Informatics 1: Data & Analysis

Lecture 17: Data Scales and Summary Statistics

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Unstructured Data

Data Retrieval

- The information retrieval problem
- The vector space model for retrieving and ranking

Statistical Analysis of Data

- Data scales and summary statistics
- Hypothesis testing and correlation
- χ^2 tests and collocations

also *chi-squared*, pronounced "kye-squared"

Analysis of Data

There are many reasons to analyse data. For example, to:

- Discover implicit structure in the data.

  For example, finding patterns in experimental data which might in turn suggest new models or experiments.
- Confirm or refute a hypothesis about the data.

  For example, to test a scientific theory against the results of an experiment.

Mathematical statistics provide a powerful toolkit for performing such analyses, with wide and effective application.

Their analytic strength cuts two ways:

- Statistics can sensitively detect information not immediately apparent within a mass of data;
- Statistics can help determine whether or not an apparent feature of data is really there.

Learn Statistics

There are lots of books for learning about statistics. Here are two, intended to be approachable introductions without requiring especially strong mathematical background.

P. Hinton.
Statistics Explained: A Guide for Social Science Students.
Routledge, second edition, 2004.

D. B. Wright and K. London.
First (and Second) Steps in Statistics.
SAGE Publications Ltd, second edition, 2009.

Statistics in Action

Here are two more books, for finding out about how statistics are used and abused. Both are easy reading. The second has amusing pictures, too.

M. Blastland and A. Dilnot.
The Tiger That Isn't: Seeing Through a World of Numbers.
Profile, 2008.

"Makes statistics far, far too interesting"

D. Huff.
How to Lie with Statistics.
W. W. Norton, 1954.

"The most widely read statistics book in the history of the world"

Data Scales

What type of statistical analysis we might apply to some data depends on:

- The reason for wishing to carry out the analysis;
- The type of data to hand.

Data may be *quantitative* (numerical) or *qualitative* (descriptive).

We can refine this further into different kinds of *data scale*:

- Qualitative data may be drawn from a *categorical* or an *ordinal* scale;
- Quantitative data may lie on an *interval* or a *ratio* scale.

Each of these supports different kinds of analyses.

Categorical Scales

Data on a categorical scale has each item of data being drawn from a fixed number of categories.

Example: A government might classify visa applications from people wishing to visit according to the nationality of the applicant. This classification is a categorical scale: the categories are all the different possible nationalities.

Example: Insurance companies classify some insurance applications (e.g., home, possessions, car) according to the alphanumeric postcode of the applicant, making different risk assessments for different postcodes. Here the categories are all existing postcodes.

Categorical scales are sometimes called *nominal*, particularly where the categories all have names.

Ordinal Scales

Data on an ordinal scale has a recognized ordering between data items, but there is no meaningful arithmetic on the values.

Example: The *European Credit Transfer and Accumulation System* (ECTS) has a grading scale where course results are recorded as A, B, C, D, E, FX and F. There are no numerical marks. The ordering is clear, but we can't add or subtract grades.

Example: The *Douglas Sea Scale* classifies the state of the sea on a scale from 0 (glassy calm) through 5 (rough) to 9 (phenomenal). This is ordered, but it makes no sense to perform arithmetic: 4 (moderate) is not the mean of 2 (smooth) and 6 (very rough).

Interval Scales

An interval scale is a numerical scale (usually with real number values) in which we are interested in *relative value* rather than *absolute value*.

Example: Moments in time are given relative to an arbitrarily chosen zero point. We can make sense of comparisons such as "date x is 17 years later than date y". But it does not make sense to say "arrival time p is twice as large as departure time q".

Example: The Celsius and Fahrenheit temperature scales are interval scales, as the choice of zero is externally imposed.

Mathematically, interval scales support the operations of subtraction and average (all kinds, possibly weighted).

Interval scales do not support either addition or multiplication.

Ratio Scales

A ratio scale is a numerical scale (again usually with real number values) in which there is a notion of *absolute value*.

Example: Most physical quantities such as mass, energy and length are measured on ratio scales. The Kelvin temperature scale is a ratio scale. So is age (of a person, for example), even though it is a time — because there is a definite zero origin.

Like interval scales, ratio scales support subtraction and weighted averages. They also support addition and multiplication by a real number (a *scalar*).

Visualising data

It is often helpful to visualise data by drawing a chart or plotting a graph of the data. Visualisations may suggest possible properties of the data, whose existence and features we can then explore mathematically with statistics.

What kind of visualisations are possible depends on the kind of data.

For a data on a categorical or ordinal scale, a natural visual representation is a *bar chart*, displaying for each category the number of times it occurs in the data.

Bars in a bar chart are all the same width, and separate.

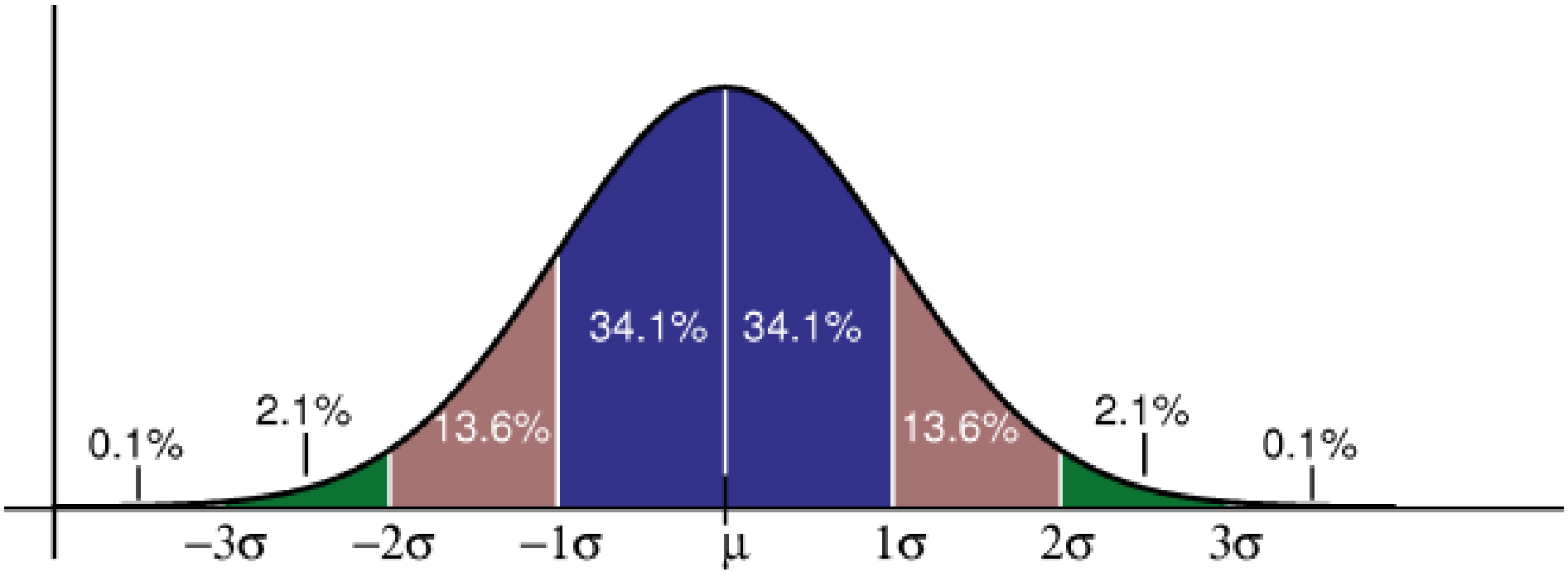
For data from an interval or ratio scale, we can collect data into bands and draw a *histogram*, giving the frequency with which values occur in the data.

In a histogram the bars are adjacent, and can be of different widths: it is their area, not height, which measures the number of values present.

Normal Distribution

In the *normal distribution*, data is clustered symmetrically around a central value with a bell-shaped frequency curve.

For sound mathematical reasons, many real-world examples of numerical data do follow a normal distribution. However, not all do so, and the name "normal" can sometimes be misleading.

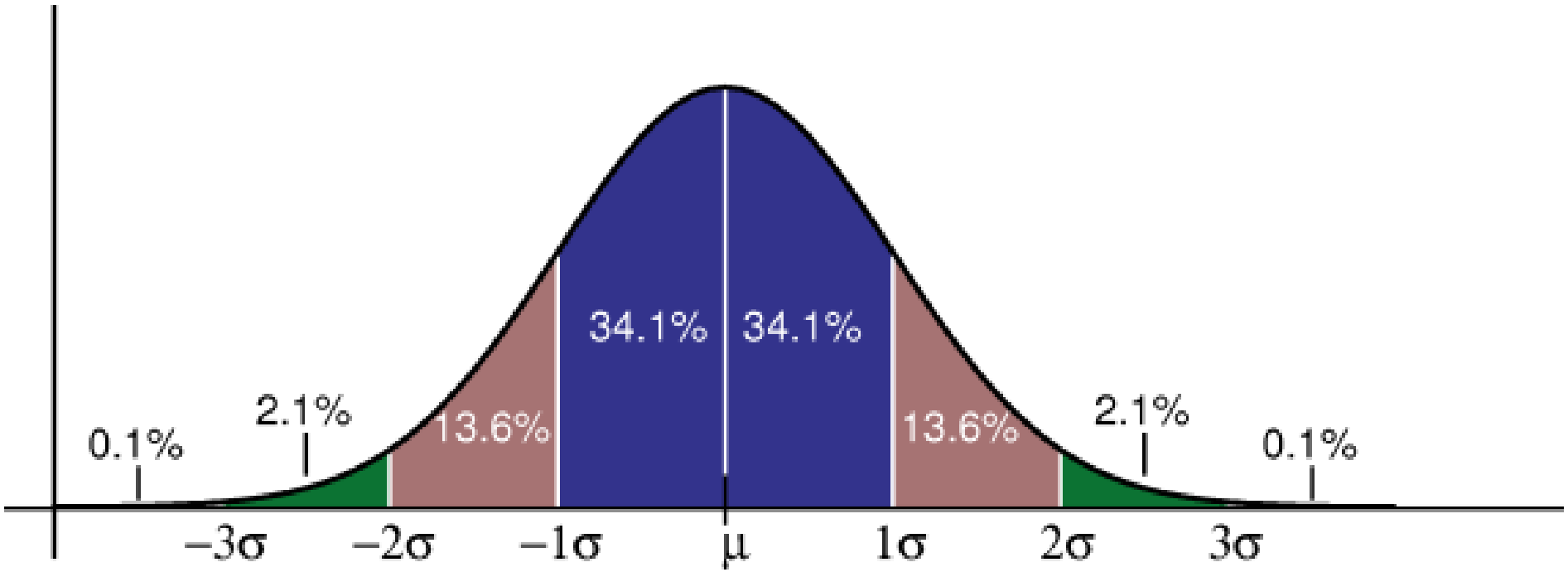


Normal Distribution

Any normal distribution is described by two parameters.

The *mean* μ (mu, said "mew") is the centre around which the data clusters.

The *standard deviation* σ (sigma) is a measure of the spread of the curve. For a normal distribution, it coincides with the *inflection point* where the curve changes from being convex to concave.



Statistics

A *statistic* is a single value computed from data that captures some overall property of the data.

For example, the mean of a normal distribution is a statistic that captures the value around which the data is clustered.

Similarly, the standard deviation of a normal distribution is a statistic that captures the degree of spread of the data around its mean.

The notion of mean and standard deviation generalise to quantitative data that is not normally distributed.

There are also other statistics, the mode and median, that are alternatives to the mean for summarising the "average value" of some data.

Mode

For any set of data the *mode* is the value which occurs most often.

Example: For the categorical data $\{\mathrm{red}, \mathrm{blue}, \mathrm{orange}, \mathrm{red}, \mathrm{yellow}\}$ the mode is red, which is the only value to occur twice.

Data may be *bimodal* (two modes) or even *multimodal* (more than two).

Example: For the integer data set $\{6, 2, 3, 6, 2, 5, 1, 7, 2, 5, 6\}$ both 2 and 6 are modes, each occurring three times.

The mode makes sense for all types of data scale. However, it is not particularly informative for quantitative data with real-number values, where it is uncommon for the same data value to occur more than once.

This is an instance of a more general phenomenon: in general it is neither useful nor meaningful to compare real-number values for equality

Median

Given data values $x_1, x_2, \ldots, x_N$ sorted into in non-decreasing order, the *median* is the middle value $x_{(N+1)/2}$, for N odd. If N is even, then any value between $x_{N/2}$ and $x_{(N/2)+1}$ inclusive is a possible median.

Example: Given the integer data set $\{6, 2, 3, 6, 2, 5, 0, 7, 2, 5, 6\}$ we can write it in non-decreasing order $\{0, 2, 2, 2, 3, 5, 5, 6, 6, 6, 7\}$ and identify the middle value as 5.

The median makes sense for qualitative ordinal data and quantitative interval and ratio data. It does not make sense for categorical data, as that has no appropriate ordering.

Median is a sensible summary statistic for data where there is a forced cutoff at one end, or the likelihood of distortion by extreme outliers. For example, typical applications include reporting income data, hospital waiting times and cancer survival times.

Mean

Given data values $\{x_1, x_2, \ldots, x_N\}$, the *mean* is their total divided by the number of values: $(\sum_{i=1}^{N} x_i)/N$.

Example: For the integer data set $\{6, 2, 3, 6, 2, 5, 0, 7, 2, 5, 6\}$, the mean is $(6+2+3+6+2+5+0+7+2+5+6)/11 = 4$.

Although the formula for the mean involves a sum, the mean makes sense for both interval and ratio scales; it does not depend on an absolute zero in the scale. Mean does not make sense for categorical or ordinal data.

A mean incorporates all the data and is a genuine summary; however, it is not always the right choice of summary statistic, and can be distorted if there are extremely high or low values.

"The mean is like a loaded gun, which in the inexperienced hand can lead to serious accidents, as means can give hopelessly distorted results"

Variance and Standard Deviation

Given data values $\{x_1, x_2, \ldots, x_N\}$ with mean μ, their *variance* σ^2 is the mean square deviation from μ:

$$\text{Variance} = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2$$

Variance measures the spread of data, but changes as the square of the data. A more common measure of spread is its square root, known as the *standard deviation* σ:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}$$

Like the mean, the standard deviation makes sense for both interval and ratio data; but has no meaning for qualitative data scales.

Example

For the integer data set $\{6, 2, 3, 6, 2, 5, 0, 7, 2, 5, 6\}$ we compute:

$$\begin{aligned}
\text{Variance} &= \frac{6^2 + 2^2 + 3^2 + 6^2 + 2^2 + 5^2 + 0^2 + 7^2 + 2^2 + 5^2 + 6^2}{11} \\
&= \frac{36 + 4 + 9 + 36 + 4 + 25 + 0 + 49 + 4 + 25 + 36}{11} \\
&= \frac{228}{11} \\
&= 20.72 \quad \text{to 2 decimal places}
\end{aligned}$$

$$\begin{aligned}
\sigma &= \sqrt{\frac{228}{11}} \\
&= \ 4.55 \quad \text{to 2 decimal places}
\end{aligned}$$

Populations and Samples

So far we have seen different statistics for a given set of data, and how to compute them exactly.

Very often, however, data is only a sample drawn from a larger population, and we really want to know — or find out some information about — the statistic on the whole population. For example:

- Experiments in social sciences where one wants to discover information about some section of society — say, university students.
- Surveys and polls — for marketing, opinion gathering, etc.
- In software design when questioning a number of potential users in order to understand general user requirements.

In such cases it is impractical to obtain exhaustive data about the population as a whole; instead, we must work with a sample.

Sampling

Sampling from a population needs to be done carefully to ensure analysis of the sample is a reliable basis for estimating properties of the whole population.

- The sample should be chosen at random from the population.
- The sample should be as large as is practically possible (given constraints on gathering data, storing data and calculating with data).

These improve the likelihood that a sample is *representative* of the population, reducing the chance of building *bias* into the sample.

Given a sample, we can calculate its statistical properties, and use that to infer information about similar properties of the whole population.

It is a significant topic in statistics, but beyond this course, to work out how to quantify and maximise the reliability of these techniques.

Estimating Population Statistics

Suppose we have a sample $\{x_1, \ldots, x_n\}$ of size n from a population of size N, where $n \ll N$ (i.e., n is much smaller than N).

We use the sample $\{x_1, \ldots, x_n\}$ to estimate statistics for the whole population. These estimates may not be correct; but knowing the sample and population size, we can often make estimates about the errors, too.

For mean, the best estimate of the population mean μ is in fact the sample mean m:

$$m = \frac{1}{n} \sum_{i=1}^{n} x_i$$

Technically, this is an *unbiased estimator*: the mean of a random sample is evenly distributed around the population mean

Estimating Population Variance

The variance and standard deviation of a sample are not appropriate estimates for the equivalent statistics on a population; they turn out to be slightly too small. The best estimate for population variance is

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - m)^2$$

from which we get an estimate for population standard deviation of

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - m)^2}$$

Note that in both cases the denominator n of the sample variance and standard deviation has been replaced by $(n-1)$; and the mean m used is that of the sample, not the (unknown) population mean μ.

Beware

The use of samples to estimate statistics of populations is so common that the formula on the previous slide is very often the one needed, rather than the sample standard deviation itself.

Its usage is so widespread that sometimes it is wrongly given as the definition of standard deviation (try a web search for *images* of "standard deviation formula").

The existence of two different formulas for calculating the standard deviation in different circumstances can lead to confusion. So take care.

Often calculators make both formulas available: as σ_n for the formula with denominator n; and σ_{n-1} for the formula with denominator $(n-1)$.