Inf2D 04: Adversarial Search

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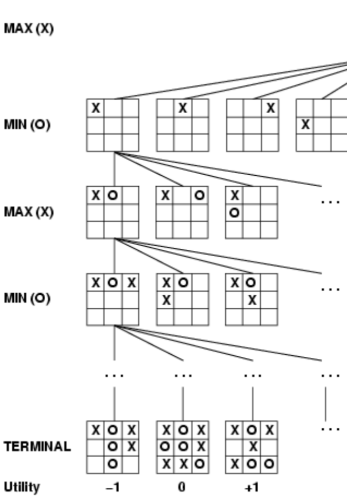
Outline

- Games
- Optimal decisions
- α-β pruning
- Imperfect, real-time decisions

Games vs. search problems

- We are (usually) interested in zero-sum games of perfect information
  - Deterministic, fully observable
  - Agents act alternately
  - Utilities at end of game are equal and opposite
- "Unpredictable" opponent ➜ specifying a move for every possible opponent reply
- Time limits ➜ unlikely to find goal, must approximate

Game tree (2-player, deterministic, turns)



– 2 players: MAX and MIN
– MAX moves first
– Tree built from MAX's POV

⟵ Utility of each terminal state from MAX's point of view.

Optimal Decisions

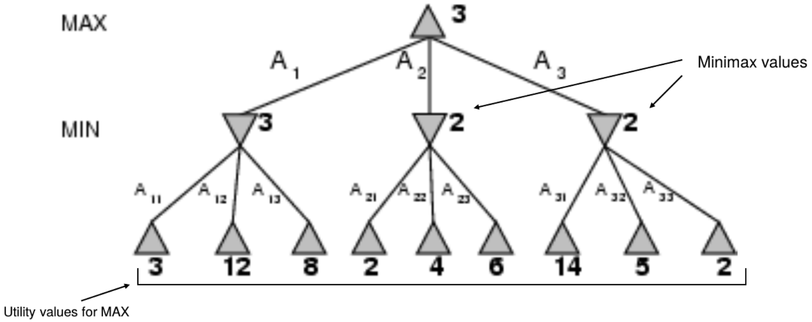
- Normal search: optimal decision is a sequence of actions leading to a goal state (i.e. a winning terminal state)
- Adversarial search:
  - MIN has a say in game
  - MAX needs to find a contingent strategy which specifies:
    - MAX's move in initial state then ...
    - MAX's moves in states resulting from every response by MIN to the move then ...
    - MAX's moves in states resulting from every response by MIN to all those moves, etc. ...

minimax value of a node=utility for MAX of being in corresponding state:

$$MINIMAX(s) = \begin{cases} UTILITY(s) & \text{if } TERMINAL\text{-}TEST(s) \\ \max_{a \in Actions(s)} MINIMAX(RESULT(s,a)) & \text{if } PLAYER(s) = \text{MAX} \\ \min_{a \in Actions(s)} MINIMAX(RESULT(s,a)) & \text{if } PLAYER(s) = \text{MIN} \end{cases}$$

Minimax

– Perfect play for deterministic games
– Idea: choose move to position with highest minimax value
  = best achievable payoff against best play
– Example: 2-ply game:



Minimax algorithm

```
function MINIMAX-DECISION(state) returns an action
   return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state, a))
-------------------------------------------------------------
function MAX-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for each a in ACTIONS(state) do
      v ← MAX(v, MIN-VALUE(RESULT(s, a)))
   return v
-------------------------------------------------------------
function MIN-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
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      v ← MIN(v, MAX-VALUE(RESULT(s, a)))
   return v
```

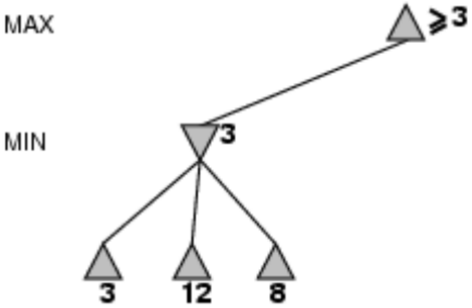
Idea: Proceed all the way down to the leaves of the tree then minimax values are backed up through tree

Properties of minimax

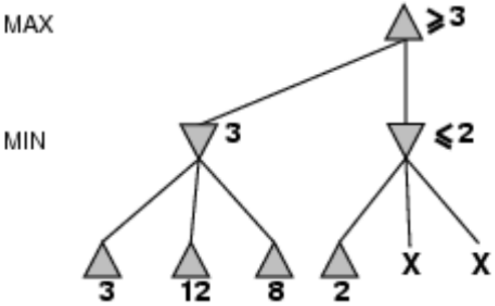
– Complete? Yes (if tree is finite)
– Optimal? Yes (against an optimal opponent)
– Time complexity? $O(b^m)$
– Space complexity? $O(bm)$ (depth-first exploration)

– For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
➜ exact solution completely infeasible!
➜ would like to eliminate (large) parts of game tree

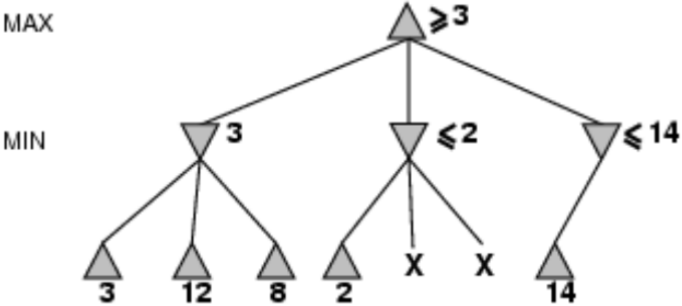
α-β pruning example



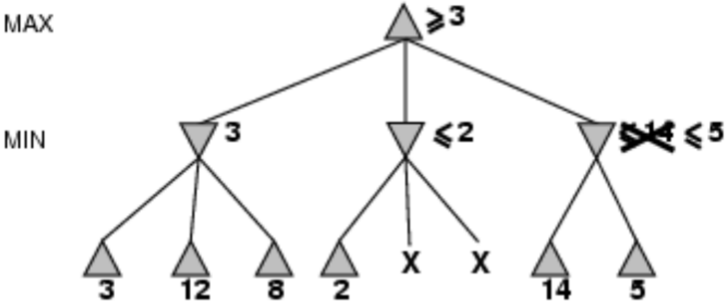
α-β pruning example



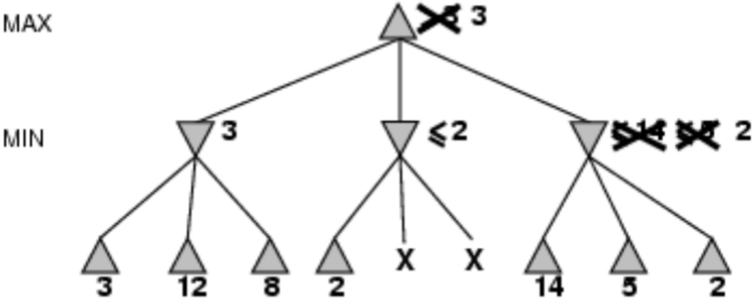
α-β pruning example



α-β pruning example



α-β pruning example



α-β pruning example

– Are minimax value of root and, hence, minimax decision independent of pruned leaves?

– Let pruned leaves have values u and v, then

$$\begin{aligned} \mathit{MINIMAX}(\mathit{root}) &= \max(\min(3,12,8),\min(2,u,v),\min(14,5,2)) \\ &= \max(3,\min(2,u,v),2) \\ &= \max(3,z,2) \text{ where } z \leq 2 \\ &= 3 \end{aligned}$$

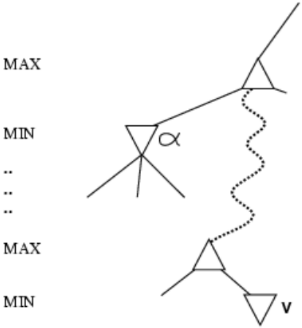
– Yes!

Properties of α-β

- Pruning does not affect final result (as we saw for example)
- Good move ordering improves effectiveness of pruning (How could previous tree be better?)
- With "perfect ordering", time complexity $O\left(b^{m/2}\right)$
  - branching factor goes from b to $\sqrt{b}$
  - (alternative view) doubles depth of search compared to minimax
- A simple example of the value of reasoning about which computations are relevant (a form of meta-reasoning)

Why is it called α-β?

- α is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for MAX
- If v is worse than α, MAX will avoid it
  ➜ prune that branch
- Define β similarly for MIN



The α-β algorithm

```
function ALPHA-BETA-SEARCH(state) returns an action
   v ← MAX-VALUE(state, −∞, +∞)
   return the action in ACTIONS(state) with value v

function MAX-VALUE(state, α, β) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for each a in ACTIONS(state) do
      v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
      if v ≥ β then return v
      α ← MAX(α, v)
   return v
```

Prune as this value is worse for MIN and so won't ever be chosen by MIN!

– α is value of the best i.e. highest-value choice found so far at any choice point along the path for MAX

– β is value of the best i.e. lowest-value choice found so far at any choice point along the path for MIN

The α-β algorithm

function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for each *a* **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

Prune as this value is worse for MAX and so won't ever be chosen by MAX!

Resource limits

– Suppose we have 100 secs, explore 10^4 nodes/sec
  ➜ 10^6 nodes per move

– Standard approach:

  - cutoff test: e.g., depth limit (perhaps add quiescence search, which tries to search interesting positions to a greater depth than quiet ones)

– evaluation function
  = estimated desirability of position

Evaluation functions

– For chess, typically linear weighted sum of features

$$EVAL(s) = w_1 f_1(s) + w_2 f_2(s) + ... + w_n f_n(s)$$

where each w_i is a weight and each f_i is a feature of state s

– Example
  - queen = 1, king = 2, etc.
  - f_i: number of pieces of type i on board
  - w_i: value of the piece of type i

Cutting off search

- Minimax Cutoff is identical to MinimaxValue except
  - TERMINAL-TEST is replaced by CUTOFF
  - UTILITY is replaced by EVAL
- Does it work in practice?
  $b^m = 10^6$, $b = 35$ ➜ $m = 4$
- 4-ply lookahead is a hopeless chess player!
  - 4-ply ≈ human novice
  - 8-ply ≈ typical PC, human master
  - 12-ply ≈ Deep Blue, Kasparov

Deterministic games in practice

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used a precomputed endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 444 billion positions.
- Chess: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.
- Othello: human champions refuse to compete against computers, who are too good.
- Go: human champions used to refuse to compete against computers, who are too bad. In Go, b ¿ 300, so most programs use pattern knowledge bases to suggest plausible moves. 2016: AlphaGo

Summary

- Games are fun to work on!
- They illustrate several important points about AI
- Perfection is unattainable ➜ must approximate
- Good idea to think about what to think about