

Automatic generation of LL(1) parsers

Informatics 2A: Lecture 12

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Recap of Lecture 11

- **LL(1) predictive parsing** reads the input string from left to right, and determines the correct production to apply purely on the basis of two pieces of information: (1) the **current input symbol**, and (2) the **current predicted nonterminal symbol** (which is kept on the head of a stack).
- The parsing algorithm is efficient and deterministic and uses a **parse table** to determine the next production.
- LL(1) parsing is suitable only for **formal languages** with **unambiguous grammars**. Even for such languages, finding an LL(1) grammar may require some thought.
(**Addendum**: Some formal languages with unambiguous grammars cannot be given an LL(1) grammar at all.)

Generating parse tables

We've seen that if a grammar \mathcal{G} happens to be LL(1) — i.e. if it admits a **parse table** — then efficient, deterministic, predictive parsing is possible with the help of a stack.

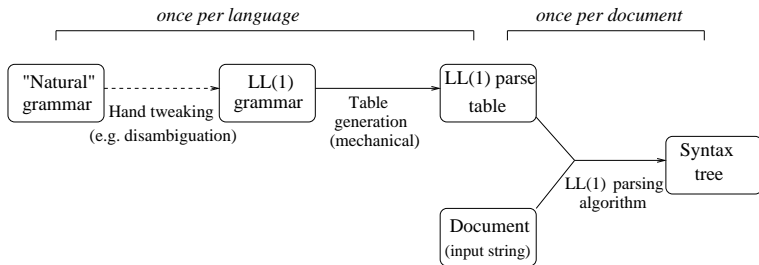
What's more, if \mathcal{G} is LL(1), \mathcal{G} is automatically **unambiguous**.

But **how do we tell** whether a grammar is LL(1)? And if it is, **how can we construct a parse table** for it?

For very small grammars, might be able to answer these questions by eye inspection. But for realistic grammars, a systematic method is needed.

In this lecture, we give an **algorithmic procedure** for answering both questions.

The overall picture



Previous lecture: the **LL(1) parsing algorithm**, which works on a parse table and a particular input string.

This lecture: algorithm for getting from a grammar \mathcal{G} to a parse table. The algorithm will succeed if \mathcal{G} is LL(1), or fail if it isn't. (As in previous lecture, assume \mathcal{G} has no 'useless nonterminals'.)

Next lecture: ways of getting from a grammar to an equivalent LL(1) grammar. (Possible quite often, but not always.)

First and Follow sets

Two steps to construct a parse table for a given grammar:

- 1 For each nonterminal X , compute two sets called $First(X)$ and $Follow(X)$, defined as follows:
 - $First(X)$ is the set of all **terminals that can appear at the start** of a phrase derived from X .
[**Convention:** if ϵ can be derived from X , also include the special symbol ϵ in $First(X)$.]
 - $Follow(X)$ is the set of all **terminals that can appear immediately after X** in some sentential form derived from the start symbol S .
[**Convention:** if X can appear at the end of some such sentential form, also include the special symbol $\$$ in $Follow(X)$.]
- 2 Use these $First$ and $Follow$ sets to fill out the parse table.

The first step is somewhat tricky. The second is easier.

Exercises

- $First(X)$ is the set of all terminals that can appear at the start of a phrase derived from X .
[Convention: if ϵ can be derived from X , also include the special symbol ϵ in $First(X)$.]

Recall our LL(1) grammar for well-matched bracket sequences:

$$S \rightarrow \epsilon \mid TS \qquad T \rightarrow (S)$$

Question. Work out each of the two sets below.

- 1 $First(T)$
- 2 $First(S)$

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Answer: $First(T) = \{(\}$.

Exercises

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$$S \rightarrow \epsilon \mid TS \qquad T \rightarrow (S)$$

Question. Work out each of the two sets below.

- 1 $First(T)$
- 2 $First(S)$

Answer: $First(T) = \{()\}$. $First(S) = \{(, \epsilon\}$.

More exercises

- $Follow(X)$ is the set of all terminals that can appear immediately after X in some sentential form derived from the start symbol S .
[Convention: if X can appear at the end of some such sentential form, also include $\$$ in $Follow(X)$.]

Again consider the same LL(1) grammar:

$$S \rightarrow \epsilon \mid TS \qquad T \rightarrow (S)$$

Question. Work out each of the two sets below.

- 1 $Follow(S)$
- 2 $Follow(T)$

More exercises

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Answer: $Follow(S) = \{), \$\}$.

More exercises

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Again consider the same LL(1) grammar:

$$S \rightarrow \epsilon \mid TS \qquad T \rightarrow (S)$$

Question. Work out each of the two sets below.

- 1 $Follow(S)$
- 2 $Follow(T)$

Answer: $Follow(S) = \{), \$\}$. $Follow(T) = \{(,), \$\}$.

Those examples again

Look again at our grammar for well-matched bracket sequences:

$$S \rightarrow \epsilon \mid TS \qquad T \rightarrow (S)$$

By inspection, we can see that

$$\begin{aligned} \textit{First}(S) &= \{ (, \epsilon \} && \text{because an } S \text{ can begin with } (\text{ or be empty} \\ \textit{First}(T) &= \{ (\} && \text{because a } T \text{ must begin with } (\\ \textit{Follow}(S) &= \{), \$ \} && \text{because within a complete phrase, an } S \\ &&& \text{can be followed by }) \text{ or appear at the end} \\ \textit{Follow}(T) &= \{ (,), \$ \} && \text{because a } T \text{ can be followed by } (\text{ or }) \\ &&& \text{or appear at the end} \end{aligned}$$

Later we'll give a systematic method for computing these sets.

Further convention: take $\textit{First}(a) = \{a\}$ for each **terminal** a .

Filling out the parse table

Once we've got these *First* and *Follow* sets, we can fill out the parse table as follows.

For each production $X \rightarrow \alpha$ of \mathcal{G} in turn:

- For each terminal a , if α 'can begin with' a , insert $X \rightarrow \alpha$ in row X , column a .
- If α 'can be empty', then for each $b \in \text{Follow}(X)$ (where b may be $\$$), insert $X \rightarrow \alpha$ in row X , column b .

If doing this leads to **clashes** (i.e. two productions fighting for the same table entry) then **conclude that the grammar is not LL(1)**.

To explain the phrases in **blue**, suppose $\alpha = x_1 \dots x_n$, where the x_i may be terminals or nonterminals.

- α **can be empty** means $\epsilon \in \text{First}(x_i)$ for every x_i .
- α **can begin with** a means that, for some i , $\epsilon \in \text{First}(x_1) \cap \dots \cap \text{First}(x_{i-1})$, and $a \in \text{First}(x_i)$.

Comments on filling out the parse table

- The case $\alpha = \epsilon$ is counted as a case in which α can be empty.

(This case is implicit in the last slide since $\alpha = \epsilon$ counts as an instance of $\alpha = x_1 \dots x_n$ by taking $n = 0$, whence the condition “ $\epsilon \in First(x_j)$ for every x_j ” is vacuously true since there are no x_j .)

- Similarly, we count $\alpha = x_1 \dots x_n$ with $a \in First(x_1)$ as one case in which α can begin with a .

(Again this is implicit in the last slide. The condition $\epsilon \in First(x_1) \cap \dots \cap First(x_{i-1})$ means that ϵ is contained in all the sets $First(x_1), First(x_2)$ up to $First(x_{i-1})$. In the case that $i = 1$, we consider the sequence x_1, \dots, x_{i-1} as being empty. Thus the condition “ $\epsilon \in First(x_1) \cap \dots \cap First(x_{i-1})$ ” is again vacuously true.)

Filling out the parse table: example

$$S \rightarrow \epsilon \mid TS \qquad T \rightarrow (S)$$

$$\begin{aligned} \text{First}(S) &= \{(\, \epsilon\} & \text{Follow}(S) &= \{), \$\} \\ \text{First}(T) &= \{(\} & \text{Follow}(T) &= \{(\,), \$\} \end{aligned}$$

Use this information to fill out the [parse table](#):

- (S) can begin with $($, so insert $T \rightarrow (S)$ in entry for $(, T$.
- TS can begin with $($, so insert $S \rightarrow TS$ in entry for $(, S$.
- ϵ can be empty, and $\text{Follow}(S) = \{), \$\}$, so insert $S \rightarrow \epsilon$ in entries for $), S$ and $\$, S$.

This gives the parse table we had in the previous lecture:

	$($	$)$	$\$$
S	$S \rightarrow TS$	$S \rightarrow \epsilon$	$S \rightarrow \epsilon$
T	$T \rightarrow (S)$		

Intermezzo: true or false?

- 1 Every LL(1) grammar is context free.
- 2 Every context-free language can be presented using an LL(1) grammar.
- 3 Every regular language can be presented using an LL(1) grammar.
- 4 Every LL(1) grammar is unambiguous.
- 5 Languages defined by LL(1) grammars can be efficiently parsed.

Intermezzo: true or false?

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- 5 Languages defined by LL(1) grammars can be efficiently parsed.

Answer: 2 is false, the others are all true.

Calculating First and Follow sets: preliminary stage

To complete the story, we'd like an algorithm for calculating *First* and *Follow* sets.

Easy first step: compute the set E of nonterminals that 'can be ϵ ':

- 1 Start by adding X to E whenever $X \rightarrow \epsilon$ is a production of \mathcal{G} .
- 2 If $X \rightarrow Y_1 \dots Y_m$ is a production and all Y_1, \dots, Y_m are already in E , add X to E .
- 3 Repeat step 2 until E stabilizes.

Example: for our grammar of well-matched bracket sequences, we have $E = \{S\}$.

Calculating First sets: the details

- 1 Set $First(a) = \{a\}$ for each $a \in \Sigma$. For each nonterminal X , initially set $First(X)$ to $\{\epsilon\}$ if $X \in E$, or \emptyset otherwise.
- 2 For each production $X \rightarrow x_1 \dots x_n$ and each $i \leq n$, if $x_1, \dots, x_{i-1} \in E$ and $a \in First(x_i)$, add a to $First(X)$.
- 3 Repeat step 2 until all $First$ sets stabilize.

Example:

- Start with $First(S) = \{\epsilon\}$, $First(T) = \emptyset$, etc.
- Consider $T \rightarrow (S)$ with $i = 1$: add (to $First(T)$.
- Now consider $S \rightarrow TS$ with $i = 1$: add (to $First(S)$.
- That's all.

Calculating Follow sets: the details

- 1 Initially set $Follow(S) = \{\$ \}$ for the start symbol S , and $Follow(X) = \emptyset$ for all other nonterminals X .
- 2 For each production $X \rightarrow \alpha$, each splitting of α as $\beta Y x_1 \dots x_n$ where $n \geq 1$, and each i with $x_1, \dots, x_{i-1} \in E$, add all of $First(x_i)$ (excluding ϵ) to $Follow(Y)$.
- 3 For each production $X \rightarrow \alpha$ and each splitting of α as βY or $\beta Y x_1 \dots x_n$ with $x_1, \dots, x_n \in E$, add all of $Follow(X)$ to $Follow(Y)$.
- 4 Repeat step 3 until all $Follow$ sets stabilize.

Example:

- Start with $Follow(S) = \{\$ \}$, $Follow(T) = \emptyset$.
- Apply step 2 to $T \rightarrow (S)$ with $i = 1$: add $)$ to $Follow(S)$.
- Apply step 2 to $S \rightarrow TS$ with $i = 1$: add $($ to $Follow(T)$.
- Apply step 3 to $S \rightarrow TS$ with $n = 1$: add $)$ and $\$$ to $Follow(T)$.
- That's all.

Parser generators

LL(1) is representative of a bunch of **classes of CFGs** that are efficiently parseable. E.g. $LL(1) \subset LALR \subset LR(1)$. These involve various tradeoffs of expressive power vs. efficiency/simplicity.

For such languages, a parser can be generated **automatically** from a suitable grammar. (E.g. for LL(1), just need parse table plus fixed 'driver' for the parsing algorithm.)

So we don't need to write parsers ourselves — just the grammar! (E.g. one can basically define the syntax of Java in about 10 pages of context-free rules.)

This is the principle behind **parser generators** like yacc ('yet another compiler compiler') and java-cup.

Reading

- **Recommended:** Some relevant lecture notes (“Note 12” in particular) and a tutorial sheet from previous years are available via the Course Schedule webpage.
- **Dragon book:** Aho, Sethi and Ullman, *Compilers: Principles, Techniques and Tools*, Section 4.4.
- **Tiger book:** Andrew Appel, *Modern Compiler Implementation in (C | Java | ML)*.
- **Turtle book:** Aho and Ullman, *Foundations of Computer Science*.