Semantic Composition
Review: compositionality, lambda expressions, and logical forms
Examples
Type raising

Semantic (Scope) Ambiguity
Definition
Semantic Scope
Approaches to Scope Ambiguity
Compositionality

How do we create FOL for sentences in a compositional manner? Through their syntactic tree.

Each node in the tree will have a lambda expression associated with it (or a logical form). That expression has a “type” – it is sometimes easier to think in terms of “what type a node should be,” instead of “what formula it should have.”

Roadmap: (1) simple case of statements such as “John loves Mary.” (2) how do we tackle sentences such as “John loves a tall woman?” (3) how do we tackle sentences such as “Every man loves a woman?”
Compositionality: The meaning of a complex expression is a function of the meaning of its parts and of the rules by which they are combined.

Do we have sufficient tools to systematically compute meaning representations according to this principle?

- The meaning of a complete sentence will hopefully be a FOPL formula, which we consider as having type $t$ (truth values).
- But the meaning of smaller fragments of the sentence will have other types. E.g.
  
  $\text{has a bone} \quad < e, t >$
  
  $\text{every dog} \quad << e, t >, t >$

- The idea is to show how to associate a meaning with such fragments, and how these meanings combine.
- To do this, we need to extend our language of FOPL with $\lambda$ expressions ($\lambda = \text{lambda}$; written as \ in Haskell).
Lambda ($\lambda$) Expressions

$\lambda$-expressions are an extension to FOPL that allows us to work with ‘partially constructed’ formulae. A $\lambda$-expression consists of:

- the Greek letter $\lambda$, followed by a variable (formal parameter);
- a FOPL expression that may involve that variable.

$$\lambda x. \text{sleep}(x) : < e, t >$$

‘The function that takes an entity $x$ to the statement sleep($x$)’

$$(\lambda x. \text{sleep}(x)) (\text{Kim}) : t$$

function argument

A $\lambda$-expression can be applied to a term.

(The above has the same truth value as sleep($Kim$).)
Lambda expressions can be nested. We can use nesting to create functions of several arguments that accept their arguments one at a time.

\[
\lambda y. \lambda x. \text{love}(x,y) : < e, < e, t >>
\]

‘The function that takes \( y \) to

(the function that takes \( x \) to the statement \( \text{love}(x,y) \))’

\[
\lambda z. \lambda y. \lambda x. \text{give}(x,y,z) : < e, < e, < e, t >>>
\]

‘The function that takes \( z \) to

(the function that takes \( y \) to

(the function that takes \( x \) to the statement \( \text{give}(x,y,z) \)))’
Beta Reduction

When a lambda expression applies to a term, a reduction operation (beta (β) reduction) can be used to replace its formal parameter with the term and simplify the result. In general:

$$(\lambda x.M)N \Rightarrow_\beta M[x \mapsto N] \quad (M \text{ with } N \text{ substituted for } x)$$

\[
(\lambda x.\text{sleep}(x)) \text{ (Kim)} \Rightarrow_\beta \text{sleep(Kim)}
\]

\[
(\lambda y.\lambda x.\text{love}(x, y)) \text{ (crabapples)} \Rightarrow_\beta \lambda x.\text{love}(x, \text{crabapples})
\]

\[
(\lambda x.\text{love}(x, \text{crabapples})) \text{ (Kim)} \Rightarrow_\beta \text{love(Kim, crabapples)}
\]
Consider the question “Who is the CEO of Microsoft?”

A possible semantic interpretation of it:
\[ \lambda x.\text{CEO}(x, \text{Microsoft}) \]

In addition, we have a large database of CEOs, in relations of the form CEO(Nadella, Microsoft), CEO(Cook, Apple), etc.

Given the above \( \lambda \) expression, we can apply it on all entities in the database and check its truth value (i.e. whether in our model of the world, as reflected by the database, the CEO relation holds between the entity and Microsoft).
Plan for Today

We will describe three grammars for representing natural language semantics

Grammar I: basic grammar that can describe propositions on entities such as “Sam loves Kim”

Grammar II: slightly improved, where we will allow propositions on general nouns that can be described using adjectives. Noun phrases will not be compositional

Grammar III: another improvement, where we will allow quantifiers (such as “every”) and compositional nouns

Things to think about (a lot of moving parts in this class!): (a) what is the type of a node? (b) How does the λ expression for a node reflect that type?
Compositional Semantics: the key idea

Grammar 1

\[
\begin{align*}
S & \rightarrow NP \ VP \quad \{VP.\text{Sem}(NP.\text{Sem})\} \quad t \\
VP & \rightarrow TV \ NP \quad \{TV.\text{Sem}(NP.\text{Sem})\} \quad <e, t> \\
NP & \rightarrow NPR \quad \{NPR.\text{Sem}\} \quad e \\
TV & \rightarrow \text{loves} \quad \{\lambda y.\lambda x.\text{love}(x,y)\} \quad <e, <e, t>> \\
NPR & \rightarrow \text{Kim} \quad \{\text{Kim}\} \quad e \\
NPR & \rightarrow \text{Sam} \quad \{\text{Sam}\} \quad e
\end{align*}
\]

- To build a compositional semantics for NL, we attach valuation functions to grammar rules (semantic attachments).
- These show how to compute the interpretation of the LHS of the rule from the interpretations of its RHS components.
- For example, VP.Sem(NP.Sem) means apply the interpretation of the VP to the interpretation of the NP.
- Types have been added to ease understanding.
Compositional Semantics: example

\[ S[\lambda x.\text{love}(x, \text{Kim})(\text{Sam}) \Rightarrow_\beta \text{love}(\text{Sam}, \text{Kim})] \]

\[ NP[\text{Sam}] \quad VP[\lambda y.\lambda x.\text{love}(x, y)(\text{Kim}) \Rightarrow_\beta \lambda x.\text{love}(x, \text{Kim})] \]

\[ NPR[\text{Sam}] \quad TV[\lambda y.\lambda x.\text{love}(x, y)] \quad NP[\text{Kim}] \]

\[ \text{loves} \quad NPR[\text{Kim}] \]

\[ \text{Sam} \quad \text{Kim} \]
Compositional Semantics: example

\[ S[\lambda x.\text{love}(x, \text{Kim})(\text{Sam}) \Rightarrow _\beta \text{love}(\text{Sam}, \text{Kim})] \]

NP[Sam]  
NPR[Sam]  
Sam

VP[\lambda y.\lambda x.\text{love}(x, y)(\text{Kim}) \Rightarrow _\beta \lambda x.\text{love}(x, \text{Kim})]  
TV[\lambda y.\lambda x.\text{love}(x, y)]  
NP[Kim]  
NPR[Kim]  
Kim

\text{loves}
Compositional Semantics: example

\[ S[\lambda x.\text{love}(x, \text{Kim})(\text{Sam}) \Rightarrow_\beta \text{love}(\text{Sam}, \text{Kim})] \]

\[
\begin{array}{c}
\text{NP}[\text{Sam}] & \text{VP}[\lambda y.\lambda x.\text{love}(x, y)(\text{Kim}) \Rightarrow_\beta \lambda x.\text{love}(x, \text{Kim})] \\
\text{NPR}[\text{Sam}] & \text{TV}[\lambda y.\lambda x.\text{love}(x, y)] & \text{NP}[\text{Kim}] \\
\text{Sam} & \text{loves} & \text{NPR}[\text{Kim}] \\
& & \text{Kim}
\end{array}
\]
Compositional Semantics: example

\[ S[\lambda x.\text{love}(x, \text{Kim})(\text{Sam}) \Rightarrow_\beta \text{love}(\text{Sam}, \text{Kim})] \]
Compositional Semantics: example

\[ S[\lambda x.\text{love}(x, Kim)(Sam) \Rightarrow_\beta \text{love}(Sam, Kim)] \]

\[ \text{NP[Sam]} \quad \text{VP[}\lambda y.\lambda x.\text{love}(x, y)(Kim) \Rightarrow_\beta \lambda x.\text{love}(x, Kim)] \]

\[ \text{NPR[Sam]} \quad \text{TV[}\lambda y.\lambda x.\text{love}(x, y)] \quad \text{NP[Kim]} \]

\[ \text{Sam} \quad \text{loves} \quad \text{NPR[Kim]} \]

\[ \text{Kim} \]
Compositional Semantics: example

\[ S[\lambda x.\text{love}(x, \text{Kim})(\text{Sam}) \Rightarrow \beta \text{love}(\text{Sam}, \text{Kim})] \]

- NP[Sam]
  - NPR[Sam]
    - Sam
- VP[\lambda y.\lambda x.\text{love}(x, y)(\text{Kim}) \Rightarrow \beta \lambda x.\text{love}(x, \text{Kim})]
  - TV[\lambda y.\lambda x.\text{love}(x, y)]
    - loves
  - NP[Kim]
    - NPR[Kim]
      - Kim
A minor variation

The following alternative semantics assigns the same overall meaning to sentences. Only the treatment of the arguments of ‘love’ is different.

**Grammar I**

\[
\begin{align*}
S &\rightarrow NP \ VP \quad \{\text{VP.Sem(NP.Sem)}\} \quad t \\
VP &\rightarrow TV \ NP \quad \{\lambda x.\text{TV.Sem}(x)(\text{NP.Sem})\} \quad < e, t > \\
NP &\rightarrow NPR \quad \{\text{NPR.Sem}\} \quad e \\
TV &\rightarrow \text{loves} \quad \{\lambda x.\lambda y.\text{love}(x,y)\} \quad < e, < e, t > > \\
NPR &\rightarrow \text{Kim} \quad \{\text{Kim}\} \quad e \\
NPR &\rightarrow \text{Sam} \quad \{\text{Sam}\} \quad e
\end{align*}
\]
What about the interpretation of an NP other than a proper name? The FOPL interpretation should often contain an existential ($\exists$) or a universal ($\forall$) quantifier:

Sam has access to a computer.
$\exists x (computer(x) \land have\_access\_to(Sam, x))$

Every student has access to a computer.
$\forall x (student(x) \rightarrow \exists y (computer(y) \land have\_access\_to(x, y)))$

Can we build such interpretations up from their component parts in the same way as with proper names?
A halfway stage.

Grammar II

\[
\begin{align*}
S & \rightarrow \text{NPR} \ \text{VP} & \{ \text{VP.Sem(NPR.Sem)} \} & t \\
\text{VP} & \rightarrow \text{TV} \ a \ \text{Nom} & \{ \lambda x. \exists y. \text{Nom.Sem}(y) \& \text{TV.Sem}(y)(x) \} & < e, t > \\
\text{Nom} & \rightarrow \text{N} & \{ \text{N.Sem} \} & < e, t > \\
\text{Nom} & \rightarrow \text{A} \ \text{Nom} & \{ \lambda x. \text{Nom.Sem}(x) \& \text{A.Sem}(x) \} & < e, t > \\
\text{NPR} & \rightarrow \text{Sam} & \{ \text{Sam} \} & e \\
\text{TV} & \rightarrow \text{loves} & \{ \lambda y. \lambda x. \text{love}(x, y) \} & < e, < e, t >> \\
\text{N} & \rightarrow \text{woman} & \{ \lambda z. \text{woman}(z) \} & < e, t > \\
\text{A} & \rightarrow \text{tall} & \{ \lambda z. \text{tall}(z) \} & < e, t >
\end{align*}
\]

- Note we haven’t given a meaning here to a tall woman.
- Could take this to have the same meaning as tall woman.
- This would be fine for this example (also in Assignment 2). But what about every tall woman?
A halfway stage.

Grammar II

\[ S \rightarrow \text{NPR} \ \text{VP} \quad \{ \ \text{VP.Sem(NPR.Sem)} \ \} \quad t \]
\[ \text{VP} \rightarrow \text{TV} \ \text{a} \ \text{Nom} \quad \{ \ \lambda x. \exists y. \text{Nom.Sem(y)} \ & \ \text{TV.Sem(y)}(x) \ \} \]
\[ \text{Nom} \rightarrow \text{N} \quad \{ \ \text{N.Sem} \ \} \quad < e, t > \]
\[ \text{Nom} \rightarrow \text{A} \ \text{Nom} \quad \{ \ \lambda x. \text{Nom.Sem}(x) \ & \ \text{A.Sem}(x) \ \} \quad < e, t > \]
\[ \text{NPR} \rightarrow \text{Sam} \quad \{ \ \text{Sam} \ \} \quad e \]
\[ \text{TV} \rightarrow \text{loves} \quad \{ \ \lambda y. \lambda x. \text{love}(x, y) \ \} \quad < e, < e, t >> \]
\[ \text{N} \rightarrow \text{woman} \quad \{ \ \lambda z. \text{woman}(z) \ \} \quad < e, t > \]
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Grammar II

\[ S \rightarrow \text{NPR VP} \quad \{ \text{VP.Sem(\text{NPR.Sem})} \} \quad t \]
\[ \text{VP} \rightarrow \text{TV a Nom} \quad \{ \lambda x. \exists y. \text{Nom.Sem}(y) \& \text{TV.Sem}(y)(x) \} \quad <e, t> \]
\[ \text{Nom} \rightarrow \text{N} \quad \{ \text{N.Sem} \} \quad <e, t> \]
\[ \text{Nom} \rightarrow \text{A Nom} \quad \{ \lambda x. \text{Nom.Sem}(x) \& \text{A.Sem}(x) \} \quad <e, t> \]
\[ \text{NPR} \rightarrow \text{Sam} \quad \{ \text{Sam} \} \quad e \]
\[ \text{TV} \rightarrow \text{loves} \quad \{ \lambda y. \lambda x. \text{love}(x, y) \} \quad <e, <e, t>> \]
\[ \text{N} \rightarrow \text{woman} \quad \{ \lambda z. \text{woman}(z) \} \quad <e, t> \]
\[ \text{A} \rightarrow \text{tall} \quad \{ \lambda z. \text{tall}(z) \} \quad <e, t> \]

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A halfway stage.

Grammar II

\[ S \rightarrow \text{NPR } \text{VP} \quad \{ \text{VP.Sem(NPR.Sem)} \} \quad t \]

\[ \text{VP} \rightarrow \text{TV a Nom} \quad \{ \lambda x. \exists y. \text{Nom.Sem}(y) \land \text{TV.Sem}(y)(x) \} \]

\[ \text{Nom} \rightarrow \text{N} \quad \{ \text{N.Sem} \} \quad <e, t> \]

\[ \text{Nom} \rightarrow \text{A Nom} \quad \{ \lambda x. \text{Nom.Sem}(x) \land \text{A.Sem}(x) \} \quad <e, t> \]

\[ \text{NPR} \rightarrow \text{Sam} \quad \{ \text{Sam} \} \quad e \]

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Computing semantics with Grammar II

Before we add more, let’s use Grammar II to compute the semantics of *Sam loves a tall woman*.

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<td>λx. ∃y. (λx. woman(x) &amp; tall(x))(y) &amp; (λyx. love(x, y))(y)(x)</td>
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<td>⇒β  λx. ∃y. (woman(y) &amp; tall(y)) &amp; love(x, y)</td>
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<td>Sam loves a tall woman</td>
<td>S</td>
<td>(λx.∃y. ···)(Sam)</td>
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Computing semantics with Grammar II

Before we add more, let’s use Grammar II to compute the semantics of *Sam loves a tall woman*.

love

tall woman

loves a tall woman

Sam loves a tall woman

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Before we add more, let’s use Grammar II to compute the semantics of *Sam loves a tall woman*.

- **loves**
  - TV: $\lambda yx. \text{love}(x, y)$
- **tall woman**
  - Nom: $\lambda x. (\lambda z. \text{woman}(z))(x) \& (\lambda z. \text{tall}(z))(x)$
  - $\Rightarrow \beta$: $\lambda x. \text{woman}(x) \& \text{tall}(x)$
- **loves a tall woman**
  - VP: $\lambda x. \exists y. (\lambda x. \text{woman}(x) \& \text{tall}(x))(y) \& (\lambda yx. \text{love}(x, y))(y)(x)$
  - $\Rightarrow \beta$: $\lambda x. \exists y. (\text{woman}(y) \& \text{tall}(y)) \& \text{love}(x, y)$
- **Sam loves a tall woman**
  - S: $(\lambda x. \exists y. \cdots)(\text{Sam})$
  - $\Rightarrow \beta$: $\exists y. \text{woman}(y) \& \text{tall}(y) \& \text{love}(\text{Sam}, y)$
Before we add more, let’s use Grammar II to compute the semantics of *Sam loves a tall woman*.

\[
\begin{align*}
\text{loves} & \quad \text{TV} \quad \lambda y x. \text{love}(x, y) \\
\text{tall woman} & \quad \text{Nom} \quad \lambda x. (\lambda z. \text{woman}(z))(x) & (\lambda z. \text{tall}(z))(x) \\
& \quad \Rightarrow \beta \quad \lambda x. \text{woman}(x) & \text{tall}(x) \\
\text{loves a tall woman} & \quad \text{VP} \quad \lambda x. \exists y. (\lambda x. \text{woman}(x) & \text{tall}(x))(y) & (\lambda y x. \text{love}(x, y))(y)(x) \\
& \quad \Rightarrow \beta \quad \lambda x. \exists y. (\text{woman}(y) & \text{tall}(y)) & \text{love}(x, y) \\
\text{Sam loves a tall woman} & \quad \text{S} \quad (\lambda x. \exists y. \cdots)(\text{Sam}) \\
& \quad \Rightarrow \beta \quad \exists y. \text{woman}(y) & \text{tall}(y) & \text{love}(\text{Sam}, y)
\end{align*}
\]
Type raising

- We’ve given Sam, Kim the semantic type $e$, and woman the semantic type $< e, t >$.
- But what type should some woman or every woman have?
- Idea: Since we wish to combine an NP.Sem with a VP.Sem (of type $< e, t >$) to get an S.Sem (of type $t$), let’s try again with NP.Sem having type $<< e, t >, t >$.

Sam $\lambda P. P(Sam)$ (type raising)

every woman $\lambda P. \forall x. \text{woman}(x) \Rightarrow P(x)$

The appropriate semantic attachment for NP VP is then

$$S \rightarrow \text{NP VP} \quad \{ \text{NP.Sem (VP.Sem)} \}$$
Semantics of determiners

- Using this approach, we can also derive the semantics of 'every woman' from that of 'every' and 'woman'.
- We’ve seen that 'woman' has semantic type $< e, t >$, and 'every woman' has semantic type $<< e, t >, t >>$.
- So the interpretation of 'every' should have type $<< e, t >, << e, t >, t >>$. Similarly for other determiners (e.g. every, a, no, not every).

$$\begin{align*}
\text{woman} & \quad \lambda x. \text{woman}(x) \quad < e, t > \\
\text{every} & \quad \lambda Q. \lambda P. \forall x. Q(x) \Rightarrow P(x) \quad << e, t >, << e, t >, t >> \\
\text{a} & \quad \lambda Q. \lambda P. \exists x. Q(x) \wedge P(x) \quad << e, t >, << e, t >, t >> \\
\text{NP} \rightarrow \text{Det N} & \quad \{ \text{Det.Sem (N.Sem)} \} \quad << e, t >, t >
\end{align*}$$

We can now compute the semantics of 'every woman' and check that it $\beta$-reduces to $\lambda P. \forall x. \text{woman}(x) \Rightarrow P(x)$. 
Example

The semantics of “every woman”: 
More on type raising

- The natural rule for VP is now $\text{VP} \rightarrow \text{TV NP}$.
- Since the semantic type for NP has now been raised to $<<e, t>, t>$, and we want VP to have semantic type $<e, t>$, what should the semantic type for TV be?
More on type raising

- The natural rule for VP is now \( VP \rightarrow TV \ NP \).

- Since the semantic type for NP has now been raised to \( <<e, t>, t> \), and we want VP to have semantic type \( <e, t> \), what should the semantic type for TV be?

It had better be \( <<<e, t>, t>, <e, t>> \).

(A 3rd order function type!)

\[
TV \rightarrow loves \quad \{ \lambda R<<e,t>,t> . \lambda z^e . R(\lambda w^e . loves(z, w)) \}
\]

\[
VP \rightarrow TV \ NP \quad \{ TV.Sem(NP.Sem) \}\]
To summarize where we’ve got to:

**Grammar III**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
<th>Sample Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S \rightarrow NP \ VP$</td>
<td>{$NP.Sem(VP.Sem)$}</td>
<td>$t$</td>
</tr>
<tr>
<td>$VP \rightarrow TV \ NP$</td>
<td>{$TV.Sem(NP.Sem)$}</td>
<td>$&lt; e, t &gt;$</td>
</tr>
<tr>
<td>$NP \rightarrow Sam$</td>
<td>{$\lambda P. P(Sam)$}</td>
<td>$&lt;&lt; e, t &gt;, t &gt;$</td>
</tr>
<tr>
<td>$NP \rightarrow Det \ Nom$</td>
<td>{$Det.Sem(Nom.Sem)$}</td>
<td>$&lt;&lt; e, t &gt;, t &gt;$</td>
</tr>
<tr>
<td>$Det \rightarrow a$</td>
<td>{$\lambda Q. \lambda P. \exists x. Q(x) \land P(x)$}</td>
<td>$&lt;&lt; e, t &gt;, &lt;&lt; e, t &gt;, t &gt;&gt;&gt;$</td>
</tr>
<tr>
<td>$Det \rightarrow every$</td>
<td>{$\lambda Q. \lambda P. \forall x. Q(x) \Rightarrow P(x)$}</td>
<td>$&lt;&lt; e, t &gt;, &lt;&lt; e, t &gt;, t &gt;&gt;&gt;$</td>
</tr>
<tr>
<td>$Nom \rightarrow N$</td>
<td>{$N.Sem$}</td>
<td>$&lt; e, t &gt;$</td>
</tr>
<tr>
<td>$Nom \rightarrow A \ Nom$</td>
<td>{$\lambda x. Nom.Sem(x) &amp; A.Sem(x)$}</td>
<td>$&lt; e, t &gt;$</td>
</tr>
<tr>
<td>$TV \rightarrow loves$</td>
<td>{$\lambda R. \lambda z. R(\lambda w. loves(z, w))$}</td>
<td>$&lt;&lt;&lt;&lt; e, t &gt;, t &gt;, &lt;&lt; e, t &gt;&gt;$</td>
</tr>
<tr>
<td>$N \rightarrow woman$</td>
<td>{$\lambda z. woman(z)$}</td>
<td>$&lt; e, t &gt;$</td>
</tr>
<tr>
<td>$A \rightarrow tall$</td>
<td>{$\lambda z. tall(z)$}</td>
<td>$&lt; e, t &gt;$</td>
</tr>
</tbody>
</table>

Can add similar entries for ‘student’, ‘computer’, ‘has access to’.
Example
The semantics for ‘every student has access to a computer’.
Example

The semantics for ‘every student has access to a computer’.

\[
\text{every student } (\lambda Q. \lambda P. \forall x. Q(x) \Rightarrow P(x))(\lambda x. \text{student}(x)) \\
\rightarrow_\beta \lambda P. \forall x. \text{student}(x) \Rightarrow P(x)
\]
Example

The semantics for ‘every student has access to a computer’.

every student  \((\lambda Q. \lambda P. \forall x. Q(x) \Rightarrow P(x))(\lambda x. \text{student}(x))\)
\[\rightarrow_{\beta} \lambda P. \forall x. \text{student}(x) \Rightarrow P(x)\]

a computer  \((\lambda Q. \lambda P. \exists x. Q(x) \land P(x))(\lambda x. \text{computer}(x))\)
\[\rightarrow_{\beta} \lambda P. \exists x. \text{computer}(x) \land P(x)\]
Example

The semantics for ‘every student has access to a computer’.

\[
\begin{align*}
\text{every student} & \quad (\lambda Q. \lambda P. \forall x. Q(x) \Rightarrow P(x))(\lambda x. \text{student}(x)) \\
& \rightarrow_{\beta} \lambda P. \forall x. \text{student}(x) \Rightarrow P(x)
\end{align*}
\]

\[
\begin{align*}
\text{a computer} & \quad (\lambda Q. \lambda P. \exists x. Q(x) \land P(x))(\lambda x. \text{computer}(x)) \\
& \rightarrow_{\beta} \lambda P. \exists x. \text{computer}(x) \land P(x)
\end{align*}
\]

\[
\begin{align*}
\text{h.a.t. a computer} & \quad \cdots \rightarrow_{\beta} \cdots \\
& \rightarrow_{\beta} \lambda z. \exists x. \text{computer}(x) \land h_a_t(z, x)
\end{align*}
\]
Example

The semantics for ‘every student has access to a computer’.

\[
\text{every student } (\lambda Q. \lambda P. \forall x. Q(x) \Rightarrow P(x))(\lambda x. \text{student}(x)) \\
\rightarrow_\beta \lambda P. \forall x. \text{student}(x) \Rightarrow P(x)
\]

\[
\text{a computer } (\lambda Q. \lambda P. \exists x. Q(x) \land P(x))(\lambda x. \text{computer}(x)) \\
\rightarrow_\beta \lambda P. \exists x. \text{computer}(x) \land P(x)
\]

\[
\text{h.a.t. a computer } \cdots \rightarrow_\beta \cdots \\
\rightarrow_\beta \lambda z. \exists x. \text{computer}(x) \land h_{\text{a.t}}(z, x)
\]

\[
(\text{whole sentence}) \cdots \rightarrow_\beta \cdots \\
\rightarrow_\beta \forall x. \text{student}(x) \Rightarrow \exists y. \text{computer}(y) \land h_{\text{a.t}}(x, y)
\]
Example

The semantics for ‘every student has access to a computer’.

every student \((\lambda Q. \lambda P. \forall x. Q(x) \Rightarrow P(x))(\lambda x. student(x))\)
\[\rightarrow_{\beta} \lambda P. \forall x. student(x) \Rightarrow P(x)\]

a computer \((\lambda Q. \lambda P. \exists x. Q(x) \land P(x))(\lambda x. computer(x))\)
\[\rightarrow_{\beta} \lambda P. \exists x. computer(x) \land P(x)\]

h.a.t. a computer \(\cdots \rightarrow_{\beta} \cdots\)
\[\rightarrow_{\beta} \lambda z. \exists x. computer(x) \land h\_a\_t(z, x)\]

(whole sentence) \(\cdots \rightarrow_{\beta} \cdots\)
\[\rightarrow_{\beta} \forall x. student(x) \Rightarrow \exists y. computer(y) \land h\_a\_t(x, y)\]

Note: In the last \(\beta\)-step, we’ve renamed ‘\(x\)’ to ‘\(y\)’ to avoid capture.
Suppose that the predicate $L(x, y)$ means $x$ loves $y$. Which of the following is not a possible representation of the meaning of *Everybody loves somebody*?

1. $\forall x. \exists y. L(x, y)$
2. $(\lambda P. \forall x. \exists y. P(x, y))(\lambda x. \lambda y. L(x, y))$
3. $(\lambda P. \forall x. \exists y. P(x, y))(\lambda x. \lambda y. L(y, x))$
4. $(\lambda P. \forall x. \exists y. P(y, x))(\lambda x. \lambda y. L(y, x))$
Semantic Ambiguity

Whilst *every student has access to a computer* is neither syntactically nor lexically ambiguous, it has two different interpretations because of its determiners:

- **every**: interpreted as $\forall$ (*universal quantifier*)
- **a**: interpreted as $\exists$ (*existential quantifier*)

**Meaning 1**

Possibly a different computer per student

$\forall x (\text{student}(x) \rightarrow \exists y (\text{computer}(y) \land \text{have_access_to}(x, y)))$

**Meaning 2**

Possibly the same computer for all students

$\exists y (\text{computer}(y) \land \forall x (\text{student}(x) \rightarrow \text{have_access_to}(x, y)))$
Scope

The ambiguity arises because *every* and *a* each has its own *scope*:

- **Interpretation 1**: every has scope over a
- **Interpretation 2**: a has scope over every

- Scope is not uniquely determined either by left-to-right order, or by position in the parse tree.
- We therefore need other mechanisms to ensure that the ambiguity is reflected by there being multiple interpretations assigned to S.
Scope ambiguity, continued

The number of interpretations grows exponentially with the number of scope operators:

**Every student at some university has access to a laptop.**

1. Not necessarily same laptop, not necessarily same university
\( \forall x (\text{stud}(x) \land \exists y (\text{univ}(y) \land \text{at}(x, y)) \rightarrow \exists z (\text{laptop}(z) \land \text{have\_access}(x, z))) \)

2. Same laptop, not necessarily same university
\( \exists z (\text{laptop}(z) \land \forall x (\text{stud}(x) \land \exists y (\text{univ}(y) \land \text{at}(x, y)) \rightarrow \text{have\_access}(x, z))) \)

3. Not necessarily same laptop, same university
\( \exists y (\text{univ}(y) \land \forall x ((\text{stud}(x) \land \text{at}(x, y)) \rightarrow \exists z (\text{laptop}(z) \land \text{have\_access}(x, z)))) \)

4. Same university, same laptop
\( \exists y (\text{univ}(y) \land \exists z (\text{laptop}(z) \land \forall x ((\text{stud}(x) \land \text{at}(x, y)) \rightarrow \text{have\_access}(x, z)))) \)

5. Same laptop, same university
\( \exists z (\text{laptop}(z) \land \exists y (\text{univ}(y) \land \forall x ((\text{stud}(x) \land \text{at}(x, y)) \rightarrow \text{have\_access}(x, z)))) \)

where 4 & 5 are equivalent

**Every student at some university does not have access to a computer.**

\( \rightarrow 18 \) interpretations
Coping with Scope: options

1. **Enumerate all interpretations.** Computationally unattractive!

2. Use an **underspecified representation** that can be further specified to each of the multiple interpretations on demand.

Sometimes the surrounding context will help us choose between interpretations:

Every student has access to a computer. It can be borrowed from the ITO. (⇒ Meaning 2)
Summary

- Syntax guides semantic composition in a systematic way.
- Lambda expressions facilitate the construction of compositional semantic interpretations.
- Logical forms can be constructed by attaching valuation functions to grammar rules.
- However, this approach is not adequate enough for quantified NPs, as LFs are not always isomorphic with syntax.
- We can elegantly handle scope by building an abstract underspecified representation and disambiguate on demand.