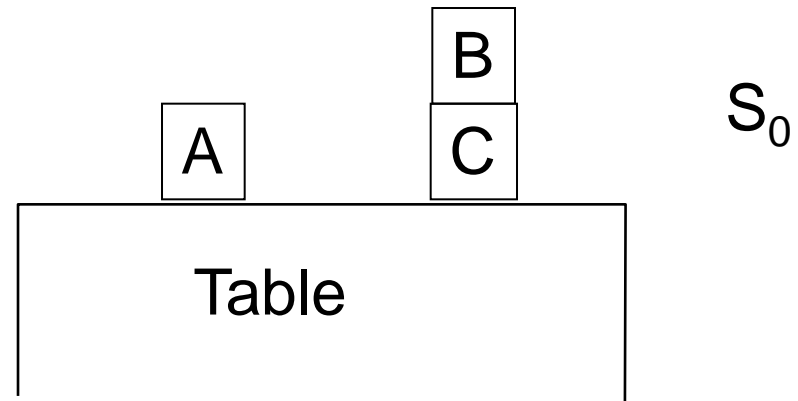
$\text{On}(B, C, S_0)$

$\text{On}(C, \text{Table}, S_0)$

$\text{Clear}(A, S_0)$

$\text{Clear}(B, S_0)$

and axioms about actions, etc.



Goal:

$\exists s' \text{On}(A, B, s') \wedge \text{On}(B, C, s')$

(Intuitively, really not harder: B already on C, and we just showed how to make A on B.)

# With Goal $\text{On}(A,B,s') \wedge \text{On}(B,C,s')$

- Suppose we try to prove the first subgoal,  $\text{On}(A,B,s')$ .
  - Use same axiom
$$\forall x,y,s. \text{Clear}(x,s) \wedge \text{Clear}(y,s) \Rightarrow \text{On}(x,y, \text{Result}(\text{Move}(x,y),s))$$
  - Again, by chaining, we can conclude  $\text{On}(A,B,\text{Result}(\text{Move}(A,B),S_0))$ .
  - Abbreviating  $\text{Result}(\text{Move}(A,B),S_0)$  as  $S_1$ , we have  $\text{On}(A,B,S_1)$ .
- Substituting for  $s'$  in our other subgoal makes that  $\text{On}(B,C,S_1)$ . If this is true, we're done.
- But we have **no reason to believe this is true!**
- Sure,  $\text{On}(B,C,S_0)$ , but how does the planner know this is still true, i.e.,  $\text{On}(B,C,S_1)$ ?
- In fact, it doesn't, so it fails to find an answer!

# The Frame Problem

- We have failed to express the fact that everything that **isn't changed by an action stays the same**.
- Can fix by adding *frame axioms*. E.g.:  
 $\forall x,y,z,s.$   
**Clear(x,s)  $\Rightarrow$  Clear(x, Result(Paint(x,y),s))**  
...
- There are *lots* of these!
- Is this a big problem?

# Better Frame Axioms

- Can fix with neater formulation:

$\forall x,y,s,a.$

$$\text{On}(x,y,s) \wedge (\forall z. a=\text{Move}(x,z) \Rightarrow y=z) \\ \Rightarrow \text{On}(x,y,\text{Result}(a,s))$$

- Can combine with effect axioms to get *successor-state axioms*:

$\forall x,y,s,a.$

$$\text{On}(x,y,\text{Result}(a,s)) \Leftrightarrow \\ \text{On}(x,y,s) \wedge (\forall z. a=\text{Move}(x,z) \Rightarrow y=z) \\ \vee (\text{Clear}(x,s) \wedge \text{Clear}(y,s) \wedge a=\text{Move}(x,y))$$

# How Does This Help Our Example?

- We want to prove

**On(B,C,Result(Move(A,B),S<sub>0</sub>))** given that **On(B,C,S<sub>0</sub>)**

Axiom says

$\forall x,y,s,a. \text{On}(x,y,\text{Result}(a,s)) \Leftrightarrow$

$\text{On}(x,y,s) \wedge (\forall z. a=\text{Move}(x,z) \Rightarrow y=z)$

$\vee (\text{Clear}(x,s) \wedge \text{Clear}(y,s) \wedge a=\text{Move}(x,y))$

- So need to show

**On(B,C,S<sub>0</sub>)**  $\wedge$  **( $\forall z. \text{Move}(A,B)=\text{Move}(B,z) \Rightarrow C=z$ )** is true, which is easy

- The first conjunct is in the KB.
- The second one is true since actions are the same only if they have the same name and involve the exact same objects i.e.

$A(x_1, \dots, x_m) = A(y_1, \dots, y_m)$  iff  $x_1 = y_1 \wedge \dots \wedge x_m = y_m$

so **Move(A,B)=Move(B,z)** is false.

Note: Another assumption in KB:  $A(x_1, \dots, x_m) \neq B(y_1, \dots, y_n)$

These are known as  
Unique Action Axioms



# For Refutation Theorem Proving: (Dual) Skolemisation

- Suppose  $\forall x. \exists y. G(x,y)$  is goal in resolution **refutation**.
- So, we need to **negate** the goal:

$$\neg \forall x. \exists y. G(x,y) \equiv \exists x. \forall y. \neg G(x,y)$$

- Then skolemise (i.e drop existential quantifier):

$$\neg G(X_0,y)$$

- Intuition:
  - $y$  is to be unified to construct **witness**.
  - $X_0$  must **not** be instantiated.
- Similar story for GMP, but goal **not** negated, i.e.  $G(X_0,y)$ , for some  $y$ , is used as the goal.

# KB and Axioms as Clauses

Constants: A, B, C, S0

Variables: a, x, y, s

## Initial State

$\text{On}(A, \text{Table}, S0)$

$\text{On}(B, C, S0)$

$\text{On}(C, \text{Table}, S0)$

$\text{Clear}(A, S0)$

$\text{Clear}(B, S0)$

## Goal

$\neg \text{On}(A, B, s') \vee \neg \text{On}(B, C, s')$

## Effect Axiom

$\neg \text{Clear}(x, s) \vee \neg \text{Clear}(y, s) \vee$

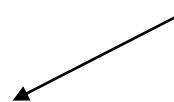
$\text{On}(x, y, \text{Result}(\text{Move}(x, y), s))$

## Frame Axioms

$\neg \text{On}(x, y, s) \vee a = \text{Move}(x, Z(x, y, z, s, a)) \vee \text{On}(x, y, \text{Result}(a, s))$

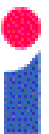
$\neg \text{On}(x, y, s) \vee \neg y = Z(x, y, z, s, a) \vee \text{On}(x, y, \text{Result}(a, s))$

Skolem function

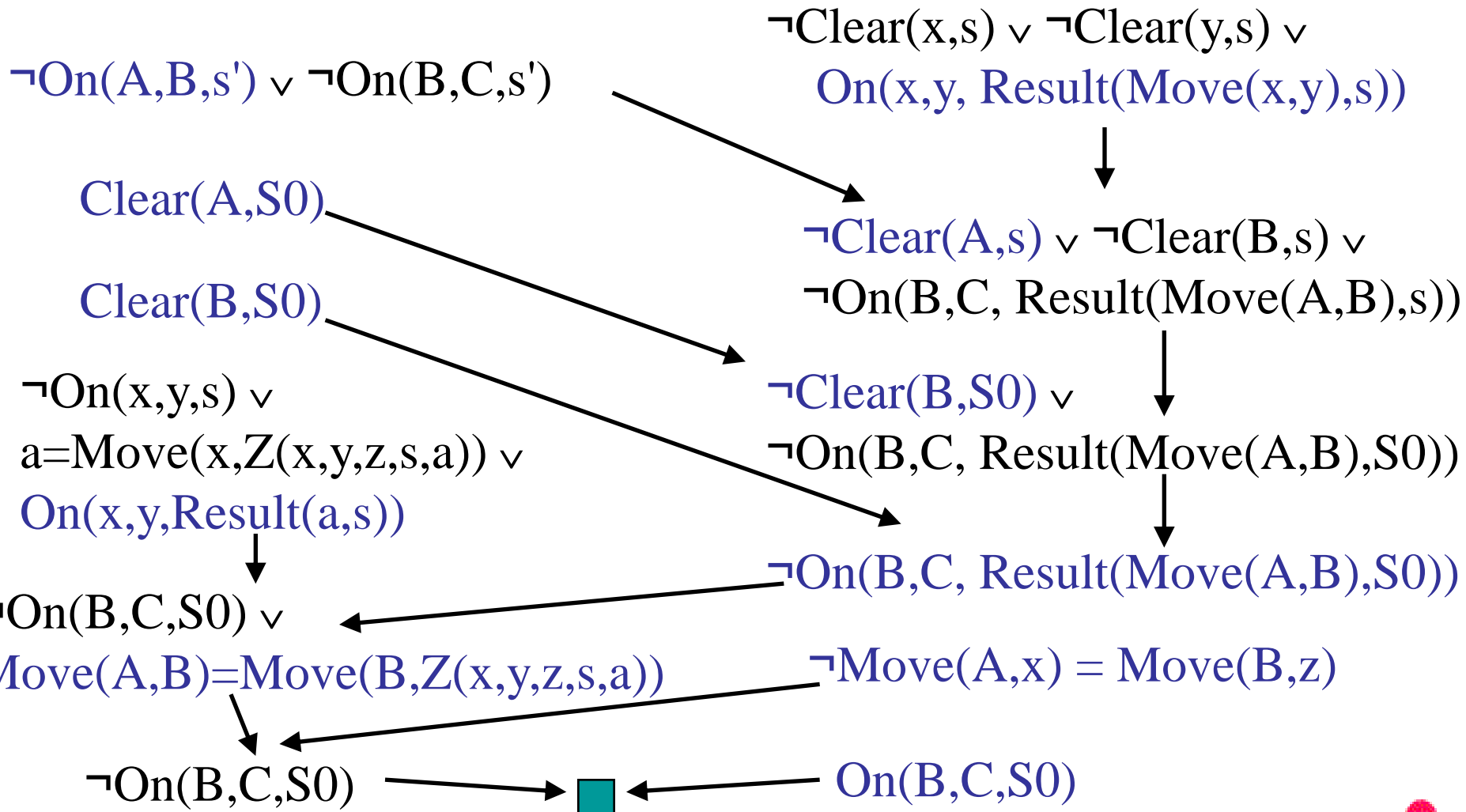


Unique Action Axioms:  $\neg \text{Move}(A, B) = \text{Move}(B, z)$ , etc

Unique Name Axiom: disequality for every pair of constants in KB



# Resolution Refutation



# Frame problem partially solved

- . This solves the representational part of the frame problem.
- . Still have to compute that everything that was true that wasn't changed is still true. Inefficient (as is general theorem proving).
- . Solution: Special purpose representations, special purpose algorithms, called *Planners*.

# Summary

- Planning
- Situations
- Frame problem