Informatics 1 - Computation & Logic: Tutorial 2

Propositional Logic: Venn Diagrams and Truth Tables

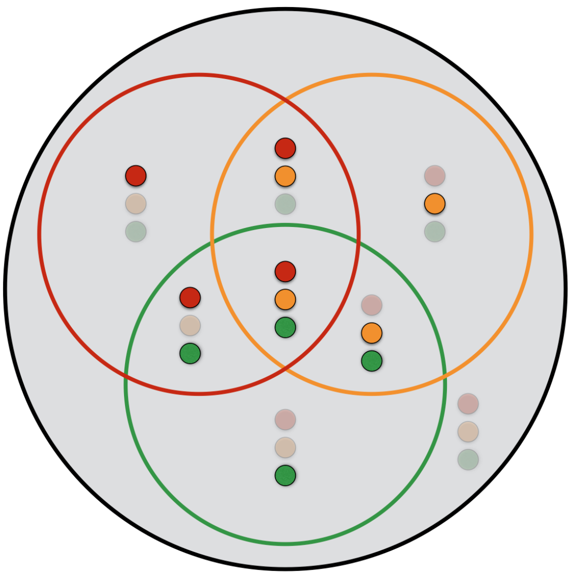
Week 4: 10-16 October 2016

Please attempt the entire worksheet in advance of the tutorial, and bring with you all work, including (if a computer is involved) printouts of code and test results. Tutorials cannot function properly unless you do the work in advance.

You may work with others, but you must understand the work; you can't phone a friend during the exam.

Assessment is formative, meaning that marks from coursework do not contribute to the final mark. But coursework is not optional. If you do not do the coursework you are unlikely to pass the exams.

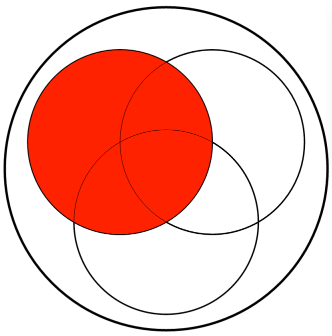
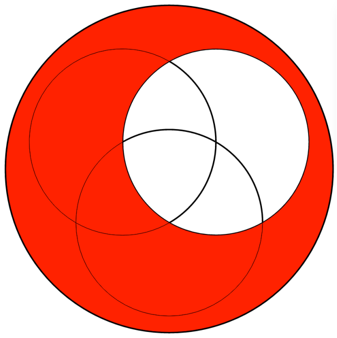
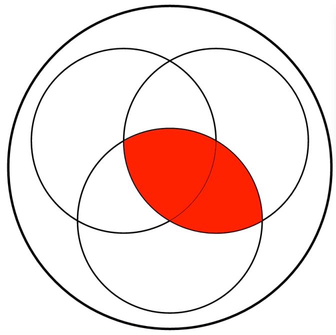
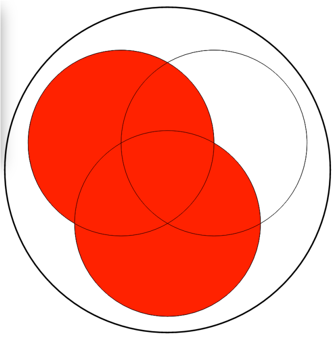
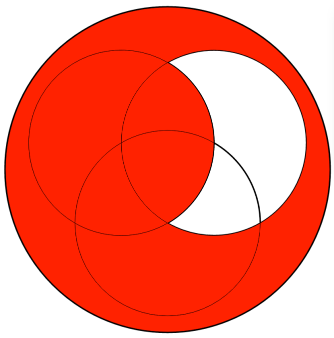
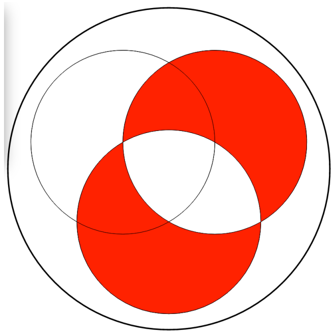
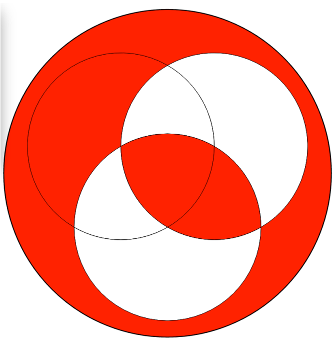
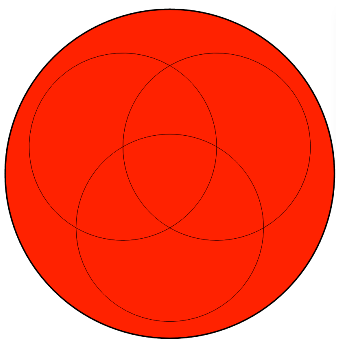
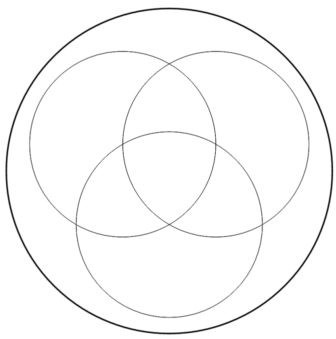
Attendance at tutorials is **obligatory**; please let your tutor know if you cannot attend.



A Venn diagram is, in essence, a visual truth table. In the blank diagrams below, each circle represents a region in which a given logical atom, is true; R, A, and G, going clockwise from top left. Where the circles overlap, two or three of the atoms are true. The diagram here represents the atoms by three lights, which may be on (true) or off (false), to show the state in each of the eight regions.

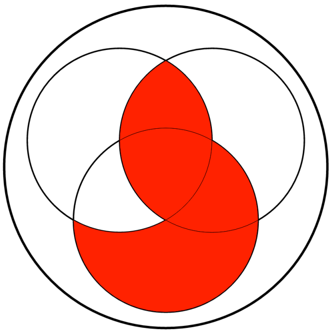
1. In this exercise, you will be asked to translate between truth tables, logical expressions, and Venn diagrams.

   (a) For the nine examples, shade in the appropriate regions of the Venn diagram to show the states in which the expression is true.

 R	 $\neg A$	 $A \wedge G$
 $G \vee R$	 $A \rightarrow R$	 $G \oplus A$
 $A \leftrightarrow G$	 $\top$	 $\perp$

You can check your answers using the Venn Diagram maker at https://www.inf.ed.ac.uk/teaching/courses/inf1/cl/tools/venn/, but please work them out for yourself first. If you read the question quite hastily, you could be forgiven for wondering if the diagrams illustrating $\wedge$ and $\vee$ were in fact transposed; after all, if the instruction is "shade in G and A", shouldn't the whole of G and A be shaded? In fact, no. The correct interpretation is that the region *that is both in* G *AND* A should be shaded. Likewise, for $G \vee R$, you are shading in the correct parts of the diagram if you are shading inside the G circle OR the R circle.

(b) Next, look at these truth tables, and shade in the Venn diagram accordingly; then, see if you can figure out corresponding logical expression; note that while there is only one correct diagram for each truth table, there are infinitely many equivalent expressions. Shorter expressions are in general to be preferred here to longer ones.



R	A	G	Exp: $(R \mathbin{?} A : G)$
$\top$	$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\bot$
$\bot$	$\top$	$\top$	$\top$
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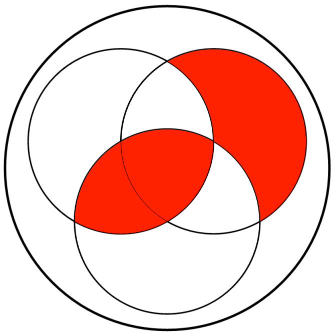

R	A	G	Exp: $(R \vee G) \mathbin{?} (R \wedge G) : A$
$\top$	$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$	$\bot$
$\top$	$\bot$	$\top$	$\top$
$\top$	$\bot$	$\bot$	$\bot$
$\bot$	$\top$	$\top$	$\bot$
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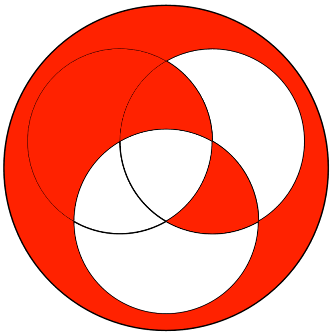

R	A	G	Exp: $G \mathbin{?} (A \wedge \neg R) : (A \rightarrow R)$
$\top$	$\top$	$\top$	$\bot$
$\top$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\top$	$\top$
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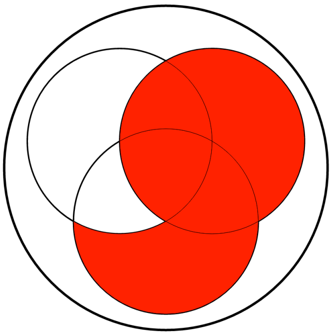
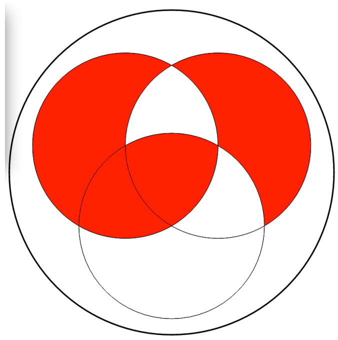
$\bot$	$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$	$\top$



R	A	G	Exp: $G \mathbin{?} \neg R \vee A : A$
$\top$	$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\bot$
$\bot$	$\top$	$\top$	$\top$
$\bot$	$\top$	$\bot$	$\top$
$\bot$	$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$	$\bot$

(c) Next, given the Venn diagram, try to complete the truth table and the expression.



R	A	G	Exp: $((R \wedge G) \vee (A \wedge \neg R \wedge \neg G)) \vee \neg(R \to A)$
$\top$	$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$	$\bot$
$\top$	$\bot$	$\top$	$\top$
$\top$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\top$	$\bot$
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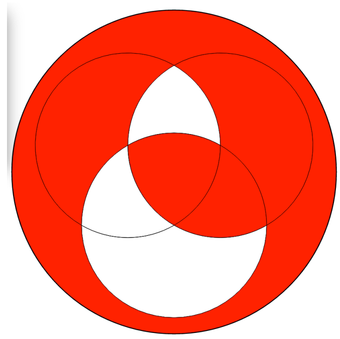
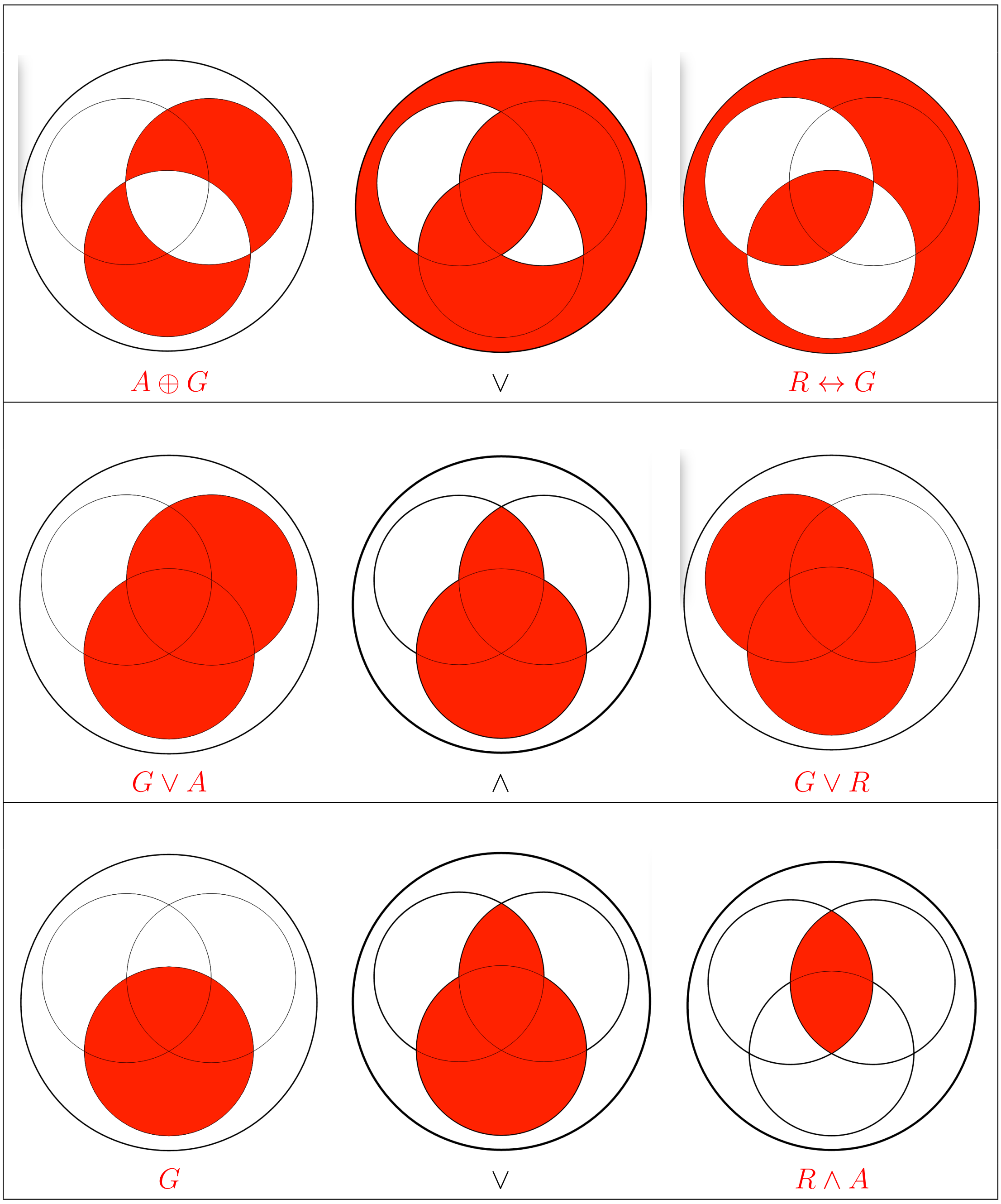


R	A	G	Exp: $(G \to A) \wedge \neg(R \wedge A \wedge \neg G)$
$\top$	$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$	$\bot$
$\top$	$\bot$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\top$	$\top$
$\bot$	$\top$	$\bot$	$\top$
$\bot$	$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$	$\top$

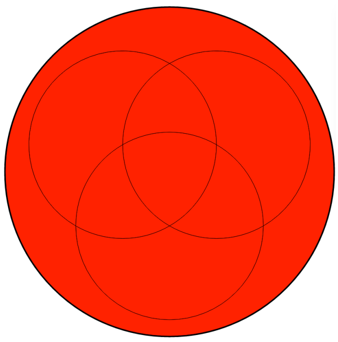
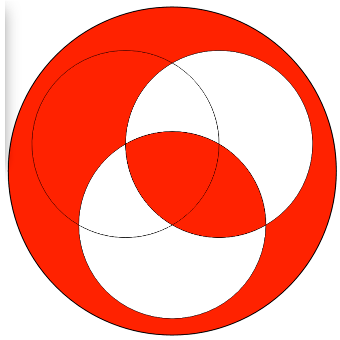
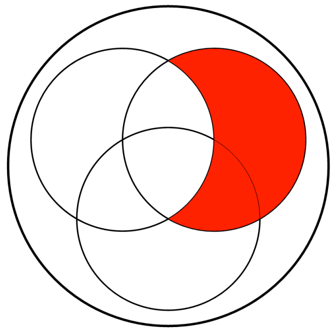
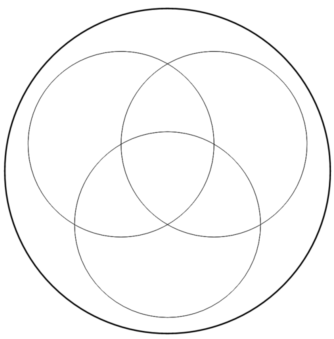
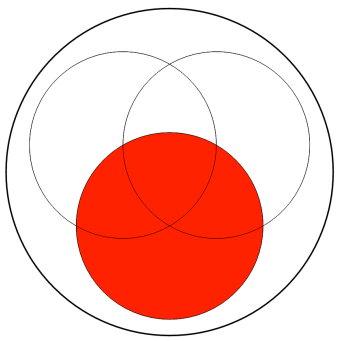
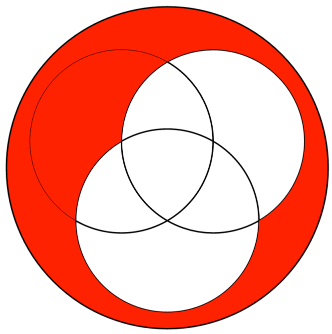
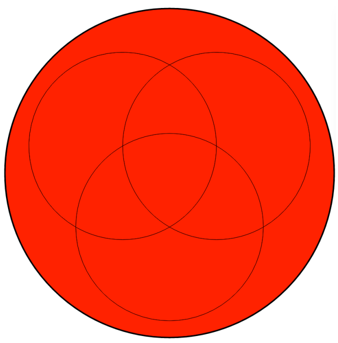
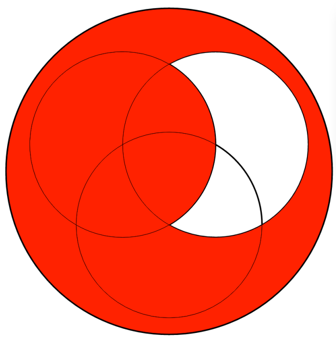
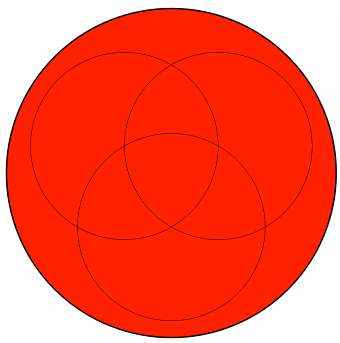
2. By now, you may be getting a feel for how Venn diagrams are combined by different operators. For this question, shade in the middle diagram to combine the left and right diagrams on the basis of the given operator. Work out the correct shading visually, then work out logical expressions and check your work using the Venn diagram generator.

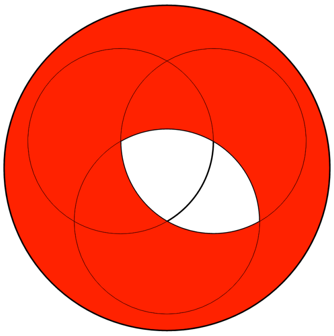


3. Now, you have probably noticed something interesting about the last two answers - well done! You've just discovered the distributivity of disjunction over conjunction! This identity is one of the laws of Boolean Algebra, which we use to reason about logical expressions.

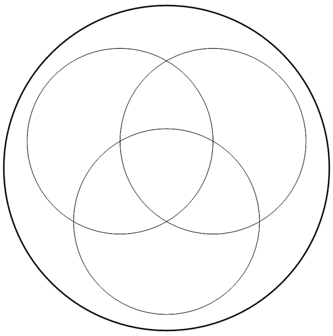
$$a \lor (b \land c) = (a \lor b) \land (a \lor c)$$

Here are some more expressions to diagram. See how many more identities you can find, among the expressions on this question, and among the other expressions on this sheet.

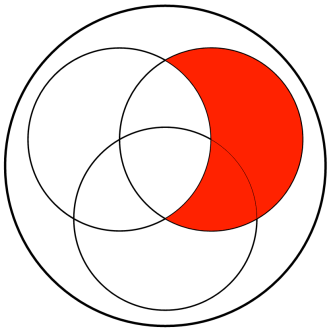
 $\neg\bot$	 $(G \to A) \land (A \to G)$	 $\neg(A \to R)$
 $A \land \neg A$	 $\neg\neg G$	 $\neg G \land \neg A$
 $G \lor \top$	 $\neg A \lor R$	 $A \land \neg A$



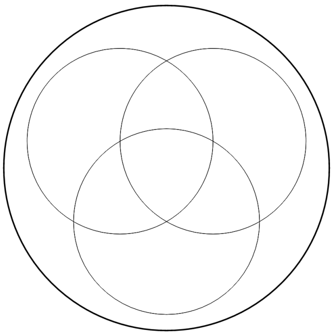
$\neg(A \wedge G)$



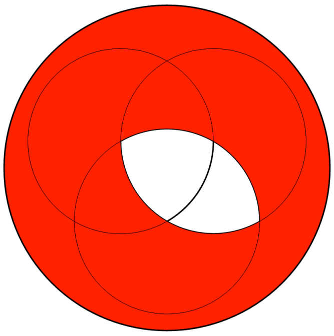
$A \wedge \bot$



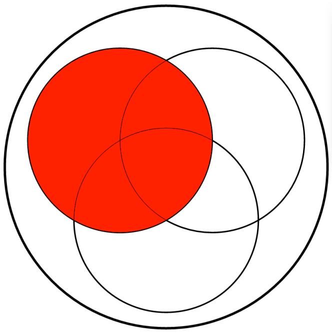
$A \wedge \neg R$



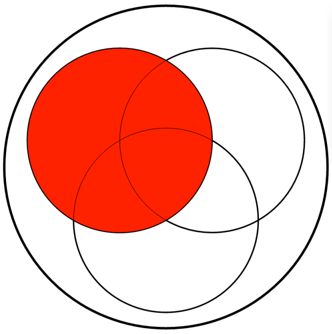
$\neg\top$



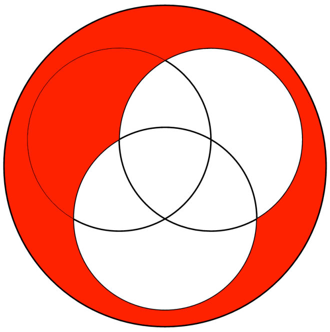
$\neg A \vee \neg G$



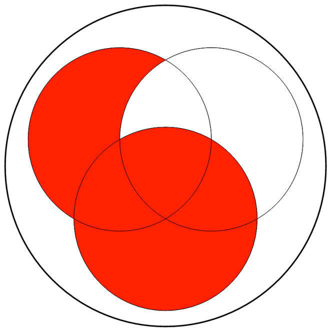
$R \vee \bot$



$R \wedge \top$



$\neg(G \vee A)$



$(R \to A) \to G$

Identities discovered:

$\neg(a \to b) = a \wedge \neg b$ $\quad a \leftrightarrow b = (a \to b) \wedge (b \to a)$ $\quad a \to b = \neg a \vee b$

$\neg(a \vee b) = \neg a \wedge \neg b$ $\quad \neg(a \wedge b) = \neg a \vee \neg b$

$\neg 0 = 1$ $\quad \neg\neg a = a$ $\quad \neg 1 = 0$

$a \vee 1 = 1$ $\quad a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ $\quad a \wedge 0 = 0$

$a \vee 0 = a$ $\quad a \vee \neg a = 1$ $\quad a \wedge \neg a = 0$ $\quad a \wedge 1 = a$

4. Look up the following terms, on the internet or elsewhere, and describe, in words, when an expression in propositional logic is:

   (a) Satisfiable:

   An expression is satisfiable IFF there exists some valuation of its atoms for which it is true

   (b) Tautologous:

   An expression is tautologous IFF it is true for all valuations of its atoms

   (c) Inconsistent:

   An expression is inconsistent IFF it is false for all valuations of its atoms

   (d) Contingent:

   An expression is contingent IFF it is satisfiable but not tautologous; that is, it is true for some valuations but not all.

5. Construct truth tables for the following expressions of propositional logic, and use these to decide whether the expressions are satisfiable, tautologous, contingent, or inconsistent:

   (a) $(R \to A) \vee (\neg A \vee \neg R)$

   Draw the truth table here:

R	A	$\neg R$	$\neg A$	$R \to A$	$(\neg R \vee \neg A)$	EXP
$\top$	$\top$	$\bot$	$\bot$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\bot$	$\top$	$\bot$	$\top$	$\top$
$\bot$	$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\bot$	$\bot$	$\top$	$\top$	$\top$	$\bot$	$\top$

   This expression is
   <u>SATISFIABLE</u>/<u>TAUTOLOGOUS</u>/CONTINGENT/INCONSISTENT

(b) $R \rightarrow (A \wedge (R \vee A))$

Draw the truth table here:

R	A	$R \vee A$	$(A \wedge (R \vee A))$	EXP
$\top$	$\top$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$	$\top$	$\top$
$\bot$	$\bot$	$\bot$	$\bot$	$\top$

This expression is
<u>SATISFIABLE</u>/TAUTOLOGOUS/<u>CONTINGENT</u>/INCONSISTENT

(c) $(\neg R \wedge A) \vee G \oplus ((R \vee \neg A) \rightarrow G)$

Draw the truth table here:

R	A	G	$\neg R$	$\neg A$	$\neg R \wedge A$	$R \vee \neg A$	$(\neg R \wedge A) \vee G$	$((R \vee \neg A) \rightarrow G)$	EXP
$\top$	$\top$	$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$\top$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$
$\top$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$
$\bot$	$\top$	$\top$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\top$	$\bot$
$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\top$	$\bot$
$\bot$	$\bot$	$\top$	$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$

This expression is
SATISFIABLE/TAUTOLOGOUS/CONTINGENT/<u>INCONSISTENT</u>

This tutorial exercise sheet was written by Dave Cochran and Michael Fourman, and includes material from an earlier version by Mark McConville, revised by Paolo Besana, Thomas French, Areti Manataki, and Michael Fourman. Send comments to `michael.fourman@ed.ac.uk`

Summary of useful symbols

Capital	Lowercase	Name
A	α	alpha
B	β	beta
Γ	γ	gamma
Δ	δ	delta
E	ϵ	epsilon
Z	ζ	zeta
H	η	eta
Θ	θ	theta
I	ι	iota
K	κ	kappa
Λ	λ	lambda
M	μ	mu
N	ν	nu
Ξ	ξ	xi
Π	π	pi
P	ρ	rho
Σ	σ	sigma
T	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
X	χ	chi
Ψ	ψ	psi
Ω	ω	omega

Symbol	Meaning	Example
$\neg$	not	$\neg A$
$\wedge$	and	$A \wedge B$
$\vee$	or	$A \vee B$
$\rightarrow$	implies	$A \rightarrow B$
$\models$	entails	$[\beta_1, ..., \beta_n] \models \alpha$
$\leftrightarrow$	equivalent	$A \leftrightarrow B$
$\vdash$	can be proved	$[\beta_1, ..., \beta_n] \vdash \alpha$