UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR08012 INFORMATICS 1 - COMPUTATION AND LOGIC

Thursday 15th December 2016

14:30 to 16:30

INSTRUCTIONS TO CANDIDATES

1. Note that ALL QUESTIONS ARE COMPULSORY.
2. CALCULATORS MAY NOT BE USED IN THIS EXAMINATION.
3. You should
   - WRITE YOUR EXAMINATION NUMBER

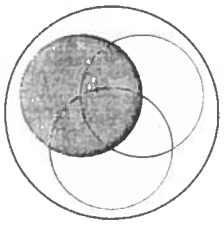
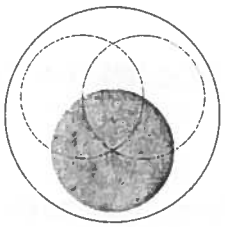
     HERE:
   - Write your answers in the spaces provided in this question paper.
   - Complete your details on the script book (which is provided for rough-working) and hand it in together with this document.

Convener: I. Simpson
External Examiner: I. Gent

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) The entailment $A \vdash C$, is invalid.

- $A \vdash C$

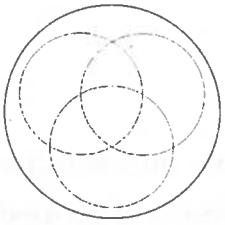
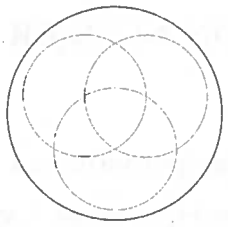
A	C	$A \vdash C$
		Valid ☐ Invalid ☒

How is this invalidity shown by comparing the two Venn diagrams above? [*4 marks*]

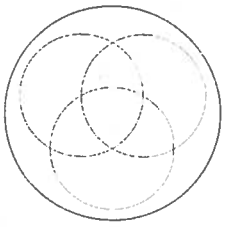
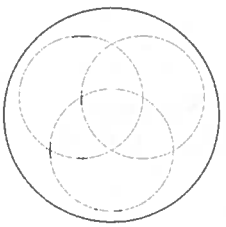
(b) For each of the following entailments, complete the two Venn diagrams to represent the assumption and conclusion, and place a mark in one of the check boxes provided to indicate whether the entailment is valid.

(You should use the same encoding as in the example above, where each circle represents one of the propositions A, B, C.)

i. $A \rightarrow (B \vee C) \vdash A \rightarrow C$ [*8 marks*]

$A \rightarrow (B \vee C)$	$A \rightarrow C$	$A \rightarrow (B \vee C) \vdash A \rightarrow C$
		Valid ☐ Invalid ☐

ii. $A \rightarrow (B \wedge C) \vdash A \rightarrow B$ [*8 marks*]

$A \rightarrow (B \wedge C)$	$A \rightarrow B$	$A \rightarrow (B \wedge C) \vdash A \rightarrow B$
		Valid ☐ Invalid ☐

2. This question concerns the 256 possible truth valuations of the following eight propositional letters A, B, C, D, E, F, G, H. For each of the following expressions, say how many of the 256 valuations satisfy the expression, and briefly explain your reasoning. For example, the expression D is satisfied by half of the valuations, that is 128 of the 256, since for each valuation that makes D true there is a matching valuation that make D false.

(a) $A \vee B$ Answer [*1 mark*]

(b) $(A \wedge B) \vee C$ Answer [*2 marks*]

(c) $A \rightarrow (B \rightarrow C)$ Answer [*2 marks*]

(d) $(A \rightarrow B) \wedge (B \rightarrow A) \wedge (C \rightarrow D) \wedge (D \rightarrow F) \wedge (E \rightarrow F) \wedge (F \rightarrow G) \wedge (G \rightarrow H)$ [*5 marks*]

Answer

(e) $(A \rightarrow B) \wedge (B \rightarrow A) \wedge (C \rightarrow D) \wedge (D \rightarrow C) \wedge (E \rightarrow G) \wedge (F \rightarrow G) \wedge (G \rightarrow H)$ [*5 marks*]

Answer

(f) $(H \rightarrow A) \wedge (A \rightarrow B) \wedge (B \vee C \rightarrow D) \wedge (D \rightarrow F) \wedge (F \rightarrow H) \wedge (F \rightarrow G)$ [*5 marks*]

Answer

3. You are given the following inference rules (Γ, Δ vary over finite sets of expressions; A, B vary over expressions):

$$\frac{}{\Gamma, A \vdash \Delta, A}\ (I)$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R)$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ (\wedge R)$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \rightarrow B \vdash \Delta}\ (\rightarrow L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \rightarrow B, \Delta}\ (\rightarrow R)$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ (\neg R)$$

(Where A and B are propositional expressions, Γ, Δ are sets of expressions, and Γ, A refers to $\Gamma \cup \{A\}$.) This question concerns the use of these rules to prove the following entailment. This is your **goal**.

$$P \rightarrow (Q \rightarrow R),\ Q \vee \neg P \vdash P \rightarrow R \tag{1}$$

(a) Which of these rules have a conclusion matching the goal (1)?
For each such rule complete a line in the table below showing the name of the rule and the bindings for Γ, Δ, A, B *[10 marks]*

Rule	Γ	Δ	A	B

(b) Use the rules given to construct a formal proof with the goal as conclusion, making any remaining assumptions as simple as possible.
Label each step in your proof with the name of the rule being applied. [*10 marks*]

$$\overline{P \to (Q \to R),\ Q \vee \neg P \vdash P \to R}$$

4. (a) What does it mean for an entailment to be valid? [*2 marks*]

Answer

(b) How can resolution be used to determine whether an entailment is valid? [*2 marks*]

Answer

(c) Use resolution to determine whether the entailment $(A \rightarrow B) \rightarrow A \vdash A$ is valid, and produce a counterexample if it is not. [*4 marks*]

A	B	C

Answer

Counterexample? [*2 marks*]

(d) Use resolution to determine whether $P \rightarrow (Q \vee R), (Q \wedge R) \rightarrow S \vdash P \rightarrow S$ is valid and produce a counter-example if it is not. [*4 marks*]

P	Q	R	S

Answer

Counterexample? [*2 marks*]

(e) Explain what it means to claim that the resolution procedure is:

i. **sound** [*2 marks*]

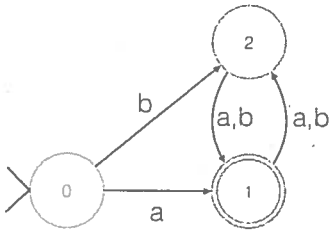
Answer

ii. **complete.** [*2 marks*]

Answer

5. Each diagram shows an FSM. In each case give a regular expression for the language accepted by the FSM, make a mark in the check box against each string that it accepts (and no mark against those strings it does not accept), make a mark in the DFA check box if it is deterministic, and draw an equivalent DFA if it is not.

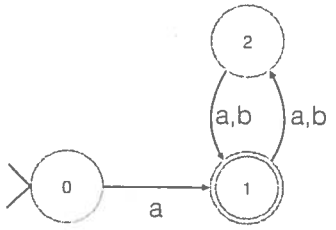
(a)



aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐

[4 marks]

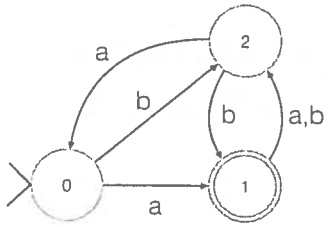
(b)



aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐

[4 marks]

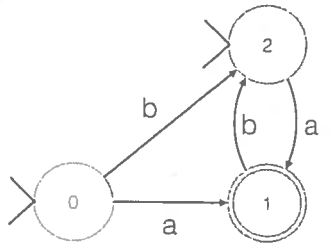
(c)



aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐

[4 marks]

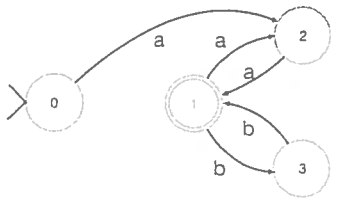
(d)



aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐

[4 marks]

(e)



aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐

[4 marks]