Part III — Unstructured Data

Data Retrieval:

III.1 Unstructured data and data retrieval

Statistical Analysis of Data:

III.2 Data scales and summary statistics

III.3 Hypothesis testing and correlation

III.4 χ^2 and collocations

Several variables

Often, one wants to relate data in several variables (i.e., multi-dimensional data).

For example, the table below tabulates, for eight students (A–H), their weekly time (in hours) spent: studying for Data & Analysis, drinking and eating. This is juxtaposed with their Data & Analysis exam results.

	A	B	C	D	E	F	G	H
Study	0.5	1	1.4	1.2	2.2	2.4	3	3.5
Drinking	25	20	22	10	14	5	2	4
Eating	4	7	4.5	5	8	3.5	6	5
Exam	16	35	42	45	60	72	85	95

Thus, we have four variables: study, drinking, eating and exam.
(This is four-dimensional data.)

Correlation

We can ask if there is any *relationship* between the values taken by two variables.

If there is no relationship, then the variables are said to be *independent*.
If there is a relationship, then the variables are said to be *correlated*.

Caution: A correlation does *not* imply a *causal relationship* between one variable and another. For example, there is a positive correlation between incidences of lung cancer and time spent watching television, but neither causes the other.

However, in cases in which there *is* a causal relationship between two variables, then there often will be an associated correlation between the variables.

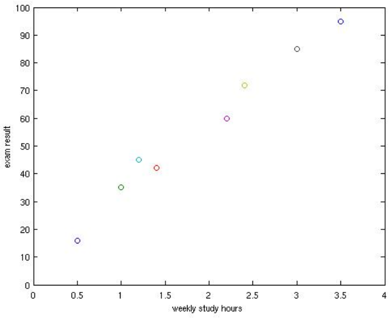
Visualising correlations

One way of discovering correlations is to visualise the data.

A simple visual guide is to draw a *scatter plot* using one variable for the x-axis and one for the y-axis.

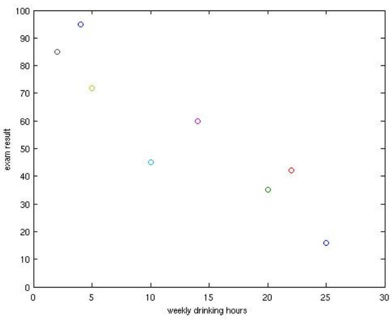
Example: In the example data on Slide III: 52, is there a correlation between study hours and exam results? What about between drinking hours and exam results? What about eating and exam results?

Studying vs. exam results



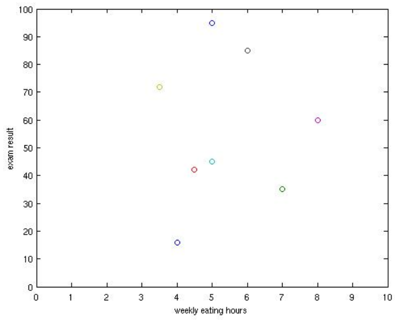
This looks like a *positive* correlation.

Drinking vs. exam results



This looks like a *negative* correlation.

Eating vs. exam results



There is no obvious correlation.

Statistical hypothesis testing

The last three slides use data visualisation as a tool for postulating hypotheses about data.

One might also postulate hypotheses for other reasons, e.g.: intuition that a hypothesis may be true; a perceived analogy with another situation in which a similar hypothesis is known to be valid; existence of a theoretical model that makes a prediction; etc.

Statistics provides the tools needed to corroborate or refute such hypotheses with scientific rigour: *statistical tests*.

The general form of a statistical test

One applies an appropriately chosen statistical test to the data and calculates the result $\boldsymbol{R}$.

Statistical tests are usually based on a *null hypothesis* that there is nothing out of the ordinary about the data.

The result $\boldsymbol{R}$ of the test has an associated *probability value* $\boldsymbol{p}$.

The value $\boldsymbol{p}$ represents the probability that we would obtain a result similar to $\boldsymbol{R}$ if the null hypothesis were true.

N.B., $\boldsymbol{p}$ is *not* the probability that the null hypothesis is true. This is not a quantifiable value.

The general form of a statistical test (continued)

The value p represents the probability that we would obtain a result similar to R if the null hypothesis were true.

If the value of p is *significantly small* then we conclude that the null hypothesis is a poor explanation for our data. Thus we *reject* the null hypothesis, and replace it with a better explanation for our data.

Standard *significance thresholds* are to require $p < 0.05$ (i.e., there is a less than $1/20$ chance that we would have obtained our test result were the null hypothesis true) or, better, $p < 0.01$ (i.e., there is a less than $1/100$ chance)

Correlation coefficient

The *correlation coefficient* is a statistical measure of how closely the data values $x_1, \ldots, x_N$ are correlated with $y_1, \ldots, y_N$.

Let μ_x and σ_x be the mean and standard deviation of the x values.
Let μ_y and σ_y be the mean and standard deviation of the y values.

The correlation coefficient $\rho_{x,y}$ is defined by:

$$\rho_{x,y} \;=\; \frac{\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)}{N\sigma_x\sigma_y}$$

If $\rho_{x,y}$ is positive this suggests x, y are *positively correlated*.
If $\rho_{x,y}$ is negative this suggests x, y are *negatively correlated*.
If $\rho_{x,y}$ is close to 0 this suggests there is no correlation.

Correlation coefficient as a statistical test

In a test for correlation between two variables x, y (e.g., exam result and study hours), we are looking for a correlation and a direction for the correlation (either negative or positive) between the variables.

The *null hypothesis* is that there is no correlation.

We calculate the correlation coefficient $\rho_{x,y}$.

We then look up significance in a *critical values table* for the correlation coefficient. Such tables can be found in statistics books (and on the Web). This gives us the associated probability value p.

The value of p tells us whether we have significant grounds for rejecting the null hypothesis, in which case our better explanation is that there *is* a correlation.

Critical values table for the correlation coefficient

The table has rows for N values and columns for p values.

N	$p = 0.1$	$p = 0.05$	$p = 0.01$	$p = 0.001$
7	0.669	0.754	0.875	0.951
8	0.621	0.707	0.834	0.925
9	0.582	0.666	0.798	0.898

The table shows that for $N = 8$ a value of $|\rho_{x,y}| > 0.834$ has probability $p < 0.01$ of occurring (that is less than a $1/100$ chance of occurring) if the null hypothesis is true.

Similarly, for $N = 8$ a value of $|\rho_{x,y}| > 0.925$ has probability $p < 0.001$ of occurring (that is less than a $1/1000$ chance of occurring) if the null hypothesis is true.

Studying vs. exam results

We use the data from III: 52 (see also III: 55), with the study values for $x_1, \ldots, x_N$, and the exam values for $y_1, \ldots, y_N$, where $N = 8$.

The relevant statistics are:

$$\begin{aligned} \mu_x &= 1.9 & \sigma_x &= 0.981 \\ \mu_y &= 56.25 & \sigma_y &= 24.979 \\ & & \rho_{x,y} &= 0.985 \end{aligned}$$

Our value of 0.985 is (much) higher than the critical value 0.925. Thus we reject the null hypothesis with very high confidence ($p < 0.001$) and conclude that there is a correlation.

It is a *positive correlation* since $\rho_{x,y}$ is positive not negative.

Drinking vs. exam results

We now use the drinking values from III: 52 (see also III: 56) as the values for $x_1, \ldots, x_8$. (The y values are unchanged.)

The new statistics are:

$$\mu_x = 12.75 \qquad \sigma_x = 8.288 \qquad \rho_{x,y} = -0.914$$

Since $|-0.914| = 0.914 > 0.834$, we can reject the null hypothesis with confidence ($p < 0.01$). This result is still significant though less so than the previous.

This time, the value -0.914 of $\rho_{x,y}$ is negative so we conclude that there is a *negative correlation*

Estimating correlation from a sample

As on slides III: 47–48, assume samples $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ from a population of size N where $n \ll N$.

Let m_x and m_y be the estimates of the means of the x and y values (V: 47)
Let s_x and s_y be the estimates of the standard deviations (V: 48)

The best estimate $r_{x,y}$ of the correlation coefficient is given by:

$$r_{x,y} \;=\; \frac{\sum_{i=1}^{n}(x_i - m_x)(y_i - m_y)}{(n-1)s_x s_y}$$

The correlation coefficient is sometimes called *Pearson's correlation coefficient*, particularly when it is estimated from a sample using the formula above.

Correlation coefficient — subtleties

The correlation coefficient measures how close a scatter plot of x, y values is to a straight line. Nonetheless, a high correlation does not mean that the relationship between x, y is linear. It just means it can be reasonably closely approximated by a linear relationship.

Critical value tables for the correlation coefficient are often given with rows indexed by *degrees of freedom* rather than by N. For the correlation coefficient, the number of *degrees of freedom* is $N - 2$, so it is easy to translate such a table into the form given here. (The notion of degree of freedom, in the case of correlation, is too advanced a concept for D&A.)

Also, critical value tables often have two classifications: one for *one-tailed tests* and one for *two-tailed tests*. Here, we are applying a *two-tailed test*: we consider both positive and negative values as significant. In a *one-tailed* test, we would be interested in just one of these possibilities.