Inf 2B: Introduction to Algorithms

Lecture 1 of ADS thread

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Algorithms and Data Structures thread

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Topics:

1: Algorithms, analysing algorithms, Asymptotic notation (for talking about running-times), Sequential Data Structures, Tree data structures, Hashing, Priority Queues, Advanced sorting.

2: Algorithms for searching graphs, applications to graph problems.

3: Algorithms for the WWW: indexing, searching.

Textbooks

For Algorithms and Data Structures (recommended, not required.):

- [GT] *Data Structures and Algorithms in Java*, by Goodrich & Tamassia (4th or 3rd ed), Wiley.
  Gentle textbook, *best* for this course (doesn't have WWW stuff).
  Java.
- [CLRS] *Introduction to Algorithms*, by Cormen, Leiserson, Rivest & Stein, MIT Press.
  Lots of Algorithms & Data Structures.
  Technical.
  No Java (or any other programing language).
  Course text for 3rd year Algorithms and Data Structures course.

If you will not take 3rd year ADS, choose [GT], *but* don't rush out to buy a book straight away.

Study advice

1. Education is done *with you* not *to* you.
2. You are here because you want to learn the subject.
3. Course consists of:
   - Lectures.
   - Tutorials.
   - Practical work (2 assignments only, 1 for each thread).
   - Private study.

   Deciding not to take an *active* part in all of these is deciding to under perform at best and fail at worst.

   It is *not* possible to coast along and revise just before the exams (unless failure seems like a good idea).

   My promise: If you ask for help I will do my utmost to provide it. But please use the channels above first when appropriate.

   Questions from you: Strongly encouraged, during lectures, after lectures or email.

Finally:

- Lectures start at 4.10, keep any eye on the clock and wind down any conversation.
- In lectures either I talk or you talk but not both!
- Laptops, tablets, phones should be put away (unless a medical condition requires the use of an aid).
  - If you have any special needs that need my cooperation please speak to me.

Our Ingredients

Algorithms Step-by-step procedure (a "recipe") for performing a task.

Data Structures Systematic way of organising data and making it accessible in certain ways.

- We are interested in the design and analysis of "good" algorithms and data structures.
- Think about very large systems and the need to have them work within acceptable time.

What you have probably seen already

Data Structures
: Arrays, linked lists, stacks, trees.

Algorithm design principles
: Recursive algorithms.

Searching and Sorting Algorithms
: Linear search and Binary search. Insertion sort, selection sort.

Other prerequisites:

- The ability to reason mathematically, spot a bad argument from a mile off.
  - Write down a mathematical argument *fluently*. It should be a pleasure to read.
- See Note 1 for advice on setting out mathematical reasoning.

Evaluating algorithms

- *Correctness*
- *Efficiency* w.r.t.
  - — running time,
  - — space (=amount of memory used),
  - — network traffic,
  - — number of times secondary storage is accessed.
- *Simplicity*

Measuring Running time

The running time of a *program* depends on a number of factors such as:

1. The *input*.
2. The running time of the algorithm.
3. The *quality of the implementation* and the *quality of the code generated by the compiler*.
4. The *machine used to execute the program*.

We will rarely be concerned with the *implementation quality*, the *code quality* or the *machine*.

- A given algorithm can be implemented by many different programs (indeed languages).

Example 1: Linear Search in JAVA

```
public static int linSearch(int[] A,int k) {
    for(int i = 0; i < A.length; i++)
        if ( A[i] == k )
            return i;
    return -1;
}
```

This is Java.

- We want to ignore implementation details, so we map this to pseudocode.

  In reality things are the other way round!

Linear Search in Pseudocode

Input: Integer array A, integer k being searched.
Output: The least index i such that $A[i] = k$; otherwise -1.

Algorithm linSearch(A, k)

```
for i ← 0 to A.length − 1 do
    if A[i] = k then
        return i
return −1
```

Suppose $A = \langle 19, 5, 6, 77, 2, 1, 90, 3, 4, 22, 1, 5, 6 \rangle$ and $k = 1$. What happens?

Worst Case Running Time

Assign a size to each possible input.

Definition
The *(worst-case) running time* of an algorithm A is the function $T_A : \mathbb{N} \to \mathbb{N}$ where $T_A(n)$ is the *maximum* number of computation steps performed by A on an input of size n.

Example: linSearch.

- Suppose the size is the length of the array A.
- Worst-case running time is a linear function of size.

Note:

- Implicit assumption that array entries are of bounded size.
- Otherwise we could take sum of all array entry sizes as measure of input size (plus size of k).

Average Running Time

In general worst-case seems overly pessimistic.

Definition
The *average running time* of an algorithm A is the function $AVT_{\mathrm{A}} : \mathbb{N} \to \mathbb{R}$ where $AVT_{\mathrm{A}}(n)$ is the *average* number of computation steps performed by A on an input of size n.

Problems with average time

- What precisely does *average* mean? What is meant by an "average" input depends on the application.
- Average time analysis is mathematically very difficult and often infeasible (OK for linSearch).

Analysis of Algorithms

A nice approach would be to combine:

Worst-Case Analysis + Experiments

We will aim for this *but*

- Java's Garbage Collection hampers the quality of our experiments.

Example 2: Binary Search

Input: Integer array A in increasing order, integers i_1, i_2, k.
Output: An index i with $i_1 \le i \le i_2$ and $A[i] = k$, if such an i exists, -1 otherwise.

Algorithm binarySearch(A, k, i_1, i_2)

```
if i2 < i1 then return −1
else
    j ← ⌊(i1+i2)/2⌋
    if k = A[j] then
        return j
    else if k < A[j] then
        return binarySearch(A, k, i1, j − 1)
    else
        return binarySearch(A, k, j + 1, i2)
```


Running-time of Binary search

Input array with $n = i_2 - i_1 + 1$ (the number of items in the region we search).

- Do at most a constant c amount of work.
- If k found done else recurse on array of size about $n/2$.
- Do a constant c amount of work.
- If k found done else recurse on array of size about $n/2^2$.

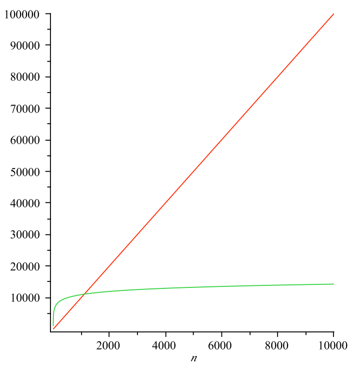
$\vdots$

- Do a constant c amount of work.
- If k found done else recurse on array of size about $n/2^r$.

Base case: $n/2^r = 1$, i.e., $r = \lg(n)$. Then one more call.
Total work done (time) no more than

$$c\big(\lg(n) + 2\big).$$

Better than linSearch?



$T_{\mathrm{linSearch}}(n) = 10n + 10,$
$T_{\mathrm{binarySearch}}(n) = 1000 \lg(n) + 1000.$

$\lg n$ versus n

Put

$$m = \lg n.$$

By definition

$$n = 2^m.$$

Now:

$$\begin{array}{ll} m \to m+1 & n \to 2n \\ m \to m+5 & n \to 32n \\ m \to m+10 & n \to 1024n \\ & \\ m \to m+c & n \to 2^c n \end{array}$$

Some Statistics

Jan 2008 on a DICE machine.

size	wc linS	avc linS	wc binS	avc binS
10	≤ 1 ms	≤ 1 ms	≤ 1 ms	≤ 1 ms
100	≤ 1 ms	≤ 1 ms	≤ 1 ms	≤ 1 ms
1000	≤ 1 ms	≤ 1 ms	≤ 1 ms	≤ 1 ms
10000	≤ 1 ms	≤ 1 ms	≤ 1 ms	≤ 1 ms
100000	≤ 1 ms	≤ 1 ms	≤ 1 ms	≤ 1 ms
200000	≤ 1 ms	≤ 1 ms	≤ 1 ms	≤ 1 ms
400000	3 ms	≤ 1 ms	≤ 1 ms	≤ 1 ms
600000	3 ms	1.3 ms	≤ 1 ms	≤ 1 ms
800000	3 ms	1.5 ms	≤ 1 ms	≤ 1 ms
1000000	5 ms	2.1 ms	≤ 1 ms	≤ 1 ms
2000000	7 ms	3.7 ms	≤ 1 ms	≤ 1 ms
4000000	12 ms	6.9 ms	≤ 1 ms	≤ 1 ms
6000000	24 ms	11.6 ms	≤ 1 ms	≤ 1 ms
8000000	24 ms	15.6 ms	≤ 1 ms	≤ 1 ms

200 repetitions for each size.

Why not just do experiments?

- Consider sorting arrays of the integers $1, 2, \ldots, 100$ held in some order.
- Just take a 1% sample of all possible inputs.
- How many experiments?

$$\begin{aligned}99! = {} & 93326215443944152681699238856266700490715968264381 \\ & 62146859296389521759999322991560894146397615651 \\ & 828625369792082722375825118521091686400000000000 \\ & 00000000000.\end{aligned}$$

Assume algorithm can sort 10^{50} instances per second(!).
How long do we need to wait?

$$\frac{99!}{60 \times 60 \times 24 \times 366 \times 10^{50}} \approx 2.951269209 \times 10^{98} \text{ years.}$$

Be seeing you!