INSTRUCTIONS TO CANDIDATES

1. Note that ALL QUESTIONS ARE COMPULSORY.

2. DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.

3. CALCULATORS MAY NOT BE USED IN THIS EXAMINATION.

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY
1. This question concerns the 64 possible truth valuations of six propositional letters, \( ABCDEF \). For each of the following expressions say how many of the 64 valuations satisfy the expression:

Use the space provided for any rough working, and to briefly explain your reasoning.

(a) \((A \Leftrightarrow B) : C)\)

Answer:

Reason: [3 marks]

(b) \((A \rightarrow B) \rightarrow C\)

Answer:

Reason: [3 marks]

(c) \(A \rightarrow (B \rightarrow C) \land (D \lor E \lor F)\)

Answer:

Reason: [3 marks]

(d) \((A \rightarrow B) \land (B \rightarrow C) \land (C \rightarrow A) \land (E \rightarrow F) \land (F \rightarrow C)\)

Answer:

Reason: [3 marks]

(e) \((A \rightarrow D) \land (C \rightarrow D) \land (\neg D \rightarrow E) \land (E \rightarrow D) \land (F \rightarrow C)\)

Answer:

Reason: [3 marks]
2. For each of the following entailments complete two Karnaugh maps, one to represent the assumption and one the conclusion, by marking the valuations that make the expression false.

Place a mark in one of the check boxes provided, to indicate whether the entailment is valid. Give a reason for your answer in the box provided.

Use your Karnaugh maps to give a simple CNF for each assumption.

(a) \( \neg(\neg A \land \neg C) \land (B \to \neg C) \models \neg(B \to A) \)

Valid □ Invalid □ [1 mark]

\[
\begin{array}{c|cccc}
\text{AB} & 00 & 01 & 11 & 10 \\
\hline
\text{assumption CD} & & & & \\
00 & & & & \\
01 & & & & \\
11 & & & & \\
10 & & & &
\end{array}
\]

\[
\begin{array}{c|cccc}
\text{CD} & 00 & 01 & 11 & 10 \\
\hline
\text{conclusion AB} & & & & \\
00 & & & & \\
01 & & & & \\
11 & & & & \\
10 & & & &
\end{array}
\]

Reason:

assumption CNF:

(b) \( (A \oplus B) \to (C \to D) \models (A \to C) \to (A \to (\neg B \to D)) \)

Valid □ Invalid □ [1 mark]

\[
\begin{array}{c|cccc}
\text{AB} & 00 & 01 & 11 & 10 \\
\hline
\text{assumption CD} & & & & \\
00 & & & & \\
01 & & & & \\
11 & & & & \\
10 & & & &
\end{array}
\]

\[
\begin{array}{c|cccc}
\text{CD} & 00 & 01 & 11 & 10 \\
\hline
\text{conclusion AB} & & & & \\
00 & & & & \\
01 & & & & \\
11 & & & & \\
10 & & & &
\end{array}
\]

Reason:

assumption CNF:
3. (a) Convert each of the following expressions to CNF

- $R \rightarrow (P \land Q)$  
- $(S \oplus P \ ? T : R)$  
- $(T \lor P \ ? Q : R))$  
- $\neg(A \leftrightarrow B) \rightarrow C$

(b) Use resolution to determine whether the entailment

$(B \rightarrow A) \rightarrow (A \rightarrow C) \vdash B \rightarrow C$

is valid, and produce a counterexample if it is not.

(c) Use resolution to determine whether

$(P \lor Q) \rightarrow (R \lor S), Q \rightarrow \neg R \vdash Q \rightarrow S$

is valid and produce a counter-example if it is not.
Gentzen Rules

Question 4 refers to these rules.

\[
\begin{align*}
\Gamma, A, B & \vdash \Delta & (I) \\
\Gamma, A \land B & \vdash \Delta & (\land L) \\
\Gamma & \vdash A, B, \Delta & (\land R) \\
\Gamma, A & \vdash \Delta & (\lor L) \\
\Gamma & \vdash A, B, \Delta & (\lor R) \\
\Gamma & \vdash A, \Delta & (\rightarrow L) \\
\Gamma & \vdash A \rightarrow B, \Delta & (\rightarrow R) \\
\Gamma & \vdash A, \Delta & (\neg L) \\
\Gamma & \vdash \neg A, \Delta & (\neg R)
\end{align*}
\]

A and B are propositional expressions, \( \Gamma, \Delta \) are sets of expressions, and \( \Gamma, A \) refers to \( \Gamma \cup \{A\} \).
4. Use the Gentzen rules, provided on the previous page, to attempt to prove the following entailment, your **goal**:

\[(P \to Q) \to R, \ S \lor P \vdash \lnot R \to (Q \to S)\]  

(goal)

(a) Which of the rules have a conclusion matching this goal? 
   For each such rule complete a line in the table below showing the name of the rule and the bindings for \(\Gamma, \Delta, A, B\)  

<table>
<thead>
<tr>
<th>Rule</th>
<th>(\Gamma)</th>
<th>(\Delta)</th>
<th>(A)</th>
<th>(B)</th>
</tr>
</thead>
</table>

(b) Use the Gentzen rules to construct a formal proof with the goal as conclusion. Label each step in your proof with the name of the rule being applied.  

\[(P \to Q) \to R, \ S \lor P \vdash \lnot R \to (Q \to S)\]
5. Give a regular expression (re) for the language accepted by each FSM. Mark the check boxes to show the strings it accepts, and whether it, together with any implicit black hole state, is deterministic. Draw an equivalent DFA if it is not.

(a) [Diagram]

```
abba □
abab □
abbb □
abba □
DFA □
```

(b) [Diagram]

```
abba □
abab □
abbb □
baab □
DFA □
```

(c) [Diagram]

```
abab □
abba □
babb □
baba □
DFA □
```

(d) [Diagram]

```
aaa □
baab □
bbab □
bbbb □
DFA □
```

(e) [Diagram]

```
aaab □
aab □
baab □
bbbb □
DFA □
```