Informatics 1

Functional Programming Lectures 3 and 4

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# Lists and Recursion

Philip Wadler

University of Edinburgh

# Part I

# List Comprehensions

# List comprehensions — Generators

```
Prelude> [ x*x | x <- [1,2,3] ]
[1,4,9]

Prelude> [ toLower c | c <- "Hello, World!" ]
"hello, world!"

Prelude> [ (x, even x) | x <- [1,2,3] ]
[(1,False),(2,True),(3,False)]
```

`x <- [1,2,3]` is called a *generator*

`<-` is pronounced *drawn from*

# List comprehensions — Guards

```
Prelude> [ x | x <- [1,2,3], odd x ]
[1,3]

Prelude> [ x*x | x <- [1,2,3], odd x ]
[1,9]

Prelude> [ x | x <- [42,-5,24,0,-3], x > 0 ]
[42,24]

Prelude> [ toLower c | c <- "Hello, World!", isAlpha c ]
"helloworld"
```

`even x` is called a *guard*

# Sum, Product

```
Prelude> sum [1,2,3]
6

Prelude> sum []
0

Prelude> sum [ x*x | x <- [1,2,3], odd x ]
10

Prelude> product [1,2,3,4]
24

Prelude> product []
1

Prelude> let factorial n = product [1..n]
Prelude> factorial 4
24
```

# Example uses of comprehensions

```
squares :: [Integer] -> [Integer]
squares xs  =  [ x*x | x <- xs ]

odds :: [Integer] -> [Integer]
odds xs  =  [ x | x <- xs, odd x ]

sumSqOdd :: [Integer] -> Integer
sumSqOdd xs  =  sum [ x*x | x <- xs, odd x ]
```

# QuickCheck, a program

```haskell
-- sumSqOdd.hs

import Test.QuickCheck

squares :: [Integer] -> [Integer]
squares xs  =  [ x*x | x <- xs ]

odds :: [Integer] -> [Integer]
odds xs  =  [ x | x <- xs, odd x ]

sumSqOdd :: [Integer] -> [Integer]
sumSqOdd xs  =  sum [ x*x | x <- xs, odd x ]

prop_sumSqOdd :: [Integer] -> Bool
prop_sumSqOdd xs  =  sum (squares (odds xs)) == sumSqOdd xs
```

# QuickCheck, running the program

```
[culross]wadler: ghci sumSqOdd.hs
GHCi, version 6.8.3: http://www.haskell.org/ghc/ :? for help
Loading package base ... linking ... done.
[1 of 1] Compiling Main             ( lect03.hs, interpreted )
*Main> quickCheck prop_sumSqOdd
Loading package old-locale-1.0.0.0 ... linking ... done.
Loading package old-time-1.0.0.0 ... linking ... done.
Loading package random-1.0.0.0 ... linking ... done.
Loading package mtl-1.1.0.1 ... linking ... done.
Loading package QuickCheck-2.1 ... linking ... done.
+++ OK, passed 100 tests.
*Main>
```

# Part II

# Lists and Recursion

# Cons and append

Cons takes an element and a list.

Append takes two lists.

```
(:)  :: a -> [a] -> [a]
(++) :: [a] -> [a] -> [a]

1 : [2,3]       =  [1,2,3]
[1] ++ [2,3]    =  [1,2,3]
[1,2] ++ [3]    =  [1,2,3]
'l' : "ist"     =  "list"
"l" ++ "ist"    =  "list"
"li" ++ "st"    =  "list"

[1] : [2,3]            -- type error!
1 ++ [2,3]             -- type error!
[1,2] ++ 3             -- type error!
"l" : "ist"            -- type error!
'l' ++ "ist"           -- type error!
```

`(:)` is pronounced *cons*, for *construct*

`(++)` is pronounced *append*

# Lists

Every list can be written using only `(:)` and `[]`.

```
[1,2,3]   =  1 : (2 : (3 : []))

"list"    =  ['l','i','s','t']
          =  'l' : ('i' : ('s' : ('t' : [])))
```

A *recursive* definition: A *list* is either

- *null*, written `[]`, or
- *constructed*, written `x:xs`, with *head* `x` (an element), and *tail* `xs` (a list).

# A list of numbers

```
Prelude> null [1,2,3]
False
Prelude> head [1,2,3]
1
Prelude> tail [1,2,3]
[2,3]
Prelude> null [2,3]
False
Prelude> head [2,3]
2
Prelude> tail [2,3]
[3]
Prelude> null [3]
False
Prelude> head [3]
3
Prelude> tail [3]
[]
Prelude> null []
True
```

# Part III

# Mapping: Square every element of a list

# Two styles of definition—squares

## Comprehension

```
squares :: [Integer] -> [Integer]
squares xs  =  [ x*x | x <- xs ]
```

## Recursion

```
squaresRec :: [Integer] -> [Integer]
squaresRec []      =  []
squaresRec (x:xs)  =  x*x : squaresRec xs
```

# Pattern matching and conditionals

## Pattern matching

```
squaresRec :: [Integer] -> [Integer]
squaresRec []        =  []
squaresRec (x:xs)    =  x*x : squaresRec xs
```

## Conditionals with binding

```
squaresCond :: [Integer] -> [Integer]
squaresCond ws   =
  if null ws then
    []
  else
    let
      x  = head ws
      xs = tail ws
    in
      x*x : squaresCond xs
```

# How recursion works—squaresRec

```
squaresRec :: [Integer] -> [Integer]
squaresRec []       =  []
squaresRec (x:xs)   =  x*x : squaresRec xs

   squaresRec [1,2,3]
```

# How recursion works—squaresRec

```
squaresRec :: [Integer] -> [Integer]
squaresRec []       =  []
squaresRec (x:xs)   =  x*x : squaresRec xs

   squaresRec [1,2,3]
=
   squaresRec (1 : (2 : (3 : [])))
```

# How recursion works—squaresRec

```
squaresRec :: [Integer] -> [Integer]
squaresRec []       =  []
squaresRec (x:xs)   =  x*x : squaresRec xs

   squaresRec [1,2,3]
=
   squaresRec (1 : (2 : (3 : [])))
=        { x = 1, xs = (2 : (3 : [])) }
   1*1 : squaresRec (2 : (3 : []))
```

# How recursion works—squaresRec

```
squaresRec :: [Integer] -> [Integer]
squaresRec []       =  []
squaresRec (x:xs)   =  x*x : squaresRec xs

   squaresRec [1,2,3]
=
   squaresRec (1 : (2 : (3 : [])))
=
   1*1 : squaresRec (2 : (3 : []))
=       { x = 2, xs = (3 : []) }
   1*1 : (2*2 : squaresRec (3 : []))
```

# How recursion works—squaresRec

```
squaresRec :: [Integer] -> [Integer]
squaresRec []       =  []
squaresRec (x:xs)   =  x*x : squaresRec xs

   squaresRec [1,2,3]
=
   squaresRec (1 : (2 : (3 : [])))
=
   1*1 : squaresRec (2 : (3 : []))
=
   1*1 : (2*2 : squaresRec (3 : []))
=        { x = 3, xs = [] }
   1*1 : (2*2 : (3*3 : squaresRec []))
```

# How recursion works—squaresRec

```
squaresRec :: [Integer] -> [Integer]
squaresRec []       =  []
squaresRec (x:xs)   =  x*x : squaresRec xs

   squaresRec [1,2,3]
=
   squaresRec (1 : (2 : (3 : [])))
=
   1*1 : squaresRec (2 : (3 : []))
=
   1*1 : (2*2 : squaresRec (3 : []))
=
   1*1 : (2*2 : (3*3 : squaresRec []))
=
   1*1 : (2*2 : (3*3 : []))
```

# How recursion works—squaresRec

```
squaresRec :: [Integer] -> [Integer]
squaresRec []       =  []
squaresRec (x:xs)   =  x*x : squaresRec xs

   squaresRec [1,2,3]
=
   squaresRec (1 : (2 : (3 : [])))
=
   1*1 : squaresRec (2 : (3 : []))
=
   1*1 : (2*2 : squaresRec (3 : []))
=
   1*1 : (2*2 : (3*3 : squaresRec []))
=
   1*1 : (2*2 : (3*3 : []))
=
   1 : (4 : (9 : []))
```

# How recursion works—squaresRec

```
squaresRec :: [Integer] -> [Integer]
squaresRec []       =  []
squaresRec (x:xs)   =  x*x : squaresRec xs

   squaresRec [1,2,3]
=
   squaresRec (1 : (2 : (3 : [])))
=
   1*1 : squaresRec (2 : (3 : []))
=
   1*1 : (2*2 : squaresRec (3 : []))
=
   1*1 : (2*2 : (3*3 : squaresRec []))
=
   1*1 : (2*2 : (3*3 : []))
=
   1 : (4 : (9 : []))
=
   [1,4,9]
```

# How recursion works—squaresRec

```
squaresRec :: [Integer] -> [Integer]
squaresRec []       =  []
squaresRec (x:xs)   =  x*x : squaresRec xs
```

```
   squaresRec [1,2,3]
=
   squaresRec (1 : (2 : (3 : [])))
=
   1*1 : squaresRec (2 : (3 : []))
=
   1*1 : (2*2 : squaresRec (3 : []))
=
   1*1 : (2*2 : (3*3 : squaresRec []))
=
   1*1 : (2*2 : (3*3 : []))
=
   1 : (4 : (9 : []))
=
   [1,4,9]
```

# QuickCheck

```
-- squares.hs
import Test.QuickCheck

squares :: [Integer] -> [Integer]
squares xs  =  [ x*x | x <- xs ]

squaresRec :: [Integer] -> [Integer]
squaresRec []       =  []
squaresRec (x:xs)   =  x*x : squaresRec xs

prop_squares :: [Integer] -> Bool
prop_squares xs   =  squares xs == squaresRec xs
```

```
[culross]wadler: ghci squares.hs
GHCi, version 6.8.3: http://www.haskell.org/ghc/ :? for help
*Main> quickCheck prop_squares
+++ OK, passed 100 tests.
*Main>
```

# Part IV

# Filtering: Select odd elements from a list

# Two styles of definition—odds

## Comprehension

```
odds :: [Integer] -> [Integer]
odds xs  =  [ x | x <- xs, odd x ]
```

## Recursion

```
oddsRec :: [Integer] -> [Integer]
oddsRec []                       =  []
oddsRec (x:xs) | odd x           =  x : oddsRec xs
               | otherwise       =  oddsRec xs
```

# Pattern matching and conditionals

## Pattern matching with guards

```
oddsRec :: [Integer] -> [Integer]
oddsRec []                         =  []
oddsRec (x:xs) | odd x             =  x : oddsRec xs
               | otherwise         =  oddsRec xs
```

## Conditionals with binding

```
oddsCond :: [Integer] -> [Integer]
oddsCond ws  =
  if null ws then
    []
  else
    let
      x  = head ws
      xs = tail ws
    in
      if odd x then
        x : oddsCond xs
      else
        oddsCond xs
```

# How recursion works—oddsRec

```
oddsRec :: [Integer] -> [Integer]
oddsRec []                      =  []
oddsRec (x:xs) | odd x          =  x : oddsRec xs
               | otherwise      =  oddsRec xs
```

```
oddsRec [1,2,3]
```

# How recursion works—oddsRec

```
oddsRec :: [Integer] -> [Integer]
oddsRec []                        =  []
oddsRec (x:xs) | odd x            =  x : oddsRec xs
               | otherwise        =  oddsRec xs
```

```
   oddsRec [1,2,3]
=
   oddsRec (1 : (2 : (3 : [])))
```

# How recursion works—oddsRec

```
oddsRec :: [Integer] -> [Integer]
oddsRec []                        =  []
oddsRec (x:xs) | odd x            =  x : oddsRec xs
               | otherwise        =  oddsRec xs

   oddsRec [1,2,3]
=
   oddsRec (1 : (2 : (3 : [])))
=        { x = 1, xs = (2 : (3 : [])), odd 1 = True }
   1 : oddsRec (2 : (3 : []))
```

# How recursion works—oddsRec

```
oddsRec :: [Integer] -> [Integer]
oddsRec []                        =  []
oddsRec (x:xs) | odd x            =  x : oddsRec xs
               | otherwise        =  oddsRec xs

   oddsRec [1,2,3]
=
   oddsRec (1 : (2 : (3 : [])))
=
   1 : oddsRec (2 : (3 : []))
=       { x = 2, xs = (3 : []), odd 2 = False }
   1 : oddsRec (3 : [])
```

# How recursion works—oddsRec

```
oddsRec :: [Integer] -> [Integer]
oddsRec []                      =  []
oddsRec (x:xs) | odd x          =  x : oddsRec xs
               | otherwise      =  oddsRec xs

   oddsRec [1,2,3]
=
   oddsRec (1 : (2 : (3 : [])))
=
   1 : oddsRec (2 : (3 : []))
=
   1 : oddsRec (3 : [])
=      { x = 3, xs = [], odd 3 = True }
   1 : (3 : oddsRec [])
```

# How recursion works—oddsRec

```
oddsRec :: [Integer] -> [Integer]
oddsRec []                        =  []
oddsRec (x:xs) | odd x            =  x : oddsRec xs
               | otherwise        =  oddsRec xs
```

```
   oddsRec [1,2,3]
=
   oddsRec (1 : (2 : (3 : [])))
=
   1 : oddsRec (2 : (3 : []))
=
   1 : oddsRec (3 : [])
=
   1 : (3 : oddsRec [])
=
   1 : (3 : [])
```

# How recursion works—oddsRec

```
oddsRec :: [Integer] -> [Integer]
oddsRec []                        =  []
oddsRec (x:xs) | odd x            =  x : oddsRec xs
               | otherwise        =  oddsRec xs
```

```
   oddsRec [1,2,3]
=
   oddsRec (1 : (2 : (3 : [])))
=
   1 : oddsRec (2 : (3 : []))
=
   1 : oddsRec (3 : [])
=
   1 : (3 : oddsRec [])
=
   1 : (3 : [])
=
   [1,3]
```

# How recursion works—oddsRec

```
oddsRec :: [Integer] -> [Integer]
oddsRec []                        =  []
oddsRec (x:xs) | odd x            =  x : oddsRec xs
               | otherwise        =  oddsRec xs
```

```
   oddsRec [1,2,3]
=
   oddsRec (1 : (2 : (3 : [])))
=
   1 : oddsRec (2 : (3 : []))
=
   1 : oddsRec (3 : [])
=
   1 : (3 : oddsRec [])
=
   1 : (3 : [])
=
   [1,3]
```

# QuickCheck

```haskell
-- odds.hs
import Test.QuickCheck

odds :: [Integer] -> [Integer]
odds xs  =  [ x | x <- xs, odd x ]

oddsRec :: [Integer] -> [Integer]
oddsRec []                      =  []
oddsRec (x:xs) | odd x          =  x : oddsRec xs
               | otherwise      =  oddsRec xs

prop_odds :: [Integer] -> Bool
prop_odds xs  =  odds xs == oddsRec xs
```

```
[culross]wadler: ghci odds.hs
GHCi, version 6.8.3: http://www.haskell.org/ghc/ :? for help
*Main> quickCheck prop_odds
+++ OK, passed 100 tests.
*Main>
```

# Part V

# Accumulation: Sum a list

# Sum

```
sum :: [Integer] -> Integer
sum []       =  0
sum (x:xs)   =  x + sum xs

    sum [1,2,3]
```

# Sum

```
sum :: [Integer] -> Integer
sum []        =  0
sum (x:xs)    =  x + sum xs

    sum [1,2,3]
=
    sum (1 : (2 : (3 : [])))
```

# Sum

```
sum :: [Integer] -> Integer
sum []       =  0
sum (x:xs)   =  x + sum xs

    sum [1,2,3]
=
    sum (1 : (2 : (3 : [])))
=       {x = 1, xs = (2 : (3 : []))}
    1 + sum (2 : (3 : []))
```

# Sum

```
sum :: [Integer] -> Integer
sum []       =  0
sum (x:xs)   =  x + sum xs

    sum [1,2,3]
=
    sum (1 : (2 : (3 : [])))
=
    1 + sum (2 : (3 : []))
=      {x = 2, xs = (3 : [])}
    1 + (2 + sum (3 : []))
```

# Sum

```
sum :: [Integer] -> Integer
sum []       =  0
sum (x:xs)   =  x + sum xs

    sum [1,2,3]
=
    sum (1 : (2 : (3 : [])))
=
    1 + sum (2 : (3 : []))
=
    1 + (2 + sum (3 : []))
=      {x = 3, xs = []}
    1 + (2 + (3 + sum []))
```

# Sum

```
sum :: [Integer] -> Integer
sum []       =  0
sum (x:xs)   =  x + sum xs

    sum [1,2,3]
=
    sum (1 : (2 : (3 : [])))
=
    1 + sum (2 : (3 : []))
=
    1 + (2 + sum (3 : []))
=
    1 + (2 + (3 + sum []))
=
    1 + (2 + (3 + 0))
```

# Sum

```
sum :: [Integer] -> Integer
sum []        =  0
sum (x:xs)    =  x + sum xs

    sum [1,2,3]
=
    sum (1 : (2 : (3 : [])))
=
    1 + sum (2 : (3 : []))
=
    1 + (2 + sum (3 : []))
=
    1 + (2 + (3 + sum []))
=
    1 + (2 + (3 + 0))
=
    6
```

# Sum

```
sum :: [Integer] -> Integer
sum []       =  0
sum (x:xs)   =  x + sum xs

    sum [1,2,3]
=
    sum (1 : (2 : (3 : [])))
=
    1 + sum (2 : (3 : []))
=
    1 + (2 + sum (3 : []))
=
    1 + (2 + (3 + sum []))
=
    1 + (2 + (3 + 0))
=
    6
```

# Sum

```
sum :: [Integer] -> Integer
sum []       =  0
sum (x:xs)   =  x + sum xs

    sum [1,2,3]
=
    sum (1 : (2 : (3 : [])))
=
    1 + sum (2 : (3 : []))
=
    1 + (2 + sum (3 : []))
=
    1 + (2 + (3 + sum []))
=
    1 + (2 + (3 + 0))
=
    6
```

# Product

```
product :: [Integer] -> Integer
product []       =  1
product (x:xs)   =  x * product xs

    product [1,2,3]
=
    product (1 : (2 : (3 : [])))
=
    1 * product (2 : (3 : []))
=
    1 * (2 * product (3 : []))
=
    1 * (2 * (3 * product []))
=
    1 * (2 * (3 * 1))
=
    6
```

Part VI

# Putting it all together:

# Sum of the squares of the odd numbers in a list

# Two styles of definition

## Comprehension

```
sumSqOdd :: [Integer] -> Integer
sumSqOdd xs  =  sum [ x*x | x <- xs, odd x ]
```

## Recursion

```
sumSqOddRec :: [Integer] -> Integer
sumSqOddRec []                    =  0
sumSqOddRec (x:xs) | odd x        =  x*x + sumSqOddRec xs
                   | otherwise    =  sumSqOddRec xs
```

# How recursion works—sumSqOddRec

```
sumSqOddRec :: [Integer] -> Integer
sumSqOddRec []                  =  []
sumSqOddRec (x:xs) | odd x      =  x*x + sumSqOddRec xs
                   | otherwise  =  sumSqOddRec xs
```

```
   sumSqOddRec [1,2,3]
```

# How recursion works—sumSqOddRec

```
sumSqOddRec :: [Integer] -> Integer
sumSqOddRec []                    =  []
sumSqOddRec (x:xs) | odd x        =  x*x + sumSqOddRec xs
                   | otherwise    =  sumSqOddRec xs
```

```
   sumSqOddRec [1,2,3]
=
   sumSqOddRec (1 : (2 : (3 : [])))
```

# How recursion works—sumSqOddRec

```
sumSqOddRec :: [Integer] -> Integer
sumSqOddRec []                       =  0
sumSqOddRec (x:xs) | odd x           =  x*x + sumSqOddRec xs
                   | otherwise       =  sumSqOddRec xs

   sumSqOddRec [1,2,3]
=
   sumSqOddRec (1 : (2 : (3 : [])))
=        { x = 1, xs = (2 : (3 : [])), odd 1 = True }
   1*1 + sumSqOddRec (2 : (3 : []))
```

# How recursion works—sumSqOddRec

```
sumSqOddRec :: [Integer] -> Integer
sumSqOddRec []                        =  0
sumSqOddRec (x:xs) | odd x            =  x*x + sumSqOddRec xs
                   | otherwise        =  sumSqOddRec xs

   sumSqOddRec [1,2,3]
=
   sumSqOddRec (1 : (2 : (3 : [])))
=
   1*1 + sumSqOddRec (2 : (3 : []))
=        { x = 2, xs = (3 : []), odd 2 = False }
   1*1 + sumSqOddRec (3 : [])
```

# How recursion works—sumSqOddRec

```
sumSqOddRec :: [Integer] -> Integer
sumSqOddRec []                      =  0
sumSqOddRec (x:xs) | odd x          =  x*x + sumSqOddRec xs
                   | otherwise      =  sumSqOddRec xs
```

```
   sumSqOddRec [1,2,3]
=
   sumSqOddRec (1 : (2 : (3 : [])))
=
   1*1 + sumSqOddRec (2 : (3 : []))
=
   1*1 + sumSqOddRec (3 : [])
=        { x = 3, xs = [], odd 3 = True }
   1*1 + (3*3 : sumSqOddRec [])
```

# How recursion works—sumSqOddRec

```
sumSqOddRec :: [Integer] -> Integer
sumSqOddRec []                     =  0
sumSqOddRec (x:xs) | odd x         =  x*x + sumSqOddRec xs
                   | otherwise     =  sumSqOddRec xs
```

```
   sumSqOddRec [1,2,3]
=
   sumSqOddRec (1 : (2 : (3 : [])))
=
   1*1 + sumSqOddRec (2 : (3 : []))
=
   1*1 + sumSqOddRec (3 : [])
=
   1*1 + (3*3 + sumSqOddRec [])
=
   1*1 + (3*3 + 0)
```

# How recursion works—sumSqOddRec

```
sumSqOddRec :: [Integer] -> Integer
sumSqOddRec []                    =  0
sumSqOddRec (x:xs) | odd x        =  x*x + sumSqOddRec xs
                   | otherwise    =  sumSqOddRec xs
```

```
   sumSqOddRec [1,2,3]
=
   sumSqOddRec (1 : (2 : (3 : [])))
=
   1*1 + sumSqOddRec (2 : (3 : []))
=
   1*1 + sumSqOddRec (3 : [])
=
   1*1 + (3*3 + sumSqOddRec [])
=
   1*1 + (3*3 + 0)
=
   1 + (9 + 0)
```

# How recursion works—sumSqOddRec

```
sumSqOddRec :: [Integer] -> Integer
sumSqOddRec []                       =  0
sumSqOddRec (x:xs) | odd x           =  x*x + sumSqOddRec xs
                   | otherwise       =  sumSqOddRec xs
```

```
   sumSqOddRec [1,2,3]
=
   sumSqOddRec (1 : (2 : (3 : [])))
=
   1*1 + sumSqOddRec (2 : (3 : []))
=
   1*1 + sumSqOddRec (3 : [])
=
   1*1 + (3*3 + sumSqOddRec [])
=
   1*1 + (3*3 + 0)
=
   1 + (9 + 0)
=
   10
```

# How recursion works—sumSqOddRec

```
sumSqOddRec :: [Integer] -> Integer
sumSqOddRec []                     =  0
sumSqOddRec (x:xs) | odd x         =  x*x + sumSqOddRec xs
                   | otherwise     =  sumSqOddRec xs
```

```
   sumSqOddRec [1,2,3]
=
   sumSqOddRec (1 : (2 : (3 : [])))
=
   1*1 + sumSqOddRec (2 : (3 : []))
=
   1*1 + sumSqOddRec (3 : [])
=
   1*1 + (3*3 + sumSqOddRec [])
=
   1*1 + (3*3 + 0)
=
   1 + (9 + 0)
=
   10
```