INFR08013 INFORMATICS 1 - FUNCTIONAL PROGRAMMING

Wednesday 19th December 2012

14:30 to 16:30

INSTRUCTIONS TO CANDIDATES

1. Note that ALL QUESTIONS ARE COMPULSORY.

2. DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.

3. This is an OPEN BOOK examination.

Convener: J Bradfield
External Examiner: A Preece

THIS EXAMINATION WILL BE MARKED ANONUMOUSLY
1. (a) Write a function \( f :: \text{Char} \rightarrow \text{String} \rightarrow \text{String} \) that replaces every second character in a string, starting with the first, with the given character. The result string should have the same length as the argument string and should begin with the given character, followed by the second character of the string, followed by the given character, followed by the fourth character of the string, followed again by the given character, and so on. For example:

\[
\begin{align*}
f \ . \ . \ "\text{abcdefg}" & = \ "\text{b.d.f.}" \\
f \ . \ . \ "\text{abcd}" & = \ "\text{b.d}" \\
f \ . \ . \ "\text{"}" & = [] \\
f \ . \ . \ "\text{a}" & = \ "\text{.}" \\
\end{align*}
\]

Use basic functions, list comprehension, and library functions, but not recursion. Credit may be given for indicating how you have tested your function. [12 marks]

(b) Write a second function \( g :: \text{Char} \rightarrow \text{String} \rightarrow \text{String} \) that behaves like \( f \), this time using basic functions and recursion, but not list comprehension or other library functions. Credit may be given for indicating how you have tested your function. [12 marks]
2. (a) Write a function \( p :: \text{[Int]} \rightarrow \text{Bool} \) that checks that every positive number in a list is divisible by 3. For example:

\[
\begin{align*}
p \ [-1,6,-15,12,9,-9] &= \text{True} \\
p \ [-1,6,-15,11,-9] &= \text{False} \\
p \ [] &= \text{True} \\
p \ [-1,-15] &= \text{True}
\end{align*}
\]

Use basic functions, list comprehension, and library functions, but not recursion. Credit may be given for indicating how you have tested your function. [16 marks]

(b) Write a second function \( q :: \text{[Int]} \rightarrow \text{Bool} \) that behaves like \( p \), this time using basic functions and recursion, but not list comprehension or library functions. Credit may be given for indicating how you have tested your function. [16 marks]

(c) Write a third function \( r :: \text{[Int]} \rightarrow \text{Bool} \) that also behaves like \( p \), this time using the following higher-order library functions:

\[
\begin{align*}
\text{map} :& : (a \rightarrow b) \rightarrow \text{[a]} \rightarrow \text{[b]} \\
\text{filter} :& : (a \rightarrow \text{Bool}) \rightarrow \text{[a]} \rightarrow \text{[a]} \\
\text{foldr} :& : (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow \text{[a]} \rightarrow b
\end{align*}
\]

Do not use recursion or list comprehension. Credit may be given for indicating how you have tested your function. [12 marks]
3. The following data type represents propositional formulas built from a single variable (\(X\)), constants true (\(T\)) and false (\(F\)), negation (\(\text{Not}\)) and conjunction (\(\land\), written using infix :&:):

\[
data \text{Prop} = X \\
| \ F \\
| \ T \\
| \ \text{Not Prop} \\
| \ \text{Prop :&: Prop}
\]

The template file includes a function \(\text{showProp} :: \text{Prop} \to \text{String}\) which converts formulas into a readable format and code that enables QuickCheck to generate arbitrary values of type \(\text{Prop}\), to aid testing.

(a) Write a function \(\text{eval} :: \text{Prop} \to \text{Bool} \to \text{Bool}\), which given a propositional formula and the truth value of the variable \(X\) returns the value of the formula. For example,

\[
\begin{align*}
\text{eval} \ (\text{Not} \ F) \ \text{True} &= \ True \\
\text{eval} \ (\text{Not} \ X) \ \text{False} &= \ True \\
\text{eval} \ (\text{Not} \ X :&: \ \text{Not} \ (\text{Not} \ X)) \ \text{True} &= \ False \\
\text{eval} \ (\text{Not} \ X :&: \ \text{Not} \ (\text{Not} \ X)) \ \text{False} &= \ False \\
\text{eval} \ (\text{Not} \ (\text{Not} \ X :&: \ T)) \ \text{True} &= \ True \\
\text{eval} \ (\text{Not} \ (\text{Not} \ X :&: \ T)) \ \text{False} &= \ False
\end{align*}
\]

Credit may be given for indicating how you have tested your function. \(16 \text{ marks}\)

(b) Write a function \(\text{simplify} :: \text{Prop} \to \text{Prop}\) that converts a propositional formula to an equivalent simpler formula by use of the following laws:

\[
\begin{align*}
\text{Not} \ T &= \ F \\
\text{Not} \ F &= \ T \\
\text{Not} \ (\text{Not} \ p) &= \ p \\
T :&: \ p &= \ p \\
F :&: \ p &= \ F \\
p :&: \ T &= \ p \\
p :&: \ F &= \ F \\
p :&: \ p &= \ p
\end{align*}
\]

For example,

\[
\begin{align*}
\text{simplify} \ (\text{Not} \ X :&: \ \text{Not} \ (\text{Not} \ X)) &= \ \text{Not} \ X :&: X \\
\text{simplify} \ (\text{Not} \ (\text{Not} \ X :&: \ F)) &= \ T \\
\text{simplify} \ (\text{Not} \ T) &= \ F \\
\text{simplify} \ (\text{Not} \ F :&: \ X) &= \ X \\
\text{simplify} \ (\text{Not} \ (\text{Not} \ (\text{Not} \ X) :&: \ X)) &= \ \text{Not} \ X
\end{align*}
\]

Credit may be given for indicating how you have tested your function. \(16 \text{ marks}\)