Inf2D 05: Informed Search and Exploration for Agents

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Outline

- Best-first search
- Greedy best-first search
- A^* search
- Heuristics
- Admissibility

Review: Tree search

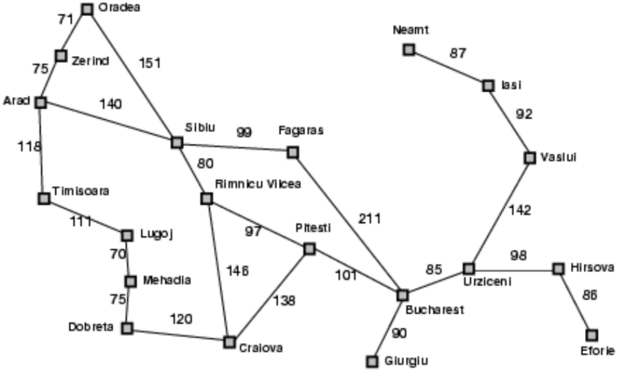
```
function TREE-SEARCH(problem) returns a solution, or failure
    initialize the frontier using the initial state of problem
    loop do
        if the frontier is empty then return failure
        choose a leaf node and remove it from the frontier
        if the node contains a goal state then return the corresponding solution
        expand the chosen node, adding the resulting nodes to the frontier
```

A search strategy is defined by picking the order of node expansion from the frontier

Best-first search

- An instance of general TREE-SEARCH or GRAPH-SEARCH
- Idea: use an evaluation function $f(n)$ for each node n
  - estimate of "desirability"
    ➜ Expand most desirable unexpanded node, usually the node with the lowest evaluation
- Implementation:
  Order the nodes in frontier in decreasing order of desirability
- Special cases:
  - Greedy best-first search
  - A^*search

Romania with step costs in km



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy best-first search

- Evaluation function $f(n) = h(n)$ (heuristic)
- $h(n)$: estimated cost of cheapest path from state at node n to a goal state
  - e.g., $h_{\mathrm{SLD}}(n)$: straight-line distance from n to goal (Bucharest)
  - Greedy best-first search expands the node that appears to be closest to goal

Added slide: Heuristic

What is a heuristic?

- From the greek word "*heuriskein*" meaning "*to discover*" or "*to find*"
- A heuristic is any method that is believed or practically proven to be useful for the solution of a given problem, although there is no guarantee that it will always work or lead to an optimal solution.
- Here we will use heuristics to guide tree search. This may not change the worst case complexity of the algorithm, but can help in the average case.
- We will introduce conditions (admissibility, consistency, see below) in order to identify good heuristics, i.e. those which actually lead to an improvement over uninformed search.
- See also: https://en.wikipedia.org/wiki/Heuristic

Greedy best-first search example

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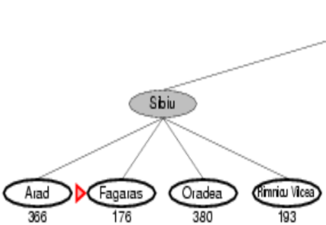
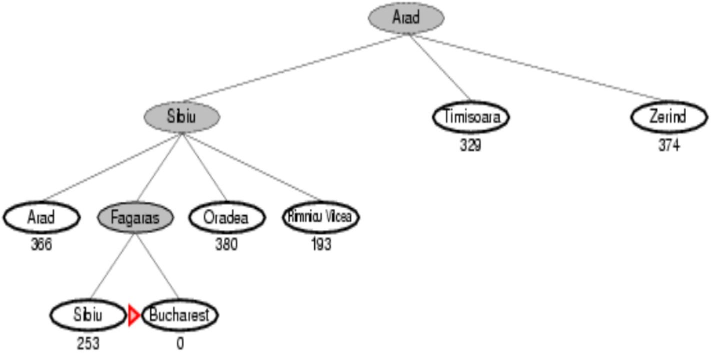


Greedy best-first search example



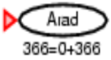
Properties of greedy best-first search

- Complete? No – can get stuck in loops
  - Graph search version is complete in finite space, but not in infinite ones
- Time? $O(b^m)$ for tree version, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ – keeps all nodes in memory
- Optimal? No

A^* search

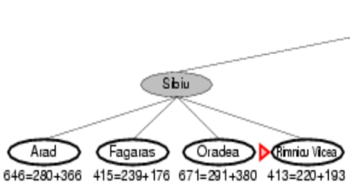
- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$: cost so far to reach n
  - $h(n)$: estimated cost from n to goal
  - $f(n)$: estimated total cost of path through n to goal
- A^* is both complete and optimal if $h(n)$ satisfies certain conditions

A^* search example

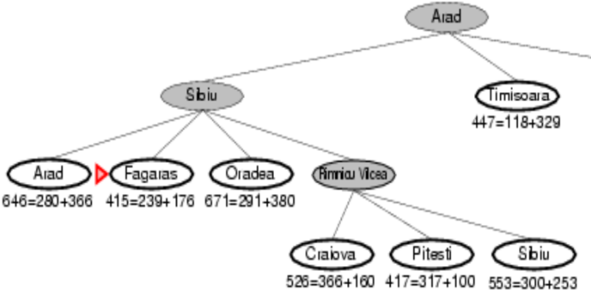


A^* search example

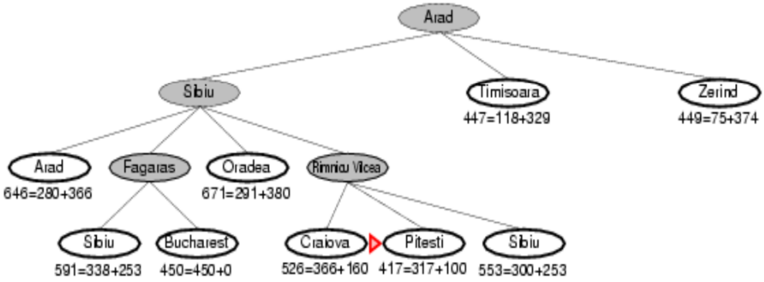
A^* search example



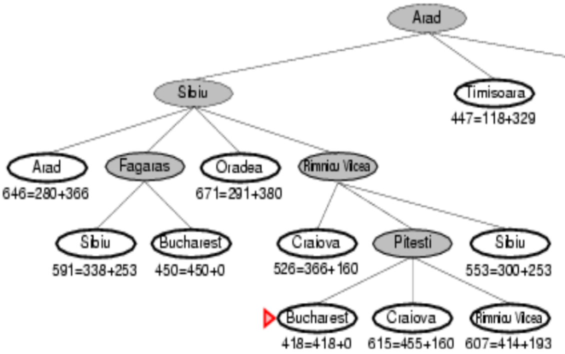
A^* search example



A^* search example



A^* search example

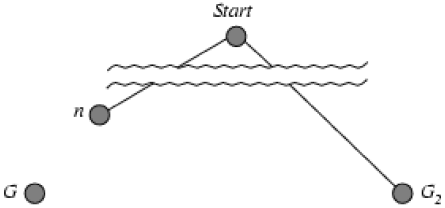


Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
  - Thus, $f(n) = g(n) + h(n)$ never overestimates the true cost of a solution
- Example: $h_{\mathrm{SLD}}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A^* using TREE-SEARCH is optimal.

Optimality of A^* (proof)

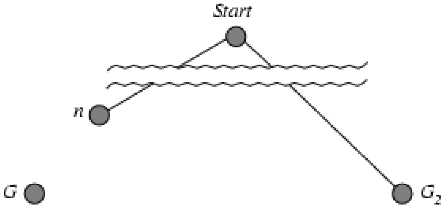
- Suppose some suboptimal goal G_2 has been generated and is in the frontier. Let n be an unexpanded node in the frontier such that n is on a shortest path to an optimal goal G.



- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $f(G) = g(G)$ since $h(G) = 0$
- $g(G_2) > g(G)$ since G_2 is suboptimal
- $f(G_2) > f(G)$ from above

Optimality of A^* (proof cntd.)

– Suppose some suboptimal goal G_2 has been generated and is in the frontier. Let n be an unexpanded node in the frontier such that n is on a shortest path to an optimal goal G.



– $f(G) < f(G_2)$ from above (G_2 is suboptimal)

– $h(n) \leq h^*(n)$ since h is admissible

– $g(n) + h(n) \leq g(n) + h^*(n) = f(G)$

– $f(n) \leq f(G)$

Hence $f(n) < f(G_2) \Rightarrow A^*$ will never select G_2 for expansion.

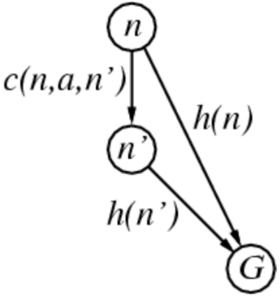
Consistent heuristics

– A heuristic is consistent if for every node n, every successor n' of n generated by any action a,

$$h(n) \leq c(n, a, n') + h(n')$$

– If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &\geq f(n) \end{aligned}$$

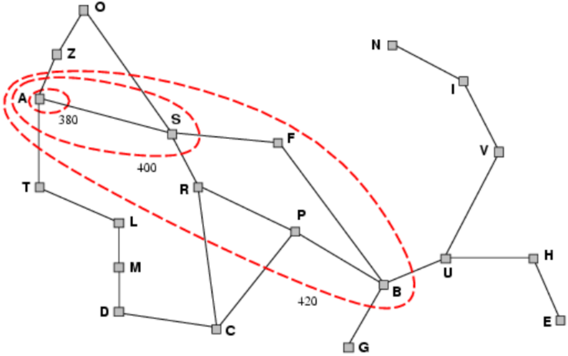


– i.e., $f(n)$ is non-decreasing along any path.

Theorem:
If $h(n)$ is consistent, A^* using GRAPH-SEARCH is optimal.

Optimality of A^*

- A^* expands nodes in order of increasing f value
- Gradually adds "f-contours" of nodes
- Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A^*

- Complete? Yes (unless there are infinitely many nodes with $f \leq f(G)$)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

Admissible heuristics

Example:

– for the 8-puzzle:

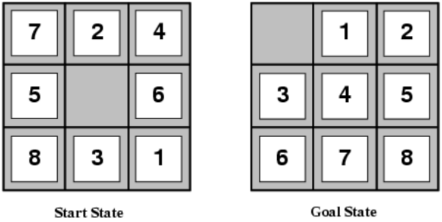
  - $h_1(n)$: number of misplaced tiles
  - $h_2(n)$: total Manhattan distance

  (i.e., no. of squares from desired location of each tile)

Exercise: Calculate these two values:

– $h_1(S) = ?$

– $h_2(S) = ?$



Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible) then
  - h_2 dominates h_1
  - h_2 is better for search
- Typical search costs (average number of nodes expanded):
  - $d = 12$ $\quad$ IDS $=$ 3,644,035 nodes
    $A^*(h_1) =$ 227 nodes
    $A^*(h_2) =$ 73 nodes
  - $d = 24$ $\quad$ IDS $=$ too many nodes
    $A^*(h_1) =$ 39,135 nodes
    $A^*(h_2) =$ 1,641 nodes

Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere,
  - then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square,
  - then $h_2(n)$ gives the shortest solution
- Can use relaxation to automatically generate admissible heuristics

Summary

Smart search based on heuristic scores.

- Best-first search
- Greedy best-first search
- A^* search
- Admissible heuristics and optimality.