Inf2b - Learning

Lecture 2: Similarity and Reocommendation systems

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http://www.inf.ed.ac.uk/teaching/courses/inf2b/
https://piazza.com/ed.ac.uk/spring2020/infr08028
Office hours: Wednesdays at 14:00-15:00 in IF-3.04

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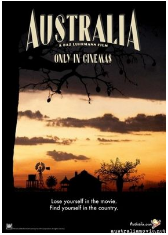
Recommender systems

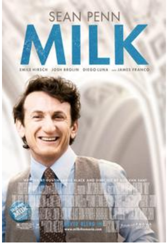
What makes recommendations good?

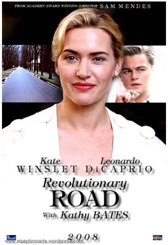
Today's schedule

1. Data and distances between entities
2. Similarity and recommendations
3. Normalisation, Pearson Correlation
4. Transposed problem

The Films in 2008







The Critics

David Denby | Todd McCarthy | Joe Morgenstern





Claudia Puig | Peter Travers | Kenneth Turan

Film review scores by critics – data

	Australia	*Body of Lies*	*Burn After*	*Hancock*	*Milk*	*Rev Road*
Denby	3	7	4	9	9	7
McCarthy	7	5	5	3	8	8
M'stern	7	5	5	0	8	4
Puig	5	6	8	5	9	8
Travers	5	8	8	8	10	9
Turan	7	7	8	4	7	8

Representation of data & notation:

$$X = \begin{pmatrix} 3 & 7 & 4 & 9 & 9 & 7 \\ 7 & 5 & 5 & 3 & 8 & 8 \\ 7 & 5 & 5 & 0 & 8 & 4 \\ 5 & 6 & 8 & 5 & 9 & 8 \\ 5 & 8 & 8 & 8 & 10 & 9 \\ 7 & 7 & 8 & 4 & 7 & 8 \end{pmatrix}$$

Score of movie m by critic c:

$$x_{cm}, \quad \mathrm{sc}_c(m)$$

Score vector by critic c:

$$\mathbf{x}_c = (x_{c1}, \ldots, x_{cM})^T$$

aka feature vector

Problem definition

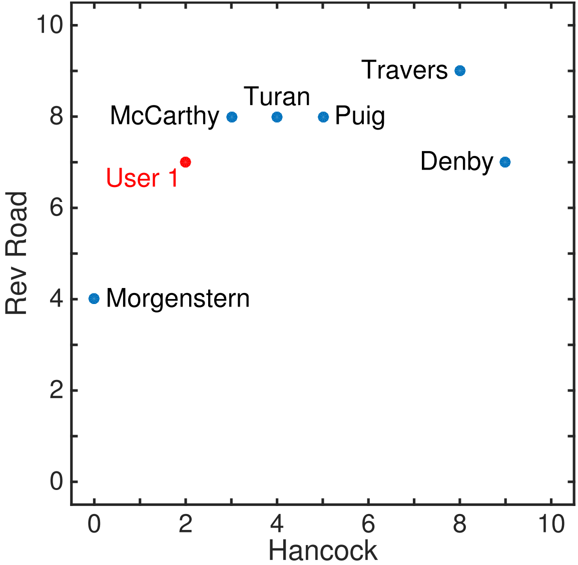
	Australia	*Body of Lies*	*Burn After*	*Hancock*	*Milk*	*Rev Road*
Denby	3	7	4	9	9	7
McCarthy	7	5	5	3	8	8
M'stern	7	5	5	0	8	4
Puig	5	6	8	5	9	8
Travers	5	8	8	8	10	9
Turan	7	7	8	4	7	8

User1	-	-	-	2	-	7
User2	-	6	9	-	-	6

Predict user's score $\hat{x}_{um}$ for unseen film m based on the film review scores by the critics. $\Rightarrow$ Film recommendation

(Fill the missing elements based on others)

A two-dimensional review space



Euclidean distance

Distance between $2D$ vectors: $\boldsymbol{u} = (u_1, u_2)^T$ and $\boldsymbol{v} = (v_1, v_2)^T$

$$r_2(\boldsymbol{u}, \boldsymbol{v}) = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2}$$

Distance between D-dimensional vectors: $\boldsymbol{u} = (u_1, \ldots, u_D)^T$ and $\boldsymbol{v} = (v_1, \ldots, v_D)^T$

$$r_2(\boldsymbol{u}, \boldsymbol{v}) = \sqrt{\sum_{k=1}^{D} (u_k - v_k)^2}$$

Measures similarities between feature vectors

i.e., similarities between digits, critics, movies, genes, ...

NB: $r_2(\ )$ denotes "2-norm", c.f. p-norm or L^p-norm. [Note 2]

cf. other distance measures, e.g. Hamming distance, city-block distance (L^1 norm).

Distances between critics

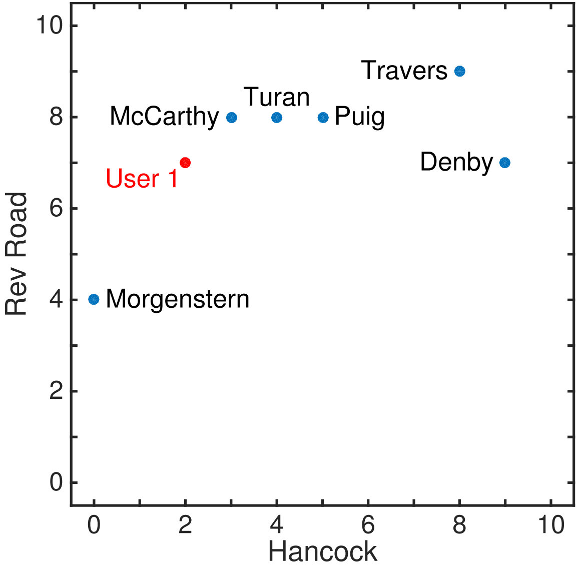
$$r_2(\boldsymbol{x}_i, \boldsymbol{x}_j) = \sqrt{\sum_{m=1}^{M} (x_{im} - x_{jm})^2}$$

	Denby	McCarthy	M'stern	Puig	Travers	Turan
Denby		7.7	10.6	6.2	5.2	7.9
McCarthy	7.7		5.0	4.4	7.2	3.9
M'stern	10.6	5.0		7.5	10.7	6.8
Puig	6.2	4.4	7.5		3.9	3.2
Travers	5.2	7.2	10.7	3.9		5.6
Turan	7.9	3.9	6.8	**3.2**	5.6	

NB: Distances measured in a 6-dimensional space ($M = 6$)

The closest pair is Puig and Turan

2D distance between User1 and critics



$$r_2(\text{User1, McCarthy})$$
$$= \sqrt{(2-3)^2 + (7-8)^2}$$
$$= \sqrt{2}$$

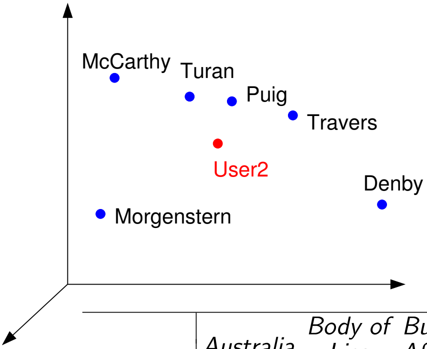
$$r_2(\text{User1, Turan})$$
$$= \sqrt{(2-4)^2 + (7-8)^2}$$
$$= \sqrt{5}$$

Simple strategy 1 for film recommendation

- Find the closest critic, c^*, to User u,
- use x_{c^*m} for $\hat{x}_{um}$.

	Australia	*Body of Lies*	*Burn After*	*Hancock*	*Milk*	*Rev Road*
Denby	3	7	4	9	9	7
McCarthy	**7**	**5**	**5**	**3**	**8**	**8**
M'stern	7	5	5	0	8	4
Puig	5	6	8	5	9	8
Travers	5	8	8	8	10	9
Turan	7	7	8	4	7	8
User1	-	-	-	**2**	-	**7**
User2	-	6	9	-	-	6

Film recommendation for User2



	Australia	*Body of Lies*	*Burn After*	*Hancock*	*Milk*	*Rev Road*	r_2(critic, User2)
Denby	3	7	4	9	9	7	$\sqrt{27} \approx 5.2$
McCarthy	7	5	5	3	8	8	$\sqrt{21} \approx 4.6$
M'stern	7	5	5	0	8	4	$\sqrt{21} \approx 4.6$
Puig	5	6	8	5	9	8	$\sqrt{5} \approx 2.2$
Travers	5	8	8	8	10	9	$\sqrt{14} \approx 3.7$
Turan	7	7	8	4	7	8	$\sqrt{6} \approx 2.4$
User2	-	6	9	-	-	6	

Strategy 2

Consider not only the closest critic but also all the critics.

Option 1: The mean or average of critic scores for film m:

$$\hat{x}_{um} = \frac{1}{C} \sum_{c=1}^{C} x_{cm}$$

Option 2: Weighted average over critics:

Weight critic scores according to the *similarity* between the critic and user.

$$\hat{x}_{um} = \frac{1}{\sum_{c=1}^{C} \mathrm{sim}(\boldsymbol{x}_u, \boldsymbol{x}_c)} \sum_{c=1}^{C} \left(\mathrm{sim}(\boldsymbol{x}_u, \boldsymbol{x}_c) \cdot x_{cm}\right)$$

cf. Weighted arithmetic mean (weighted average) in maths:

$$\bar{x} = \frac{w_1 x_1 + w_2 x_2 + \cdots + w_n x_n}{w_1 + w_2 + \cdots w_n} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

NB: if every x_i has the same value, so does $\bar{x}$.

Similarity measures

There's a choice. For example:

$$\mathrm{sim}(\boldsymbol{u}, \boldsymbol{v}) = \frac{1}{1 + r_2(\boldsymbol{u}, \boldsymbol{v})}$$

Can now predict scores for User 2 (see notes)

Good measure?

- Consider distances 0, ∞, and in between.
- What if some critics rate more highly than others?
- What if some critics have a wider spread than others?
- What if not all critics have seen the same movies? (missing data problem)

Critic review score statistics

	Australia	*Body of Lies*	*Burn After*	*Hancock*	*Milk*	*Rev Road*	mean	std.
Denby	3	7	4	9	9	7	**6.5**	**2.5**
McCarthy	7	5	5	3	8	8	**6.0**	**2.0**
M'stern	7	5	5	0	8	4	**4.8**	**2.8**
Puig	5	6	8	5	9	8	**6.8**	**1.7**
Travers	5	8	8	8	10	9	**8.0**	**1.7**
Turan	7	7	8	4	7	8	**6.8**	**1.5**

Normalisation

Sample mean and **sample standard deviation** of critic c's scores:

$$\bar{x}_c = \frac{1}{M}\sum_{m=1}^{M} x_{cm}$$

$$s_c = \sqrt{\frac{1}{M-1}\sum_{m=1}^{M}\left(x_{cm} - \bar{x}_c\right)^2}$$

Different means and spreads make reviewers look different.

$\Rightarrow$ Create 'standardised score' with mean zero and st. dev. 1.
Standard score:

$$z_{cm} = \frac{x_{cm} - \bar{x}_c}{s_c}$$

Many learning systems work better with standardised features / outputs

Normalisation of critics review scores

	Australia	*Body of Lies*	*Burn After*	*Hancock*	*Milk*	*Rev Road*
Denby	3	7	4	9	9	7
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M'stern	7	5	5	0	8	4
Puig	5	6	8	5	9	8
Travers	5	8	8	8	10	9
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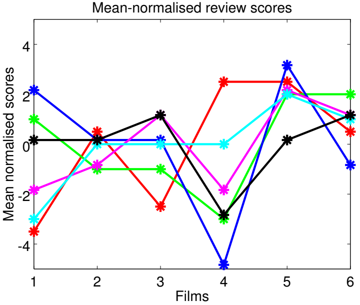
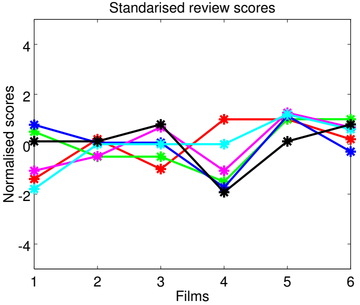




Pearson correlation coefficient

Estimate of 'correlation' between critics c and d:

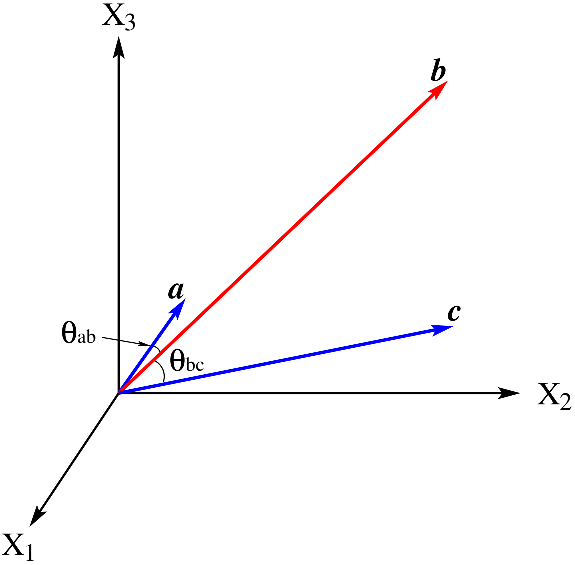
$$
\begin{aligned}
r_{cd} &= \frac{1}{M-1} \sum_{m=1}^{M} z_{cm} \, z_{dm} \\
&= \frac{1}{M-1} \sum_{m=1}^{M} \left(\frac{x_{cm} - \bar{x}_c}{s_c} \right) \left(\frac{x_{dm} - \bar{x}_d}{s_d} \right).
\end{aligned}
$$

- Based on standard scores
  (a shift and stretch of a reviewer's scale makes no difference – shift/scale invariant)
- $-1 \leq r_{cd} \leq 1$
- How r_{cd} can be used as a similarity measure?

Used in the mix by the winning netflix teams:

https://www.netflixprize.com/assets/GrandPrize2009_BPC_BellKor.pd

Pearson correlation coefficient (*cont.*)



And a trick: transpose your data matrix and run your code again. The result is sometimes interesting.

Transposed problem

Another strategy — based on distance between Movies

	Australia	*Body of Lies*	*Burn After*	*Hancock*	*Milk*	*Rev Road*
Australia		5.8	5.3	10.9	8.9	7.2
Body of Lies	5.8		3.7	6.6	5.9	4.0
Burn After	5.3	3.7		8.9	7.0	4.5
Hancock	10.9	6.6	8.9		10.9	8.4
Milk	8.9	5.9	7.0	10.9		4.8
Rev. Road	7.2	4.0	4.5	8.4	4.8	

Run the same code for distance between critics, simply **transpose the data matrix** first

Transpose of `data` in numpy is `data.T`, in Matlab/Octave it's `data'`

The Netflix million dollar prize

$C = 480,189$ users/critics

$M = 17,770$ movies

$C \times M$ matrix of ratings $\in \{1, 2, 3, 4, 5\}$

(ordinal values)

Full matrix $\sim$ 10 billion cells

$\sim 1\%$ cells filled (100,480,507 ratings available)

References *(NE)*

- https://www.netflixprize.com
- https://doi.org/10.1109/MSPEC.2009.4907383
- https://doi.org/10.1109/MC.2009.263

Further reading *(NE)*

- J. Bobadilla, F. Ortega, A. Hernando, A. Gutiérrez,
  Recommender systems survey,
  Knowledge-Based Systems, Volume 46, 2013, pp.109-132.
  https://doi.org/10.1016/j.knosys.2013.03.012

- Jie Lu, Dianshuang Wu, Mingsong Mao, Wei Wang, Guangquan Zhang,
  Recommender system application developments: A survey,
  Decision Support Systems, Volume 74, 2015, pp.12-32.
  https://doi.org/10.1016/j.dss.2015.03.008

- Shuai Zhang, Lina Yao, Aixin Sun, Yi Tay
  Deep Learning based Recommender System: A Survey and New Perspectives,
  ACM Computing Surveys (CSUR), February 2019, Article No.: 5.
  https://doi.org/10.1145/3285029

Quizzes

Q1: Give examples for $r_{cd} \approx -1,\ 0,\ $ and 1.

Q2: Show the Pearson correlation coefficient can be rewritten as

$$r_{cd} = \frac{\sum_{m=1}^{M}(x_{cm} - \bar{x}_c)(x_{dm} - \bar{x}_d)}{\sqrt{\sum_{m=1}^{M}(x_{cm} - \bar{x}_c)^2}\sqrt{\sum_{m=1}^{M}(x_{dm} - \bar{x}_d)^2}}$$

Q3: How the missing data of critics scores should be treated?

Q4: What if a user provides scores for a few films only?

Summary

- **Rating prediction:** fill in entries of a $C \times M$ matrix
- A row is a feature vector of a critic
- Guess cells based on weighted average of similar rows
- Similarity based on distance and Pearson correlation coef.
- Could transpose matrix and run same code!
- NB: we considered a very simple case only.
- Try the exercises in Note 2, and do programming in Lab 2.

Drop-in labs for Learning

- Lab1 on 21th at 11:10-13:00, 22nd Jan. at 13:10-15:00 in AT-6.06.

  "Similarity and recommender systems"

- Lab worksheet available from the course web page.

- Questions outside the lab hours:
  http://piazza.com/ed.ac.uk/spring2019/infr08009inf2blearning

Matlab/Octave version

```
c_scores = [
    3 7 4 9  9 7;
    7 5 5 3  8 8;
    7 5 5 0  8 4;
    5 6 8 5  9 8;
    5 8 8 8 10 9;
    7 7 8 4  7 8]; % CxM
u2_scores = [6 9 6];
u2_movies = [2 3 6]; % one-based indices

% The next line is complicated. See also next slide:
d2 = sum(bsxfun(@minus, c_scores(:,u2_movies), u2_scores).^2, 2)';
r2 = sqrt(d2);
sim = 1./(1 + r2); % 1xC
pred_scores = (sim * c_scores) / sum(sim) % 1xM = 1xC * CxM
```

Matlab/Octave square distances

Other ways to get square distances:

```
% The next line is like the Python, but not valid Matlab.
% Works in recent builds of Octave.
d2 = sum((c_scores(:,u2_movies) - u2_scores).^2, 2)';

% Old-school Matlab way to make sizes match:
d2 = sum((c_scores(:,u2_movies) - ...
        repmat(u2_scores, size(c_scores,1), 1)).^2, 2)';

% Sq. distance is common; I have a general routine at:
% homepages.inf.ed.ac.uk/imurray2/code/imurray-matlab/square_dist.m
d2 = square_dist(u2_scores', c_scores(:,u2_movies)');
```

Or you could write a for loop and do it as you might in Java.
Worth doing to check your code.

NumPy programming example

```
from numpy import *

c_scores = array([
    [3, 7, 4, 9,  9, 7],
    [7, 5, 5, 3,  8, 8],
    [7, 5, 5, 0,  8, 4],
    [5, 6, 8, 5,  9, 8],
    [5, 8, 8, 8, 10, 9],
    [7, 7, 8, 4,  7, 8]]) # C,M
u2_scores = array([6, 9, 6])
u2_movies = array([1, 2, 5]) # zero-based indices

r2 = sqrt(sum((c_scores[:,u2_movies] - u2_scores)**2, 1).T) # C,
sim = 1/(1 + r2) # C,
pred_scores = dot(sim, c_scores) / sum(sim)
print(pred_scores)

# The predicted scores has predictions for all movies,
# including ones where we know the true rating from u2.
```