Lecture 2: Data Representation

- Data need to be represented in a convenient way that simplifies:
  - common operations: addition, comparison, etc.
    - How would an algorithm for adding Roman numbers look like?
  - hardware implementation (cheap, fast, reliable computers)

- Data representation is a major part of the software-hardware interface

Lecture outline

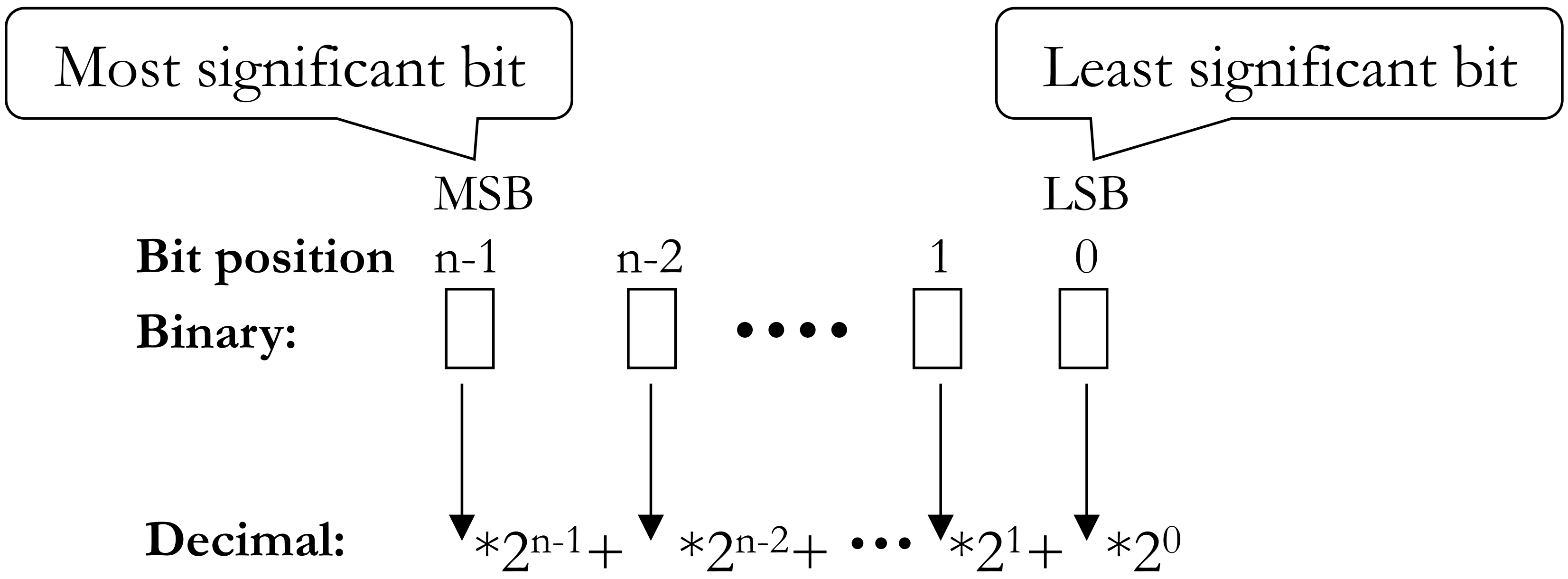
- The bit – atomic unit of data
- Representing numbers
- Representing text

The bit

- Acronym for Binary digiT
- The smallest amount of meaningful info, according to information theory
  - If only 1 value is possible, there is no information
- Disadvantages: too little information per bit, must use many of them
- Advantages: easy to do computation, very reliable, simple circuits

Natural numbers representation

- Positive (unsigned) integers are very simple to represent in binary



Basic operations

- Addition, subtraction with binary numbers is easy:

```
  1 1 1 1            0 0 1 0
   01101   (13)       01101
  +01011   (11)      -01011
  ------             ------
   11000   (24)       00010   (2)
```

Fixed bit-length arithmetic

- Hardware cannot handle infinite long bit sequences
- We end up with a few fixed sized data types
  – Byte: always 8 bits
  – Word: the 'natural' unit of access, usually 32 bits
- Overflow happens when a result does not fit
  – Numbers wrap-around when they become too large
  – Comp. arithmetic is modulo 2^n, n=number of bits

What about negative numbers?

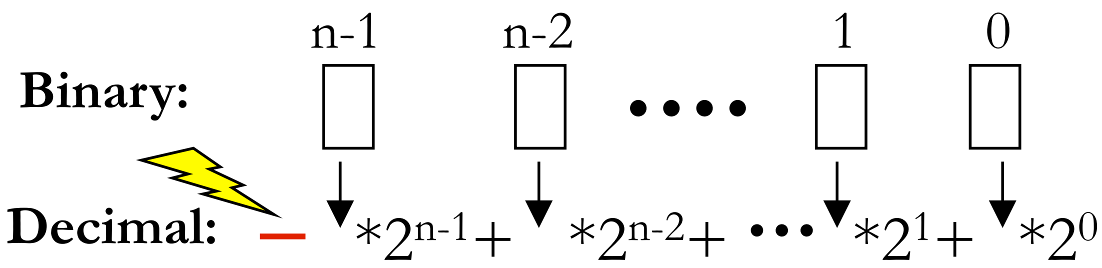
- No special symbols, e.g. +, −, available
- Sign-magnitude representation:
  – Use 1st bit (MSB) as the sign: 1-negative, 0-positive
- Complicates the arithmetic operations
  – The actual operation depends on the sign
- There is a better way

Two's complement representation

- What is the result of 000 – 001?

  …111

- The MSB has negative weighting:



- Arithmetic operations do not depend on the operands' signs

2's complement quirks

- The MSB is the sign
- Range is asymmetric: -2^{n-1} to $2^{n-1}-1$
- There are two kinds of overflows:
  - – Positive overflow produces a negative number
  - – Negative underflow produces a positive number
- To negate a number

  Invert all bits ($0 \leftrightarrow 1$) and add 1, at the LSB

  $-(-2^{n-1})$ overflows!
- A-B = A + 2's complement of B

Converting between data types

- Converting from a smaller to a larger representation is done by sign extension

Example: from byte to short (16 bits):

2 = 0 0 0 0 0 0 1 0 ⇒ ? ? ? ? ? ? ? ? 0 0 0 0 0 0 1 0

-2 = 1 1 1 1 1 1 1 0 ⇒ ? ? ? ? ? ? ? ? 1 1 1 1 1 1 1 0

-2 = 11111110 ⇒ 1111111111111110 (byte) (short)

2 = 00000010 ⇒ 0000000000000010 (byte) (short)

Shifting

- Shifting: move the bits of a data type left or right
  - Data bits falling off the edge are lost
- 0s fill up the empty bit places for left shifts
- For right shifts, two options:
  - Fill with 0: for non-numerical data (or positive integers)
  - Fill with the MSB: for 2's complement numbers
- Shift left by n is equivalent to multiplying by 2^n
- Shift right by n is equivalent to dividing by 2^n
- Example 6 = 0 0 0 0 0 1 1 0 >> 2 → 0 0 0 0 0 0 0 1 = 1

  -8 = 1 1 1 1 1 0 0 0 >> 2 → 1 1 1 1 1 1 1 0 = -2

Hexadecimal notation

- Binary numbers (and other data) are too long and tedious for us to use
- Hexadecimal (base 16) is very commonly used in computer programming
- Hex digits: 0-9 and A-F
  - A=10, B=11, …, F=15
- Conversion to/from binary is very easy:

  Every 4 bits correspond to 1 hex digit:

  1 1 1 1 1 0 0 0 = 0xF8

  F(15) 8

Hex is just a convenience, computers use the binary form

Real numbers - floating point

- Java's `float` **(32 bits)**
  `double` **(64 bits)**
- IEEE 754:
  – example 0.75 in base 10 ⇒ 0.11 in base 2

    $(2^{-1} + 2^{-2} = 0.5 + 0.25 = 0.75)$

  – Normalized:

    mantissa **exponent**

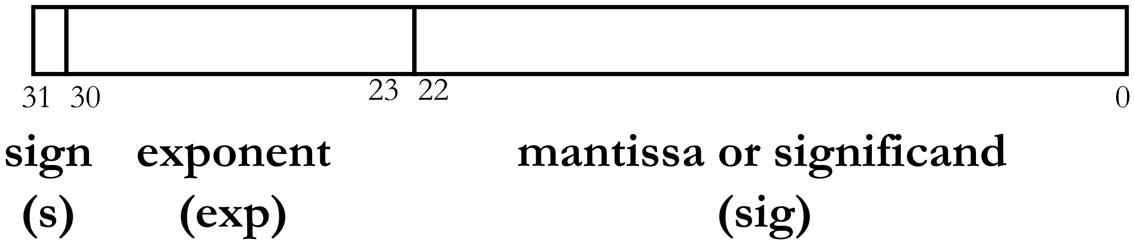
    $0.11 \Rightarrow 1.1 \times 2^{-1}$

    implict (always 1)

  – example: 25 in base 10 ⇒ 11001 in base 2 ⇒ 1.1001×2^{4}

Floating Point

- 32 bit:



$$(-1)^s \times (1.sig) \times 2^{exp-127}$$

e.g.,

$(0.75)_{10} \rightarrow (0.11)_2 \rightarrow (1.1\text{x}2^{-1})_2 \rightarrow$ 0 01111110 10000000000000000000000

- 64 bit:
  – exponent = 11 bits; significand = 52 bits
- Note: processors usually have specialized floating point units and extra fp registers to perform fp arithmetic

Representing characters, strings

- Characters need to be encoded in binary too
- Operations on characters have simpler requirements than on numbers, so the encoding choice is not crucial
- Most common representation is ASCII
  – Each character is held in a byte
  – E.g. ‘0’ is 0x30, ‘A’ is 0x41, ‘a’ is 0x61
- Java uses Unicode which can encode characters from many (all?) languages
  – 16 bits per character required
- Words, sentences, etc. are just strings of characters
  – A special character, encoded as 0x00, shows where the string ends (in C)
  – Or the string length is kept with the string itself (in Java)

Summary

- Computers use binary representation
- 2's complement
- Floating point
- Characters and strings