# Informatics 1: Data & Analysis

## Lecture 6: Tuple Relational Calculus

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# Lecture Plan

## Data Representation

This first course section starts by presenting two common data representation models.

- The *entity-relationship (ER)* model
- The *relational* model

## Data Manipulation

This is followed by some methods for manipulating data in the relational model and using it to extract information.

- *Relational algebra*
- The *tuple-relational calculus*
- The query language *SQL*

# The State We're In

## Relational models

- Relations: Tables matching schemas
- Schema: A set of field names and their domains
- Table: A set of tuples of values matching these fields

## Relational algebra

A high-level mathematical language of operations on relational tables. Each operation takes one or more tables, and returns another.

selection $\sigma$, projection $\pi$, renaming $\rho$, union $\cup$, difference $-$, cross-product $\times$, intersection $\cap$ and different kinds of join $\bowtie$

## Tuple relational calculus (TRC)

A declarative mathematical notation for writing queries: specifying information to be drawn from the linked tables of a relational model.

# Simple Example

## All records for students more than 18 years old

$$\{\, S \mid S \in \text{Students} \ \wedge\ S.\text{age} > 18 \,\}$$

The set of tuples $S$ such that $S$ is in the table “Students” and has component “age” least 18.

This is like list comprehension in Haskell

```
[ s | s <- students, age s > 18 ]
```

and similar constructions in other languages.

All are based on “comprehensions” in set theory

# Tuple Relational Calculus Basics

Queries in TRC have the general form

$$\{\, \mathrm{T} \mid \mathrm{P}(\mathrm{T}) \,\}$$

where T is a *tuple variable* and $\mathrm{P}(\mathrm{T})$ is a logical formula.

Every tuple variable such as T has a *schema*, like rows in a relational table, with fields and their domains. In practice, the details of the schema are usually inferred from the way T appears in $\mathrm{P}(\mathrm{T})$.

A tuple variable ranges over all possible tuple values matching its schema.

The result of the query

$$\{\, \mathrm{T} \mid \mathrm{P}(\mathrm{T}) \,\}$$

is then the set of all possible tuple values for T such that $\mathrm{P}(\mathrm{T})$ is true.

# Another Example

## Names and ages of all students over 20

$$\{\, \mathsf{T} \mid \exists S \,.\, S \in \mathsf{Students} \;\wedge\; S.\mathsf{age} > 20$$
$$\wedge\; \mathsf{T.name} = S.\mathsf{name} \;\wedge\; \mathsf{T.age} = S.\mathsf{age} \,\}$$

The set of tuples T such that there is an $S$ in table "Students" with component "age" at least 20 and where $S$ and T have the same values for "name" and "age".

- Tuple variable $S$ has schema matching the table "Students".
- Tuple variable T has (only) fields "name" and "age", with domains to match those of $S$.
- Even if $S$ has other fields, they do not appear in T or the overall result.

# Formula Syntax

Inside TRC expression $\{T \mid P(T)\}$ the logical formula $P(T)$ may be quite long, but is built up from standard logical components.

- Simple assertions: $(T \in \text{Table})$, $(T.\text{age} > 65)$, $(S.\text{name} = T.\text{name})$, ...
- Logical combinations: $(P \vee Q)$, $(P \wedge Q \wedge \neg Q')$, ...
- Quantification:

| | |
|---|---|
| $\exists S \,.\, P(S)$ | There exists a tuple $S$ such that $P(S)$ |
| $\forall T \,.\, Q(T)$ | For all tuples $T$ it is true that $Q(T)$ |

For convenience, we require that for $\exists S \,.\, P(S)$ the variable $S$ must actually appear in $P(S)$; and the same for $\forall T \,.\, Q(T)$. We also write:

$$\exists S \in \text{Table} \,.\, P(S) \quad \text{to mean} \quad \exists S \,.\, S \in \text{Table} \wedge P(S)$$

# Students and Courses (1/5)

| mn | code | mark |
|---|---|---|
| s0456782 | inf1 | 80 |
| s0412375 | geo1 | 78 |
| s0412375 | inf1 | 56 |
| s0189034 | math1 | 62 |

Referencing relation

| mn | name | age | email |
|---|---|---|---|
| s0456782 | John | 18 | john@inf |
| s0412375 | Mary | 18 | mary@inf |
| s0378435 | Helen | 20 | helen@phys |
| s0189034 | Peter | 22 | peter@math |

| code | name | year |
|---|---|---|
| inf1 | Informatics 1 | 1 |
| math1 | Mathematics 1 | 1 |
| geo1 | Geology 1 | 1 |
| dbs | Database Systems | 3 |
| adbs | Advanced Databases | 4 |

Referenced relations

Primary key

Foreign key

# Students and Courses (1/5)

## Students taking Informatics 1

$$\{\, R \mid \exists S \in \mathsf{Students} \,.\, \exists T \in \mathsf{Takes} \,.\, \exists C \in \mathsf{Courses} \,.$$
$$C.\mathsf{name} = \text{"Informatics 1"} \;\wedge\; C.\mathsf{code} = T.\mathsf{code}$$
$$\wedge\; T.\mathsf{mn} = S.\mathsf{mn} \;\wedge\; S.\mathsf{name} = R.\mathsf{name} \,\}$$

Schema for $S$, $T$ and $C$ match those of the tables from which they are drawn. The schema for result $R$ is a single field "name" with string domain, because that's all that appears here.

One way to compute this in relational algebra:

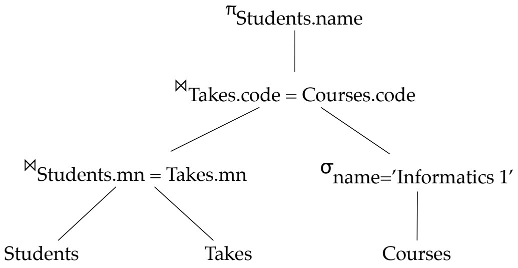
$$\pi_{\mathsf{name}}((\mathsf{Students} \bowtie \mathsf{Takes}) \bowtie (\sigma_{\mathsf{name}=\text{"Informatics 1"}}(\mathsf{Courses})))$$

# Relational Algebra

The relational algebra expression can be rearranged without changing its value, but possibly affecting the time and memory needed for computation:

$$\pi_{\mathsf{name}}((\mathsf{Students} \bowtie \mathsf{Takes}) \bowtie (\sigma_{\mathsf{name}=\text{"Informatics 1"}}(\mathsf{Courses})))$$
$$\pi_{\mathsf{name}}(\mathsf{Students} \bowtie (\mathsf{Takes} \bowtie (\sigma_{\mathsf{name}=\text{"Informatics 1"}}(\mathsf{Courses}))))$$
$$\pi_{\mathsf{name}}(\mathsf{Students} \bowtie ((\sigma_{\mathsf{name}=\text{"Informatics 1"}}(\mathsf{Courses})) \bowtie \mathsf{Takes}))$$

We can also visualise this as rearrangements of a tree:



# Students and Courses (2/5)

## Courses taken by students called “Joe”

$$\{\, R \mid \exists S \in \text{Students}, T \in \text{Takes}, C \in \text{Courses} \;.\; S.\text{name} = \text{"Joe"} \;\wedge\; S.\text{mn} = T.\text{mn} \;\wedge\; C.\text{code} = T.\text{code} \;\wedge\; C.\text{name} = R.\text{name} \,\}$$

Note the slightly abbreviated syntax for multiple quantification: we use comma-separated $\exists..,..,..$ instead of $\exists..\exists..\exists..$

Computing this in relational algebra:

$$\pi_{\text{name}}((\text{Courses} \bowtie \text{Takes}) \bowtie (\sigma_{\text{name}=\text{"Joe"}}(\text{Students})))$$

# Students and Courses (3/5)

## Students taking Informatics 1 or Geology 1

$$\{\, R \mid \exists S \in \text{Students}, T \in \text{Takes}, C \in \text{Courses} \,.\\ (C.\text{name} = \text{"Informatics 1"} \vee C.\text{name} = \text{"Geology 1"})\\ \wedge\; C.\text{code} = T.\text{code} \;\wedge\; T.\text{mn} = S.\text{mn} \;\wedge\; S.\text{name} = R.\text{name} \,\}$$

Now the logical formula becomes a little more elaborate.

Computing this in relational algebra:

$$\pi_{\text{name}}((\text{Students} \bowtie \text{Takes}) \bowtie (\sigma_{\text{name}=\text{"Informatics 1"}}(\text{Courses})))\\ \cup\, \pi_{\text{name}}((\text{Students} \bowtie \text{Takes}) \bowtie (\sigma_{\text{name}=\text{"Geology 1"}}(\text{Courses})))$$

$$\pi_{\text{name}}((\text{Students} \bowtie \text{Takes}) \bowtie (\sigma_{(\text{name}=\text{"Informatics 1"} \vee \text{name}=\text{"Geology 1"})}(\text{Courses}))$$

# Students and Courses (4/5)

## Students taking both Informatics 1 and Geology 1

$$
\begin{aligned}
\{\, R \mid \exists S \in \text{Students}, T, T' \in \text{Takes}, C, C' \in \text{Courses}\ .\\
&C.\text{name} = \text{"Informatics 1"} \ \wedge\ C.\text{code} = T.\text{code} \ \wedge\ T.\text{mn} = S.\text{mn}\\
&C'.\text{name} = \text{"Geology 1"} \ \wedge\ C'.\text{code} = T'.\text{code} \ \wedge\ T'.\text{mn} = S.\text{mn}\\
&\ \wedge\ S.\text{name} = R.\text{name} \,\}
\end{aligned}
$$

Computing this in relational algebra:

$$
\begin{aligned}
&\pi_{\text{name}}((\text{Students} \bowtie \text{Takes}) \bowtie (\sigma_{\text{name}=\text{"Informatics 1"}}(\text{Courses})))\\
&\qquad \cap\ \pi_{\text{name}}((\text{Students} \bowtie \text{Takes}) \bowtie (\sigma_{\text{name}=\text{"Geology 1"}}(\text{Courses})))
\end{aligned}
$$

# Students and Courses (5/5)

## Students taking no courses

$$\{\, R \mid \exists S \in \mathsf{Students}\,.\, S.\mathsf{name} = R.\mathsf{name} \;\wedge\; \forall T \in \mathsf{Takes}\,.\, T.\mathsf{mn} \neq S.\mathsf{mn}$$

Computing this in relational algebra:

$$\pi_{\mathsf{name}}(\mathsf{Students} - \pi_{\mathsf{name,mn}}(\mathsf{Students} \bowtie \mathsf{Takes}))$$

⋆ Challenge: why not one of these instead?

$$\pi_{\mathsf{name}}(\mathsf{Students} - (\mathsf{Students} \bowtie \mathsf{Takes}))$$

$$\pi_{\mathsf{name}}(\mathsf{Students}) - \pi_{\mathsf{name}}(\mathsf{Students} \bowtie \mathsf{Takes}))$$

# Relational Algebra vs. Tuple Relational Calculus

Codd gave a proof that relational algebra and TRC are equally expressive: anything expressed in one language can also be written in the other.

So why have both?

They give different perspectives and allow the following approach:

- Use relational calculus to specify the information wanted;
- Translate into relational algebra to give a procedure for computing it;
- Rearrange the algebra to make that procedure efficient.

The database language SQL is based on the calculus: well-suited to giving logical specifications, independent of any eventual implementation.

The algebra beneath it is good for rewriting, equations, and calculation.

# Domain-Specific Languages


Charles V, 1500–1558
Holy Roman Emperor, King of Spain, Archduke of Austria

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"I speak Spanish to God, Italian to women, French to men and German to my horse."

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Attributed, but even Wikipedia is sceptical.

# Query Optimization

- ... Rearrange the algebra to make that procedure efficient.

This last part is central to the viability of modern large databases. An effective *query optimizer* will draw up a list of possible *query plans* and compare the costs of all of them, taking account of:

- How much data there is, where it is, how it is arranged;
- What indexes are available, for which tables, and where they are;
- Selectivity: estimates of how many rows a subquery will return;
- Estimated size of any intermediate tables;
- What parts can be done in parallel;
- What I/O and computing resources are available;
- . . .