Inf2D 06: Logical Agents: Knowledge Bases and the Wumpus World

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Outline

- Knowledge-based agents
- Wumpus world
- Logic in general – models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability

Knowledge bases

Inference engine	$\longleftarrow$	domain-independent algorithms
Knowledge base	$\longleftarrow$	domain-specific content

- Knowledge base (KB): Set of sentences in a formal language
- Declarative approach to building a KB:
  - `Tell` it what it needs to know
- Then the agent can `Ask` the KB what to do
  - answers should follow from the KB
- KB can be part of agent or be accessible to many agents
- The agent's KB can be viewed at the knowledge level i.e., what it knows, regardless of how implemented
- Or at the implementation level
  - i.e., data structures in KB and algorithms that manipulate them

A simple knowledge-based agent

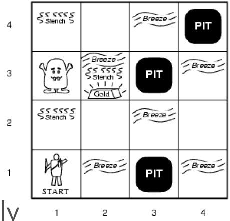
```
function KB-AGENT(percept) returns an action
  persistent  KB, a knowledge base
              t, a counter, initially 0, indicating time

  TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
  action ← ASK(KB, MAKE-ACTION-QUERY(t))
  TELL(KB, MAKE-ACTION-SENTENCE(action, t))
  t ← t + 1
  return action
```

– The agent must be able to:

- represent states, actions, etc.
- incorporate new percepts
- update internal representations of the world
- deduce hidden properties of the world
- deduce appropriate actions

Wumpus World PEAS description

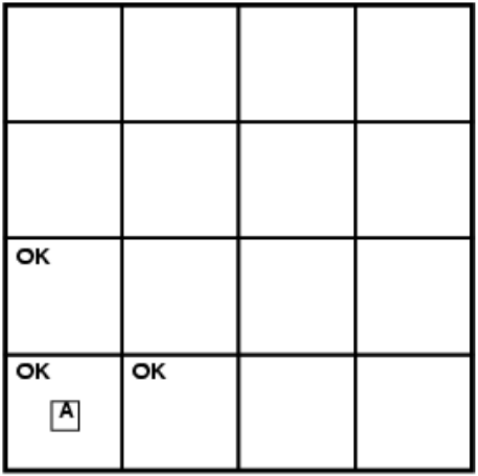


– Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using arrow
– Environment
  - Squares adjacent to Wumpus are smelly
  - Squares adjacent to pits are breezy
  - Glitter iff gold is in the same square
  - Shooting kills Wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square
– Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot
– Sensors: Stench, Breeze, Glitter, Bump, Scream

Wumpus world characterization

- Fully Observable? No – only local perception
- Deterministic? Yes – outcomes exactly specified
- Episodic? No – sequential at the level of actions
- Static? Yes – Wumpus and Pits do not move
- Discrete? Yes
- Single-agent? Yes – Wumpus is essentially a natural feature

Exploring a Wumpus world



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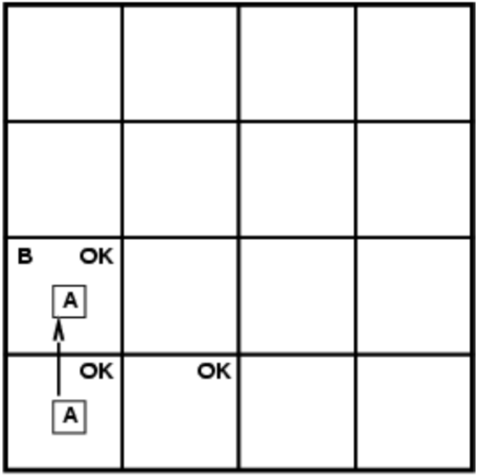

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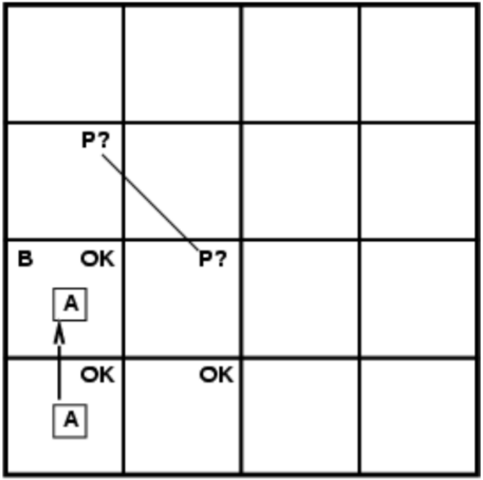

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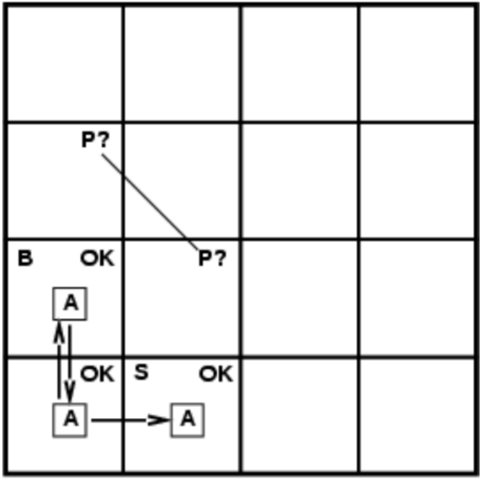

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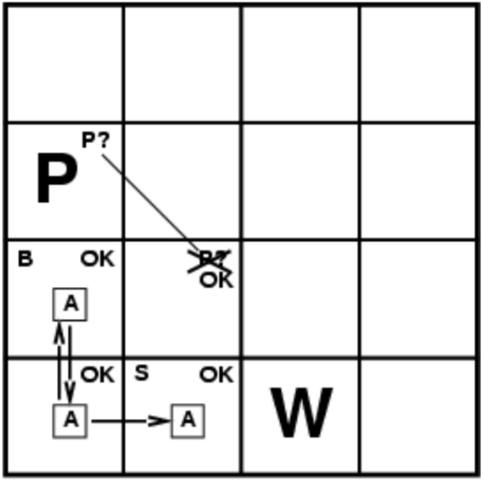

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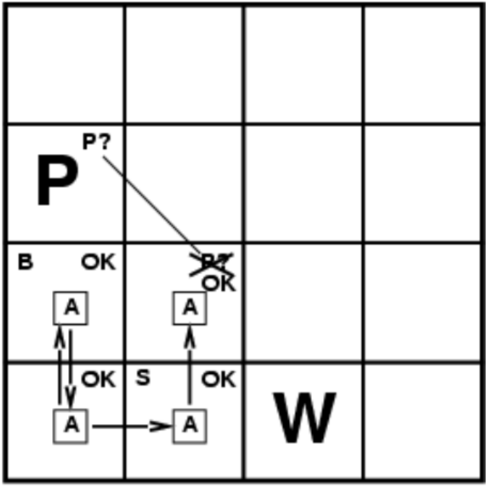

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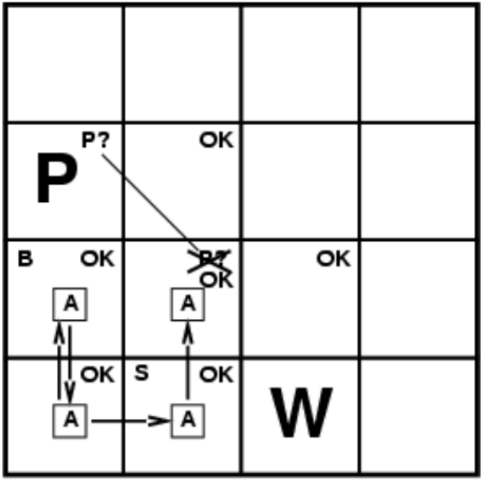

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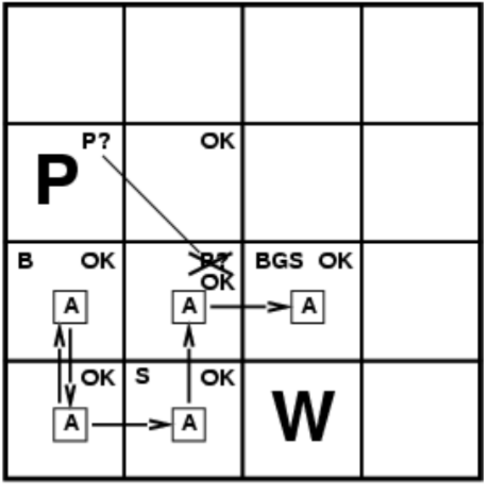


Logic in general

- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics defines the “meaning” of sentences
  - i.e., define truth of a sentence in a world
- E.g., the language of arithmetic
  - $x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence
  - $x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y
  - $x + 2 \geq y$ is true in a world where $x = 7$, $y = 1$
  - $x + 2 \geq y$ is false in a world where $x = 0$, $y = 6$

Entailment

- Entailment means that one thing follows from another:

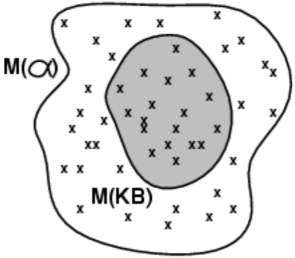
$$KB \models \alpha$$

- Knowledge base KB entails sentence α if and only if α is true in all worlds where KB is true
  - e.g., the KB containing "Celtic won" and "Hearts won" entails "Celtic won or Hearts won"
  - Considering only worlds where Celtic plays Hearts (and no draws) it entails "Either Celtic won or Hearts won"
  - e.g., $x + y = 4$ entails $4 = x + y$
  - Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

Models

- Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated
- We say m **is a model** of a sentence α if α is true in m

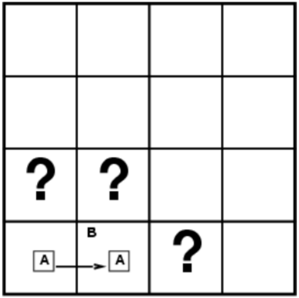
- $M(\alpha)$ is the set of all models of α
- Then $KB \models \alpha$ iff $M(KB) \subseteq M(\alpha)$
- The *stronger* an assertion, the fewer models it has.



Entailment in the Wumpus world

Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

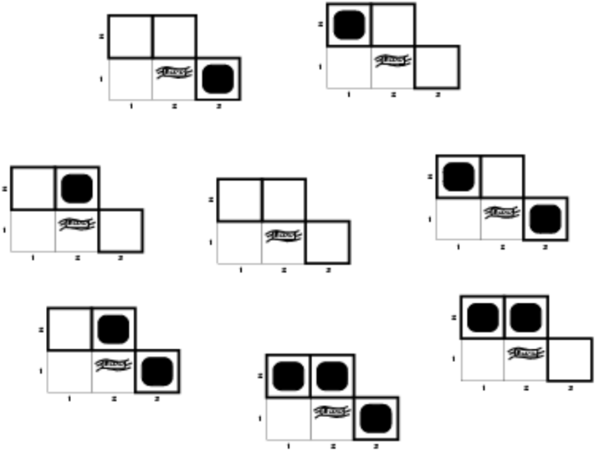
Consider possible models for KB assuming only pits



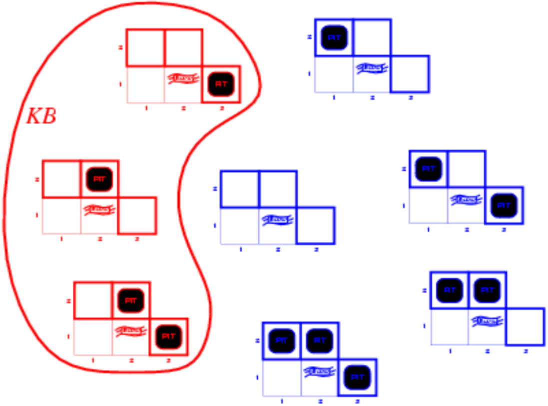
3 Boolean choices $\Longrightarrow$ 8 possible models

Mid-lecture Exercise: What are these 8 models?

Wumpus models

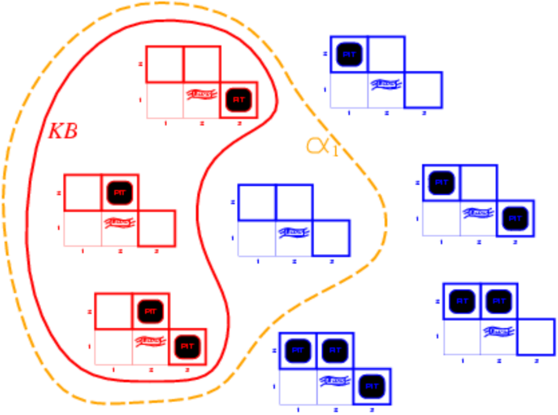


Wumpus models



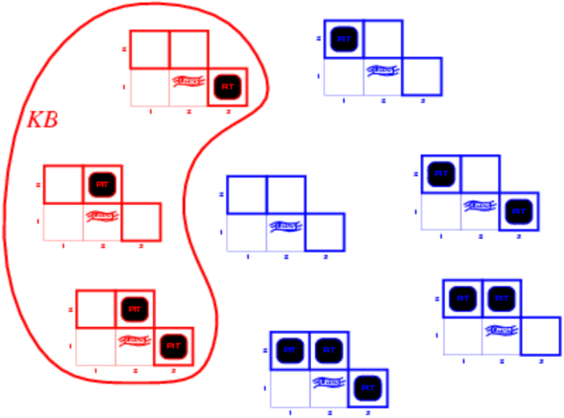
– KB = Wumpus-world rules + observations

Wumpus models



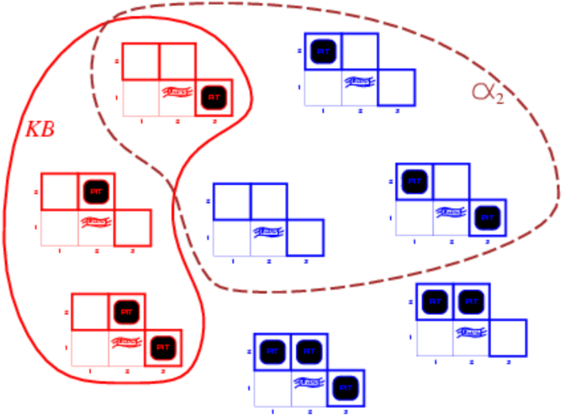
- KB = Wumpus-world rules + observations
- α_1= "[1,2] has no pit", $KB \models \alpha_1$, proved by model checking
  - In every model in which KB is true, α_1 is also true

Wumpus models



– KB = Wumpus-world rules + observations

Wumpus models



- KB = Wumpus-world rules + observations
- α_2= "[2,2] has no pit", $KB \not\models \alpha_2$
  - In some models in which KB is true, α_2 is also true

Inference

- $KB \vdash_i \alpha$: sentence α can be derived from KB by an inference procedure i
- Soundness: i is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$
- Completeness: i is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$
- Preview: we will define first-order logic:
  - expressive enough to say almost anything of interest,
  - sound and complete inference procedure exists.
  - But first...

Propositional logic: Syntax

Propositional logic is the simplest logic – illustrates basic ideas:

- The proposition symbols P_1, P_2 etc. are sentences
- If S is a sentence, $\neg S$ is a sentence (negation)
- If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
- If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
- If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
- If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

– Each model specifies true/false for each proposition symbol

e.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.

– Rules for evaluating truth with respect to a model m:

$\neg S$	is true	iff	S is false
$S_1 \wedge S_2$	is true	iff	S_1 is true and S_2 is true
$S_1 \vee S_2$	is true	iff	S_1 is true or S_2 is true
$S_1 \Rightarrow S_2$	is true	iff	S_1 is false or S_2 is true
i.e.	is false	iff	S_1 is true and S_2 is false
$S_1 \Leftrightarrow S_2$	is true	iff	$S_1 \Rightarrow S_2$ is true and $S_2 \Rightarrow S_1$ is true

– Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \text{true} \wedge (\text{true} \vee \text{false}) = \text{true} \wedge \text{true} = \text{true}$$

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
true	false	false	false	true	false	false
true	true	false	true	true	true	true

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i,j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.

$$\neg P_{1,1}$$
$$\neg B_{1,1}$$
$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$
$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

Recall: α_1= `‘[1,2] has no pit”,

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	α_1
false	false	false	false	false	false	false	false	true
false	false	false	false	false	false	true	false	true
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	false	false	false	false	false	false	true
false	true	false	false	false	false	true	true	true
false	true	false	false	false	true	false	true	true
false	true	false	false	false	true	true	true	true
false	true	false	false	true	false	false	false	true
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	false

Inference by enumeration

- Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, α) returns true or false
   symbols ← a list of the proposition symbols in KB and α
   return TT-CHECK-ALL(KB, α, symbols, [])

function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
   if EMPTY?(symbols) then
      if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
      else return true
   else do
      P ← FIRST(symbols); rest ← REST(symbols)
      return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
             TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

- PL-TRUE? returns true if a sentence holds within a model
- EXTEND(P, *val*, *model*)returns a new partial model in which P has value *val*
- For n symbols, time complexity: $O(2^n)$, space complexity: $O(n)$

Logical equivalence

– Two sentences are logically equivalent iff true in the same models: $\alpha \equiv \beta$ iff $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) \quad \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) \quad \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) \quad \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) \quad \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \lor \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg\alpha \lor \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) \quad \text{de Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) \quad \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) \quad \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

Validity and satisfiability

– A sentence is valid if it is true in all models,

$$\text{e.g. true, } A \lor \neg A,\ A \Rightarrow A,\ (A \land (A \Rightarrow B)) \Rightarrow B$$

– Validity is connected to inference via the Deduction Theorem:

$$KB \models \alpha \text{ if and only if } (KB \Rightarrow \alpha) \text{ is valid}$$

– A sentence is satisfiable if it is true in some model

$$\text{e.g., } A \lor B,\ \ C$$

– A sentence is unsatisfiable if it is true in no models

$$\text{e.g., } A \land \neg A$$

– Satisfiability is connected to inference via the following:

$$KB \models \alpha \text{ if and only if } (KB \land \neg\alpha) \text{ is unsatisfiable}$$

Proof methods

Proof methods divide into (roughly) two kinds:

- Application of inference rules
  - Legitimate (sound) generation of new sentences from old
  - Proof = a sequence of inference rule applications.
    Can use inference rules as operators in a standard search algorithm
  - Typically require transformation of sentences into a normal form
  - Example: resolution
- Model checking
  - truth table enumeration (always exponential in n)
  - improved backtracking, e.g., Davis-Putnam-Logemann-Loveland (DPLL) method
  - heuristic search in model space (sound but incomplete)
    e.g., min-conflicts-like hill-climbing algorithms

Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences w.r.t. models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Does propositional logic provide enough expressive power for statements about the real world?