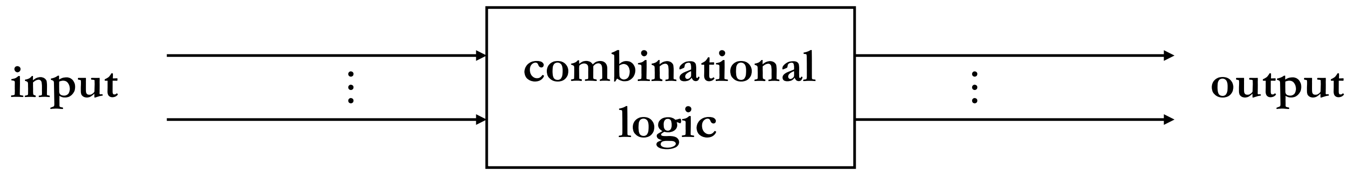
Lecture 7: Logic design

- Binary digital logic circuits:
  - Two voltage levels (ground and supply voltage) for 0 and 1
  - Built from transistors used as on/off switches
  - Analog circuits not very suitable for generic computing
  - Digital logic with more than two states is not practical

  Combinational logic: output depends only on the current inputs (no memory of past inputs)

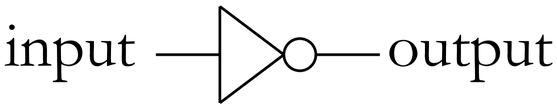


  Sequential logic: output depends on the current inputs as well as (some) previous inputs

Combinational logic circuits

- Inverter (or NOT gate): 1 input and 1 output

  "invert the input signal"

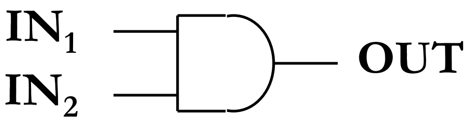


IN	OUT
0	1
1	0

$OUT = \overline{IN}$

- AND gate: minimum 2 inputs and 1 output

  "output 1 only if both inputs are 1"

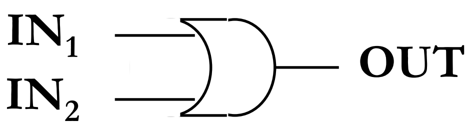


IN_1	IN_2	OUT
0	0	0
0	1	0
1	0	0
1	1	1

$OUT = IN_1 \, . \, IN_2$

Combinational logic circuits

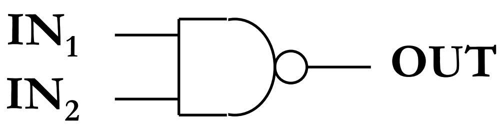
- OR gate:
  - "output 1 if at least one input is 1"



IN_1	IN_2	OUT
0	0	0
0	1	1
1	0	1
1	1	1

$$OUT = IN_1 + IN_2$$

- NAND gate:
  - "output 1 if both inputs are not 1" (NOT AND)



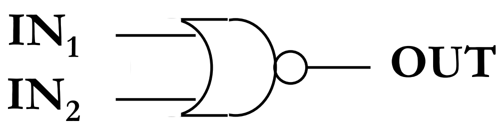
IN_1	IN_2	OUT
0	0	1
0	1	1
1	0	1
1	1	0

$$OUT = \overline{IN_1 . IN_2}$$

Combinational logic circuits

- NOR gate:

  "output 1 if no input is 1" (NOT OR)

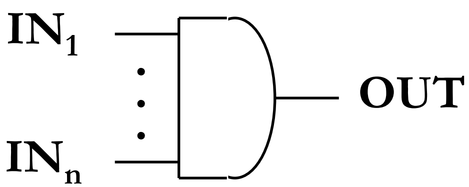


IN_1	IN_2	OUT
0	0	1
0	1	0
1	0	0
1	1	0

$$OUT = \overline{IN_1 + IN_2}$$

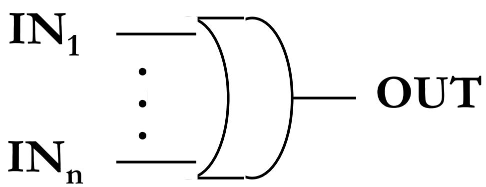
- Multiple-input gates:

AND



OUT = 1 if <u>all</u> IN_i=1

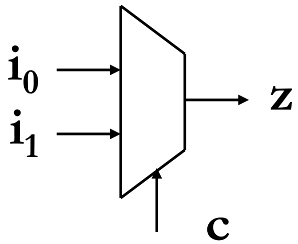
OR



OUT = 1 if <u>any</u> IN_i=1

Multiplexer

- Multiplexer: a circuit for selecting one of many inputs



$$z = \begin{cases} i_0, \text{ if } c=0 \\ i_1, \text{ if } c=1 \end{cases}$$

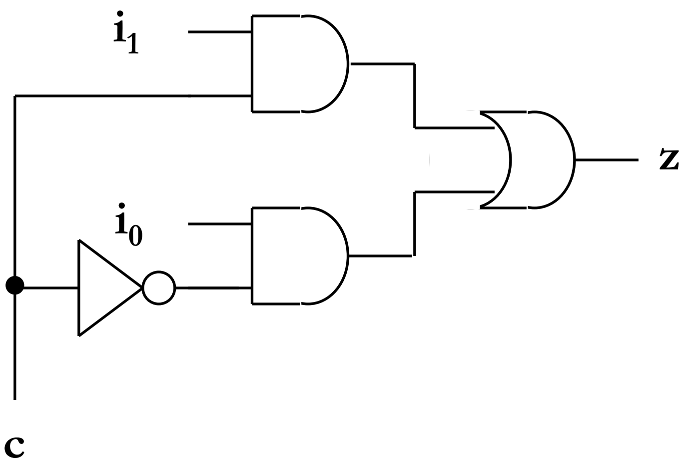
i_0	i_1	c	z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$$\begin{aligned} z &= \overline{i_0}.i_1.c + i_0.\overline{i_1}.\overline{c} + i_0.i_1.\overline{c} + i_0.i_1.c \\ &= \overline{i_0}.i_1.c + i_0.i_1.c + i_0.i_1.\overline{c} + i_0.\overline{i_1}.\overline{c} \\ &= (\overline{i_0} + i_0).i_1.c + i_0.(\overline{i_1} + i_1).\overline{c} \\ &= i_1.c + i_0.\overline{c} \end{aligned}$$

"sum of products form"

A multiplexer implementation

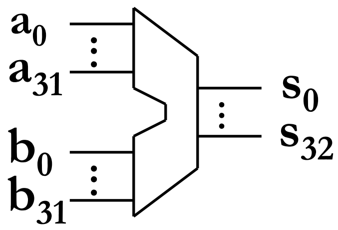
- Sum of products form: $\mathbf{i_1.c + i_0.\overline{c}}$
  - Can be implemented with 1 inverter, 2 AND gates and 1 OR gate:



- Sum of products is not practical for circuits with large number of inputs (n)
  - The number of possible products can be proportional to 2^n

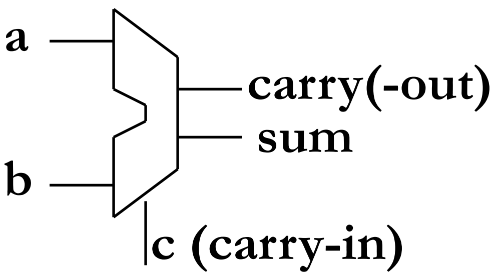
Arithmetic circuits

- 32-bit adder



64 inputs → too complex for sum of products

- Full adder:



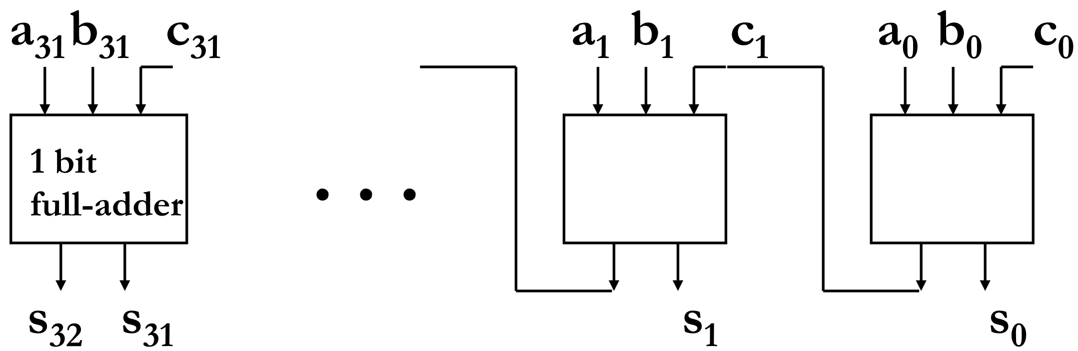
$$sum = \bar{a}.\bar{b}.c + \bar{a}.b.\bar{c} + a.\bar{b}.\bar{c} + a.b.c$$

$$carry = b.c + a.c + a.b$$

a	b	c	carry	sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Ripple carry adder

- 32-bit adder: chain of 32 full adders

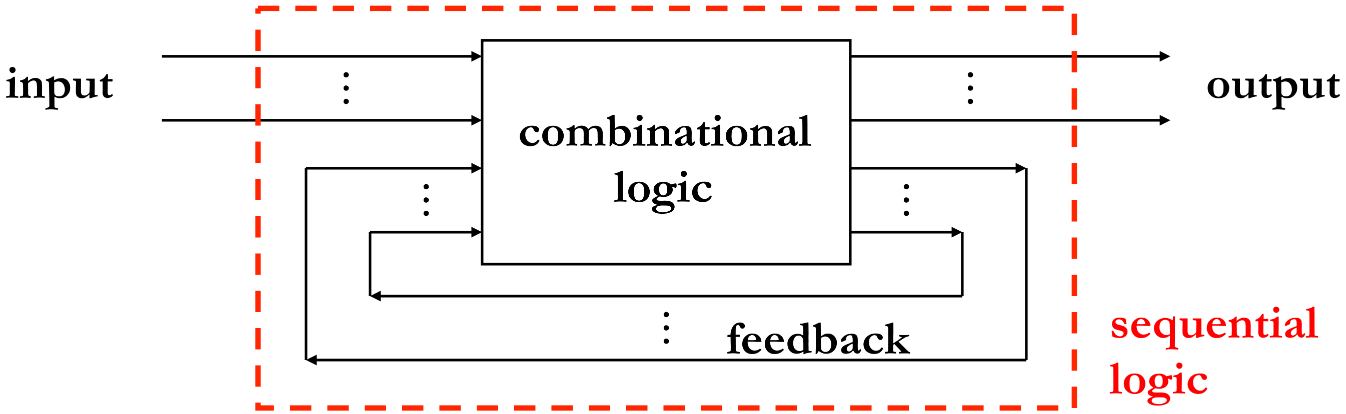


  – Carry bits c_i are computed in sequence c_1, c_2, … , c_{32} (where $c_{32} = s_{32}$), as c_i depends on c_{i-1}
  – Since sum bits s_i also depend on c_i, they too are computed in sequence

Propagation Delays

- Propagation delay = time delay between input signal change and output signal change at the other end
- Delay depends on technology (transistor, wire capacitance, etc.) and number of gates driven by the gate's output (**fan out**)
- e.g.: Sum of products circuits: 3 2-input gate delays (inverter, AND, OR) → very fast!
- e.g.: 32-bit ripple carry adder: 65 2-input gate delays (1 AND + 1 OR for each of 31 carries to propagate; 1 inverter + 1 AND + 1 OR for S_{31}) → slow

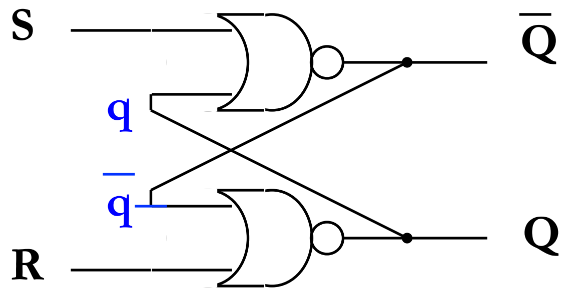
Sequential logic circuits



- Output depends on current inputs as well as past inputs
  – The circuit has memory
- Sequences of inputs generate sequences of outputs ⇒ sequential logic

Sequential logic circuits

- For a fixed input and *n* feedback signals, the circuit can have up to 2^n stable states
  - E.g. n=1 → one state if feedback signal = 0
    one state if feedback signal = 1
- Example: SR latch
  - Inputs: R, S
  - Feedback: q, $\overline{q}$
  - Output: Q

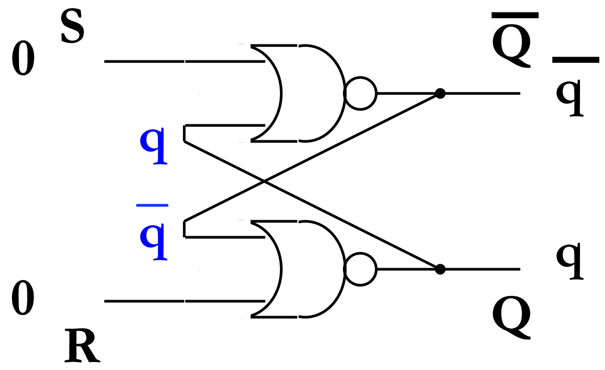


SR Latch

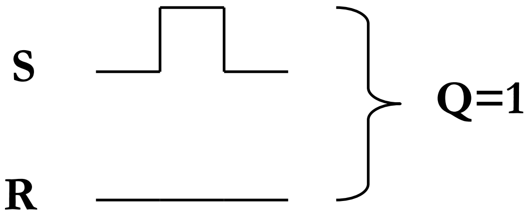
- Truth table:

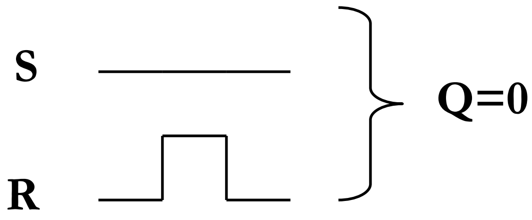
S	R	Q_i
0	0	Q_{i-1}
0	1	0
1	0	1
1	1	u

u=unused



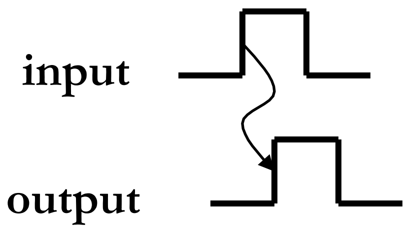
- Usage: 1-bit memory
  - Keep the value in memory by maintaining S=0 and R=0
  - Set the value in memory to 0 (or 1) by setting R=1 (or S=1) for a short time



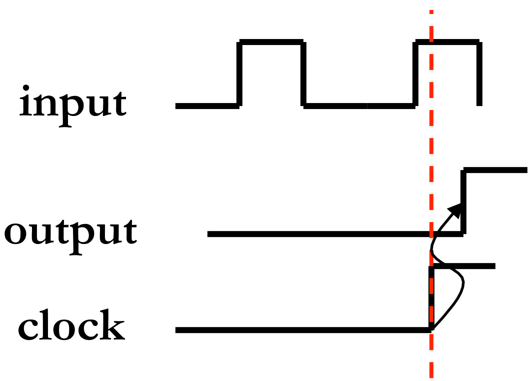


Timing of events

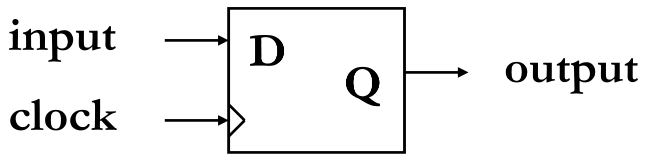
- Asynchronous sequential logic
  - State (and possibly output) of circuit changes whenever inputs change

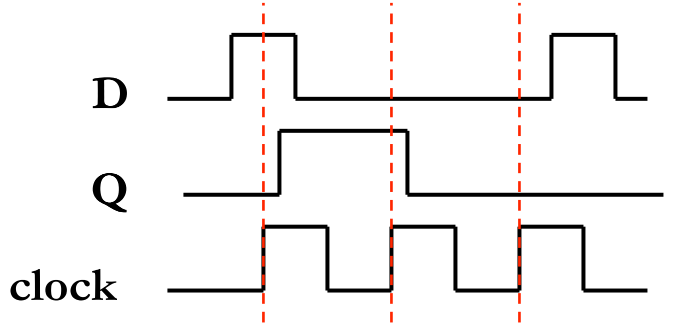


- Synchronous sequential logic
  - State (and possibly output) can only change at times synchronized to an external signal → the **clock**

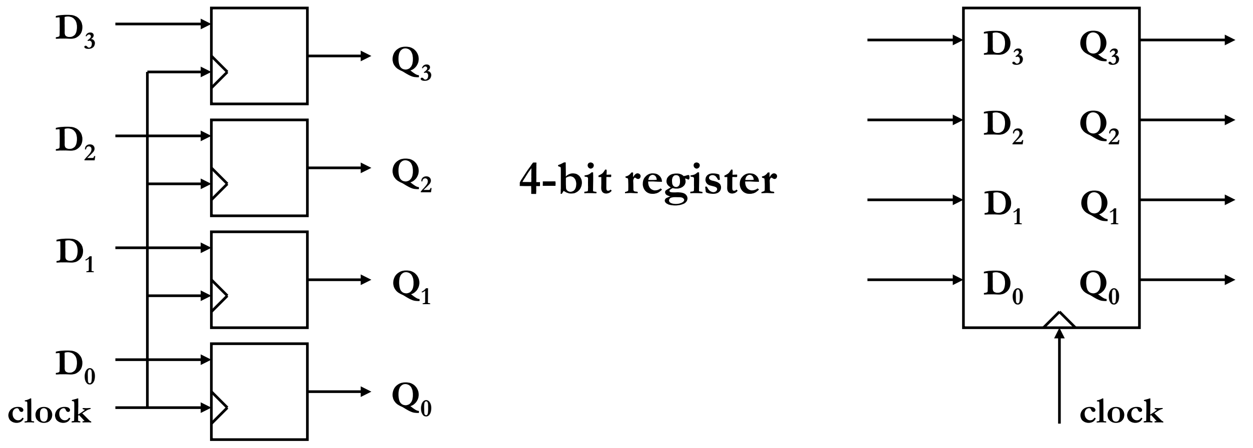


D flip-flop

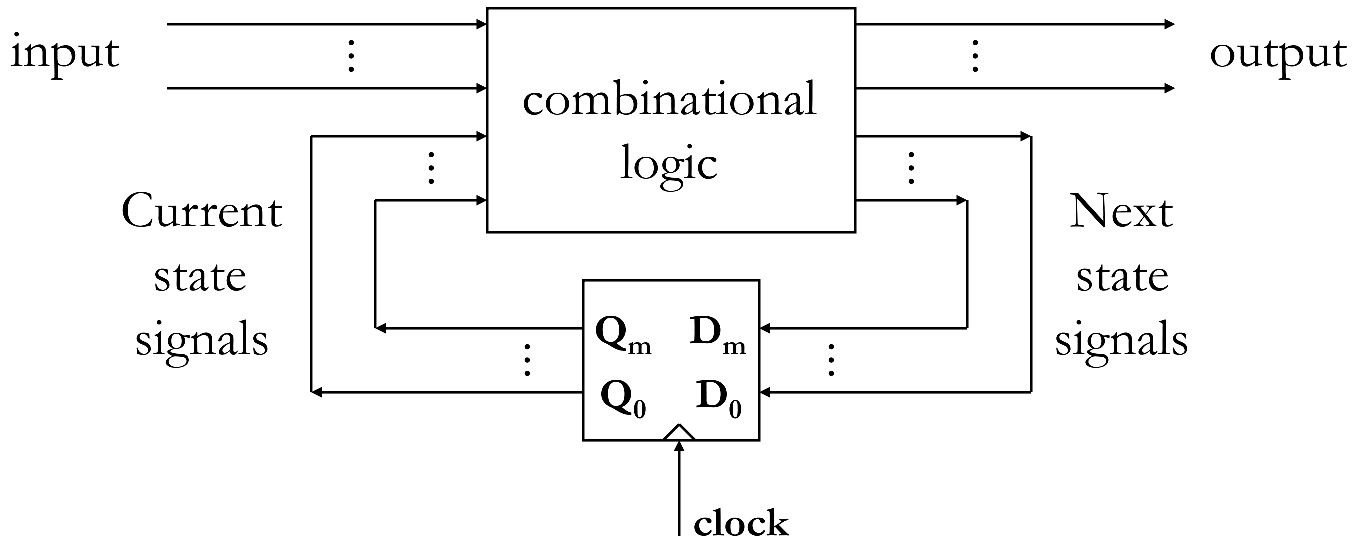




- Edge-triggered flip-flop: on a +ve clock edge, D is copied to Q
- Can be used to build registers:



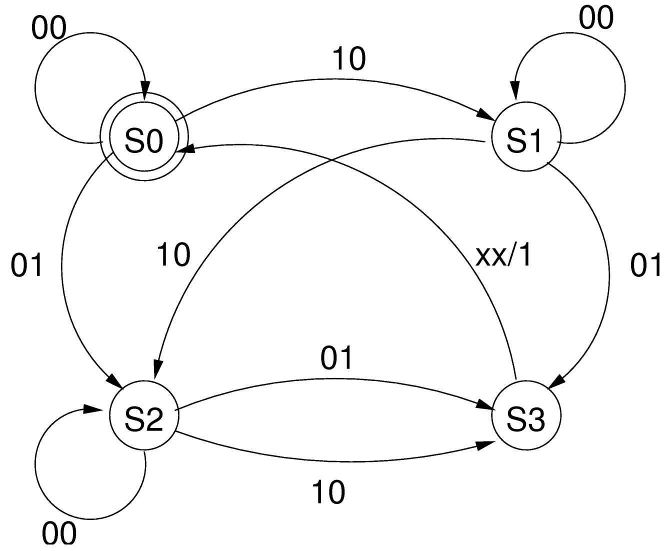
General sequential logic circuit



- Operation:
  - At every rising clock edge next state signals are propagated to current state signals
  - Current state signals plus inputs work through combinational logic and generate output and next state signals

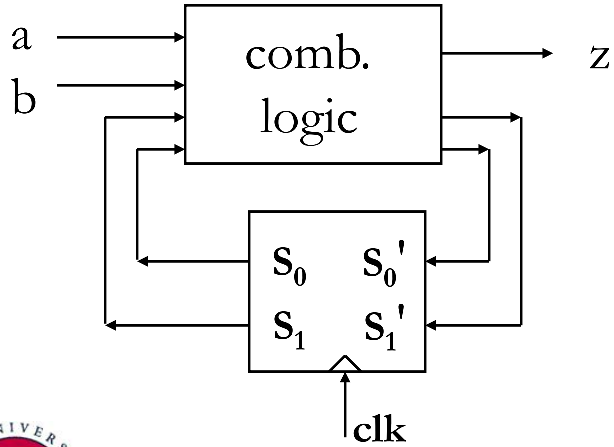
Hardware FSM

- A sequential circuit is a (deterministic) Finite State Machine – FSM
- Example: Vending machine
  - Accepts 10p, 20p coins, sells one product costing 30p, no change given
  - Coin reader has 2 outputs: a,b for 10p, 20p coins respectively
  - Output z asserted when 30p or more has been paid in



FSM implementation

- Methodology:
  - Choose encoding for states, e.g S0=00, …, S3=11
  - Build truth table for the next state s_1', s_0' and output z
  - Generate logic equations for s_1', s_0', z
  - Design comb logic from logic equations and add state-holding register



s_1	s_0	a	b	s_1'	s_0'	z
0	0	0	0	0	0	
0	0	0	1	1	0	0
0	0	1	0	0	1	
0	1	0	0	0	1	
0	1	0	1	1	1	0
0	1	1	0	1	0	

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