Informatics 1

Functional Programming Lectures 15 and 16

Monday 18 and Tuesday 19 November 2013

IO and Monads

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Mock exam: this week!

Each student has an assigned slot

Mon 18 Nov, 3:00-5:00pm, Appleton Tower room 5.05 (mostly)

Tue 19 Nov, 2:00-4:00pm, Appleton Tower room 5.05 (mostly)

Wed 20 Nov, 2:00-4:00pm, Appleton Tower room 5.05 (mostly)

Thu 21 Nov, 2:00-4:00pm, Appleton Tower room 5.05 (mostly)

Fri 22 Nov, 3:00-5:00pm, Appleton Tower room 5.05 (mostly)

Check the course webpage for your slot

Tutorials

No tutorial this week!

No revision tutorial this week!

Final tutorial next week, usual time/place

Revision tutorial next week:

Monday 1–2pm in AT 5.07

Wednesday 2–3pm in AT 5.05

Do the revision tutorial exercise and bring your solution

The 2013 Informatics 1 Competition

- First prize: A bottle of champagne or equivalent in book tokens. And glory!
- Previous year entries are online:

  www.inf.ed.ac.uk/teaching/courses/inf1/fp/#competition

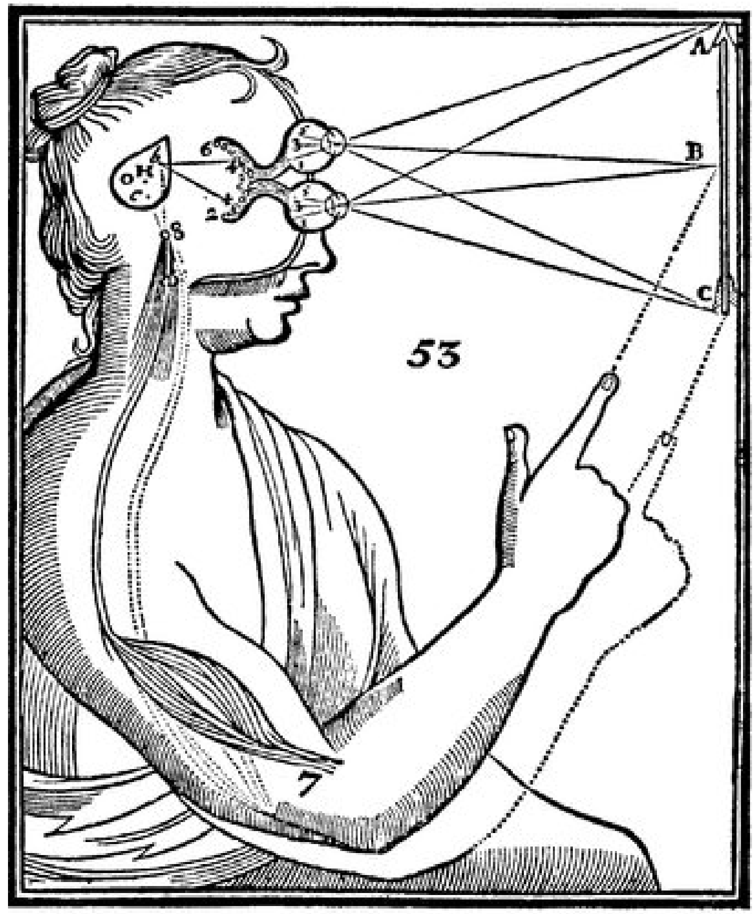
- Number of prizes depend on number/quality of entries.
- Sponsored by Galois (galois.com)
- Send code and image(s), list everyone who contributed.
- E-mail your entry to Chris Banks `<C.Banks@ed.ac.uk>`
- *Deadline: midnight, Wednesday 20 November 2013*

Part I

The Mind-Body Problem

The Mind-Body Problem

THE MECHANICAL PHILOSOPHY



Part II

Commands

Print a character

```
putChar :: Char -> IO ()
```

For instance,

```
putChar '!'
```

denotes the command that, *if it is ever performed*, will print an exclamation mark.

Combine two commands

```
(>>) :: IO () -> IO () -> IO ()
```

For instance,

```
putChar '?' >> putChar '!'
```

denotes the command that, *if it is ever performed*, prints a question mark followed by an exclamation mark.

Do nothing

```
done :: IO ()
```

The term `done` doesn't actually do nothing; it just specifies the command that, *if it is ever performed*, won't do anything. (Compare thinking about doing nothing to actually doing nothing: they are distinct enterprises.)

Print a string

```
putStr :: String -> IO ()
putStr []       =  done
putStr (x:xs)   =  putChar x >> putStr xs
```

So `putStr "?!"` is equivalent to

```
putChar '?' >> (putChar '!' >> done)
```

and both of these denote a command that, *if it is ever performed*, prints a question mark followed by an exclamation mark.

Higher-order functions

More compactly, we can define `putStr` as follows.

```
putStr  :: String -> IO ()
putStr  =  foldr (>>) done . map putChar
```

The operator >> has identity `done` and is associative.

```
m >> done        =  m
done >> m        =  m
(m >> n) >> o    =  m >> (n >> o)
```

Main

By now you may be desperate to know *how is a command ever performed?* Here is the file `Confused.hs`:

```
module Confused where

main :: IO ()
main =  putStr "?!"
```

Running this program prints an indicator of perplexity:

```
[melchior]dts: runghc Confused.hs
?![melchior]dts:
```

Thus `main` is the link from Haskell's mind to Haskell's body — the analogue of Descartes's pineal gland.

Print a string followed by a newline

```
putStrLn :: String -> IO ()
putStrLn xs  =  putStr xs >> putChar '\n'
```

Here is the file `ConfusedLn.hs`:

```
module ConfusedLn where

main :: IO ()
main =  putStrLn "?!"
```

This prints its result more neatly:

```
[melchior]dts: runghc ConfusedLn.hs
?!
[melchior]dts:
```

Part III

Equational reasoning

Equational reasoning lost

In languages with side effects, this program prints "`haha`" as a side effect.

```
print "ha"; print "ha"
```

But this program only prints "`ha`" as a side effect.

```
let x = print "ha" in x; x
```

This program again prints "`haha`" as a side effect.

```
let f () = print "ha" in f (); f ()
```

Equational reasoning regained

In Haskell, the term

```
(1+2) * (1+2)
```

and the term

```
let x = 1+2 in x * x
```

are equivalent (and both evaluate to 9).

In Haskell, the term

```
putString "ha" >> putString "ha"
```

and the term

```
let m = putString "ha" in m >> m
```

are also entirely equivalent (and both print `"haha"`).

Part IV

Commands with values

Read a character

Previously, we wrote `IO ()` for the type of commands that yield no value.

Here, `()` is the trivial type that contains just one value, which is also written `()`.

We write `IO Char` for the type of commands that yield a value of type `Char`.

Here is a function to read a character.

```
getChar :: IO Char
```

Performing the command `getChar` when the input contains `"abc"` yields the value `'a'` and remaining input `"bc"`.

Do nothing and return a value

More generally, we write `IO a` for commands that return a value of type `a`.

The command

```
return :: a -> IO a
```

is similar to `done`, in that it does nothing, but it also returns the given value.

Performing the command

```
return [] :: IO String
```

when the input contains `"bc"` yields the value `[]` and an unchanged input `"bc"`.

Combining commands with values

We combine command with an operator written >>= and pronounced "bind".

```
(>>=) :: IO a -> (a -> IO b) -> IO b
```

For example, performing the command

```
getChar >>= \x -> putChar (toUpper x)
```

when the input is `"abc"` produces the output `"A"`, and the remaining input is `"bc"`.

The “bind” operator in detail

```
(>>=) :: IO a -> (a -> IO b) -> IO b
```

If

```
m :: IO a
```

is a command yielding a value of type `a`, and

```
k :: a -> IO b
```

is a function from a value of type `a` to a command yielding a value of type `b`, then

```
m >>= k :: IO b
```

is the command that, *if it is ever performed*, behaves as follows:

first perform command `m` yielding a value `x` of type `a`;
then perform command `k x` yielding a value `y` of type `b`;
then yield the final value `y`.

Reading a line

Here is a program to read the input until a newline is encountered, and to return a list of the values read.

```
getLine :: IO String
getLine =  getChar >>= \x ->
           if x == '\n' then
             return []
           else
             getLine >>= \xs ->
             return (x:xs)
```

For example, given the input `"abc\ndef"` This returns the string `"abc"` and the remaining input is `"def"`.

Commands as a special case

The general operations on commands are:

```
return  :: a -> IO a
(>>=)   :: IO a -> (a -> IO b) -> IO b
```

The command `done` is a special case of `return`,
and the operator >> is a special case of >>=.

```
done    :: IO ()
done    =  return ()

(>>)    :: IO () -> IO () -> IO ()
m >> n  =  m >>= \() -> n
```

An analogue of “let”

Although it may seem odd at first sight, this combinator is reassuringly similar to the familiar Haskell “let” expression. Here is a type rule for “let”.

$$\frac{\texttt{E} \vdash \texttt{m :: a} \qquad \texttt{E, x :: a} \vdash \texttt{n :: b}}{\texttt{E} \vdash \textbf{let}\ \texttt{x = m}\ \textbf{in}\ \texttt{n :: b}}$$

Typically, “bind” is combined with lambda expressions in a way that resembles “let” expressions. Here is the corresponding type rule.

$$\frac{\texttt{E} \vdash \texttt{m :: IO a} \qquad \texttt{E, x :: a} \vdash \texttt{n :: IO b}}{\texttt{E} \vdash \texttt{m >>= \textbackslash x -> n :: IO b}}$$

Echoing input to output

This program echoes its input to its output, putting everything in upper case, until an empty line is entered.

```
echo :: IO ()
echo =  getLine >>= \line ->
        if line == "" then
          return ()
        else
          putStrLn (map toUpper line) >>
          echo

main :: IO ()
main =  echo
```

Testing it out

```
[melchior]dts: runghc Echo.hs
One line
ONE LINE
And, another line!
AND, ANOTHER LINE!
[melchior]dts:
```

Part V

“Do” notation

Reading a line in "do" notation

```
getLine :: IO String
getLine =  getChar >>= \x ->
           if x == '\n' then
             return []
           else
             getLine >>= \xs ->
             return (x:xs)
```

is equivalent to

```
getLine :: IO String
getLine =  do {
             x <- getChar;
             if x == '\n' then
               return []
             else do {
               xs <- getLine;
               return (x:xs)
             }
           }
```

Echoing in “do” notation

```
echo :: IO ()
echo =  getLine >>= \line ->
        if line == "" then
          return ()
        else
          putStrLn (map toUpper line) >>
          echo
```

is equivalent to

```
echo :: IO ()
echo =  do {
          line <- getLine;
          if line == "" then
            return ()
          else do {
            putStrLn (map toUpper line);
            echo
          }
        }
```

"Do" notation in general

Each line `x <- e; ...` becomes `e >>= \x -> ...`

Each line `e; ...` becomes `e >> ...`

For example,

```
do { x1 <- e1;
     x2 <- e2;
     e3;
     x4 <- e4;
     e5;
     e6 }
```

is equivalent to

```
e1 >>= \x1 ->
e2 >>= \x2 ->
e3 >>
e4 >>= \x4 ->
e5 >>
e6
```

Part VI

Monads

Monoids

A *monoid* is a pair of an operator `(@@)` and a value `u`, where the operator has the value as identity and is associative.

```
u @@ x           =  x
x @@ u           =  x
(x @@ y) @@ z    =  x @@ (y @@ z)
```

Examples of monoids:

`(+)` and `0`

`(*)` and `1`

`(||)` and `False`

`(&&)` and `True`

`(++)` and `[]`

`(>>)` and `done`

Monads

We know that `(>>)` and `done` satisfy the laws of a *monoid*.

```
done >> m          =  m
m >> done          =  m
(m >> n) >> o      =  m >> (n >> o)
```

Similarly, `(>>=)` and `return` satisfy the laws of a *monad*.

```
return v >>= \x -> m             =  m[x:=v]
m >>= \x -> return x             =  m
(m >>= \x -> n) >>= \y-> o       =  m >>= \x -> (n >>= \y -> o)
```

Laws of Let

We know that `(>>)` and `done` satisfy the laws of a *monoid*.

```
done >> m        =  m
m >> done        =  m
(m >> n) >> o    =  m >> (n >> o)
```

Similarly, `(>>=)` and `return` satisfy the laws of a *monad*.

```
return v >>= \x -> m          =  m[x:=v]
m >>= \x -> return x          =  m
(m >>= \x -> n) >>= \y-> o    =  m >>= \x -> (n >>= \y -> o)
```

The three monad laws have analogues in “let” notation.

```
let x = v in m    =   m[x:=v]
let x = m in x    =   m
let y = (let x = m in n) in o
                  =   let x = m in (let y = n in o)
```

“Let” in languages with and without effects

```
let x = v in m    =   m[x:=v]
let x = m in x    =   m
let y = (let x = m in n) in o
                  =   let x = m in (let y = n in o)
```

These laws hold even in languages with side effects. For the first law to be true, `v` must be not an arbitrary term but a *value*, such as a constant or a variable (but not a function application). A value immediately evaluates to itself, hence it can have no side effects.

While in such languages one only has the above three laws for “let”, in Haskell one has a much stronger law, where one may replace a variable by any term, rather than by any value.

```
let x = n in m   =   m[x:=n]
```

Part VII

Roll your own monad—IO

The Monad type class

```
class Monad m where
  return :: a -> m a
  (>>=)  :: m a -> (a -> m b) -> m b
```

My own IO monad (1)

```
module MyIO(MyIO, myPutChar, myGetChar, convert) where

type Input = String
type Remainder = String
type Output = String

data MyIO a  =  MyIO (Input -> (a, Remainder, Output))

apply :: MyIO a -> Input -> (a, Remainder, Output)
apply (MyIO f) inp  =  f inp
```

Note that the type `MyIO` is abstract. The only operations on it are the monad operations, `myPutChar`, `myGetChar`, and `convert`. The operation `apply` is not exported from the module.

My own IO monad (2)

```
myPutChar :: Char -> MyIO ()
myPutChar c  =  MyIO (\inp -> ((), inp, [c]))

myGetChar :: MyIO Char
myGetChar =  MyIO (\(ch:rem) -> (ch, rem, ""))
```

For example,

```
apply myGetChar "abc"  ==  ('a', "bc", "")
apply myGetChar "bc"   ==  ('b', "c", "")
apply (myPutChar 'A') "def"  ==  ((), "def", "A")
apply (myPutChar 'B') "def"  ==  ((), "def", "B")
```

My own IO monad (3)

```
instance Monad MyIO where
  return x   =  MyIO (\inp -> (x, inp, ""))
  m >>= k    =  MyIO (\inp ->
                  let (x, rem1, out1) = apply m inp in
                  let (y, rem2, out2) = apply (k x) rem1 in
                  (y, rem2, out1++out2))
```

For example

```
apply
  (myGetChar >>= \x -> myGetChar >>= \y -> return [x,y])
  "abc"
==  ("ab", "c", "")

apply
  (myPutChar 'A' >> myPutChar 'B')
  "def"
==  ((), "def", "AB")

apply
  (myGetChar >>= \x -> myPutChar (toUpper x))
  "abc"
== ((), "bc", "A")
```

My own IO monad (4)

```
convert :: MyIO () -> IO ()
convert m  =  interact (\inp ->
                 let (x, rem, out) = apply m inp in
                 out)
```

Here

```
interact :: (String -> String) -> IO ()
```

is part of the standard prelude. The entire input is converted to a string (lazily) and passed to the function, and the result from the function is printed as output (also lazily).

Using my own IO monad (1)

```
module MyEcho where

import Char
import MyIO

myPutStr :: String -> MyIO ()
myPutStr =  foldr (>>) (return ()) . map myPutChar

myPutStrLn :: String -> MyIO ()
myPutStrLn s  =  myPutStr s >> myPutChar '\n'
```

Using my own IO monad (2)

```
myGetLine :: MyIO String
myGetLine =  myGetChar >>= \x ->
             if x == '\n' then
               return []
             else
               myGetLine >>= \xs ->
               return (x:xs)

myEcho :: MyIO ()
myEcho =  myGetLine >>= \line ->
          if line == "" then
            return ()
          else
            myPutStrLn (map toUpper line) >>
            myEcho

main :: IO ()
main =  convert myEcho
```

Trying it out

```
[melchior]dts: runghc MyEcho
This is a test.
THIS IS A TEST.
It is only a test.
IT IS ONLY A TEST.
Were this a real emergency, you'd be dead now.
WERE THIS A REAL EMERGENCY, YOU'D BE DEAD NOW.

[melchior]dts:
```

You can use “do” notation, too

```
myGetLine :: MyIO String
myGetLine =  do {
               x <- myGetChar;
               if x == '\n' then
                 return []
               else do {
                 xs <- myGetLine;
                 return (x:xs)
               }
             }

myEcho :: MyIO ()
myEcho =  do {
            line <- myGetLine;
            if line == "" then
              return ()
            else do {
              myPutStrLn (map toUpper line);
              myEcho
            }
          }
```

Part VIII

The monad of lists

The monad of lists

In the standard prelude:

```
class Monad m where
  return :: a -> m a
  (>>=)  :: m a -> (a -> m b) -> m b

instance Monad [] where

  return      :: a -> [a]
  return x    =  [ x ]

  (>>=)       :: [a] -> (a -> [b]) -> [b]
  m >>= k     =  [ y | x <- m, y <- k x ]
```

Equivalently, we can define:

```
    [] >>= k        =  []
    (x:xs) >>= k    =  (k x) ++ (xs >>= k)
```

or

```
    m >>= k   =  concat (map k m)
```

‘Do’ notation and the monad of lists

```
pairs :: Int -> [(Int, Int)]
pairs n  =  [ (i,j) | i <- [1..n], j <- [(i+1)..n] ]
```

is equivalent to

```
pairs' :: Int -> [(Int, Int)]
pairs' n  =  do {
                 i <- [1..n];
                 j <- [(i+1)..n];
                 return (i,j)
               }
```

For example,

```
[melchior]dts: ghci Pairs
GHCi, version 6.10.4: http://www.haskell.org/ghc/  :? for help
Pairs> pairs 4
[(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
Pairs> pairs' 4
[(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
```

Monads with plus

In the standard prelude:

```
class Monad m => MonadPlus m where
  mzero :: m a
  mplus :: m a -> m a -> m a

instance MonadPlus [] where

  mzero  :: [a]
  mzero  =  []

  mplus  :: [a] -> [a] -> [a]
  mplus  =  (++)

guard :: MonadPlus => Bool -> m ()
guard False   =  mzero
guard True    =  return ()

msum :: MonadPlus => [m a] -> m a
msum =  foldr mplus mzero
```

Using guards

```
pairs'' :: Int -> [(Int, Int)]
pairs'' n  =  [ (i,j) | i <- [1..n], j <- [1..n], i < j ]
```

is equivalent to

```
pairs''' :: Int -> [(Int, Int)]
pairs''' n  =  do {
                  i <- [1..n];
                  j <- [1..n];
                  guard (i < j);
                  return (i,j)
                }
```

For example,

```
[melchior]dts: ghci Pairs
GHCi, version 6.10.4: http://www.haskell.org/ghc/  :? for help
Pairs> pairs'' 4
[(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
Pairs> pairs''' 4
[(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
```

Part IX

Parsers

Parser type

First attempt:

```
type Parser a = String -> a
```

Second attempt:

```
type Parser a = String -> (a, String)
```

Third attempt:

```
type Parser a = String -> [(a, String)]
```

A parser for things
is a function from strings
to lists of pairs
Of things and strings

—Graham Hutton

Module Parser

```
module Parser(Parser,apply,parse,char,spot,token,
  star,plus,parseInt) where

import Char
import Monad

-- The type of parsers
data Parser a = Parser (String -> [(a, String)])

-- Apply a parser
apply :: Parser a -> String -> [(a, String)]
apply (Parser f) s  =  f s

-- Return parsed value, assuming at least one successful parse
parse :: Parser a -> String -> a
parse m s  =  head [ x | (x,t) <- apply m s, t == "" ]
```

Parser is a Monad

```
-- Parsers form a monad

--    class Monad m where
--      return :: a -> m a
--      (>>=) :: m a -> (a -> m b) -> m b

instance Monad Parser where
  return x  =  Parser (\s -> [(x,s)])
  m >>= k   =  Parser (\s ->
                 [ (y, u) |
                   (x, t) <- apply m s,
                   (y, u) <- apply (k x) t ])
```

Parser is a Monad with Plus

```
-- Some monads have additional structure

--    class MonadPlus m where
--      mzero :: m a
--      mplus :: m a -> m a -> m a

instance MonadPlus Parser where
  mzero       =  Parser (\s -> [])
  mplus m n   =  Parser (\s -> apply m s ++ apply n s)
```

Parsing characters

```
-- Parse a single character
char :: Parser Char
char =  Parser f
  where
  f []      =  []
  f (c:s)   =  [(c,s)]

-- Parse a character satisfying a predicate (e.g., isDigit)
spot :: (Char -> Bool) -> Parser Char
spot p  =  Parser f
  where
  f []                  =  []
  f (c:s) | p c         =  [(c, s)]
          | otherwise   =  []

-- Parse a given character
token :: Char -> Parser Char
token c  =  spot (== c)
```

Parsing characters

```
-- Parse a single character
char :: Parser Char
char =  Parser f
  where
  f []      =  []
  f (c:s)   =  [(c,s)]

-- Parse a character satisfying a predicate (e.g., isDigit)
spot :: (Char -> Bool) -> Parser Char
spot p  =  do { c <- char; guard (p c); return c }

-- Parse a given character
token :: Char -> Parser Char
token c  =  spot (== c)
```

Parsing a string

```
match :: String -> Parser String
match []       =  return []
match (x:xs)   =  do
                    y <- token x;
                    ys <- match xs;
                    return (y:ys)
```

Parsing a sequence

```
-- match zero or more occurrences
star :: Parser a -> Parser [a]
star p  =  plus p `mplus` return []

-- match one or more occurrences
plus :: Parser a -> Parser [a]
plus p  =  do { x <- p;
                xs <- star p;
                return (x:xs) }
```

Parsing an integer

```
-- match a natural number
parseNat :: Parser Int
parseNat =  do { s <- plus (spot isDigit);
                 return (read s) }

-- match a negative number
parseNeg :: Parser Int
parseNeg =  do { token '-';
                 n <- parseNat
                 return (-n) }

-- match an integer
parseInt :: Parser Int
parseInt =  parseNat `mplus` parseNeg
```

Module Exp

```
module Exp where

import Monad
import Parser

data Exp = Lit Int
         | Exp :+: Exp
         | Exp :*: Exp
         deriving (Eq,Show)

evalExp :: Exp -> Int
evalExp (Lit n)    =  n
evalExp (e :+: f)  =  evalExp e + evalExp f
evalExp (e :*: f)  =  evalExp e * evalExp f
```

Parsing an expression

```
parseExp :: Parser Exp
parseExp  =  parseLit `mplus` parseAdd `mplus` parseMul
  where
  parseLit = do { n <- parseInt;
                  return (Lit n) }
  parseAdd = do { token '(';
                  d <- parseExp;
                  token '+';
                  e <- parseExp;
                  token ')';
                  return (d :+: e) }
  parseMul = do { token '(';
                  d <- parseExp;
                  token '*';
                  e <- parseExp;
                  token ')';
                  return (d :*: e) }
```

Testing the parser

```
[melchior]dts: ghci Exp.hs
GHCi, version 6.10.4: http://www.haskell.org/ghc/  :? for help
[1 of 2] Compiling Parser         ( Parser.hs, interpreted )
[2 of 2] Compiling Exp            ( Exp.hs, interpreted )
Ok, modules loaded: Parser, Exp.
*Exp> parse parseExp "(1+(2*3))"
Lit 1 :+: (Lit 2 :*: Lit 3)
*Exp> evalExp (parse parseExp "(1+(2*3))")
7
*Exp> parse parseExp "((1+2)*3)"
(Lit 1 :+: Lit 2) :*: Lit 3
*Exp> evalExp (parse parseExp "((1+2)*3)")
9
*Exp>
```