Informatics 1 - Computation & Logic: Tutorial 1

Propositional Logic: An Introduction

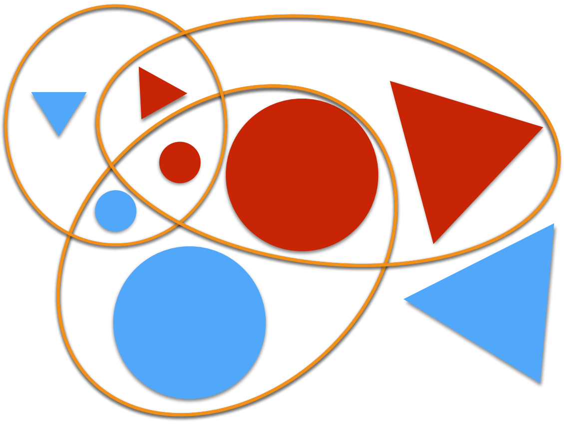
Week 3: 5–9 October 2015

Please attempt the entire worksheet in advance of the tutorial, and bring with you all work, including (if a computer is involved) printouts of code and test results. Tutorials cannot function properly unless you do the work in advance.

You may work with others, but you must understand the work; you can't phone a friend during the exam.

Assessment is formative, meaning that marks from coursework do not contribute to the final mark. But coursework is not optional. If you do not do the coursework you are unlikely to pass the exams.

Attendance at tutorials is **obligatory**; please let your tutor know if you cannot attend.

1. 

Exercise 1.1

There are 8 regions in the diagram. How many subsets of this set of 8 regions are there?

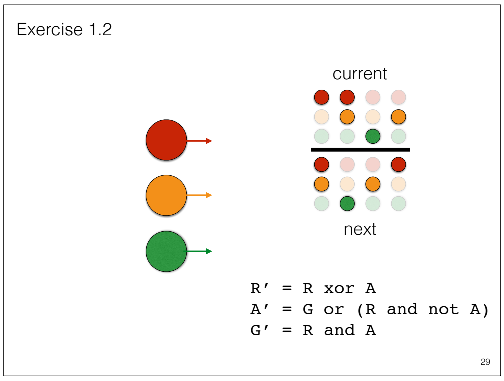
Given any subset of the eight regions can you write a complex proposition to which it corresponds (using **and**, **or**, and **not** as connectives)?

17

You may want to come back to this question after you have completed the rest of the tutorial.

To describe an arbitrary subset you must, for each region, specify whether it is included or not; thus, there are two possibilities for each region, and eight regions. Hence, there are $2^8 = 256$ subsets.

The second part of the question can be answered by characterising each region in the subset by a conjunction of literals, and then taking the disjunction of these. (There are also other ways.)



2.

Slide 25 (lecture 1) shows an implementation of the traffic light controller.

We could have designed our logic differently.

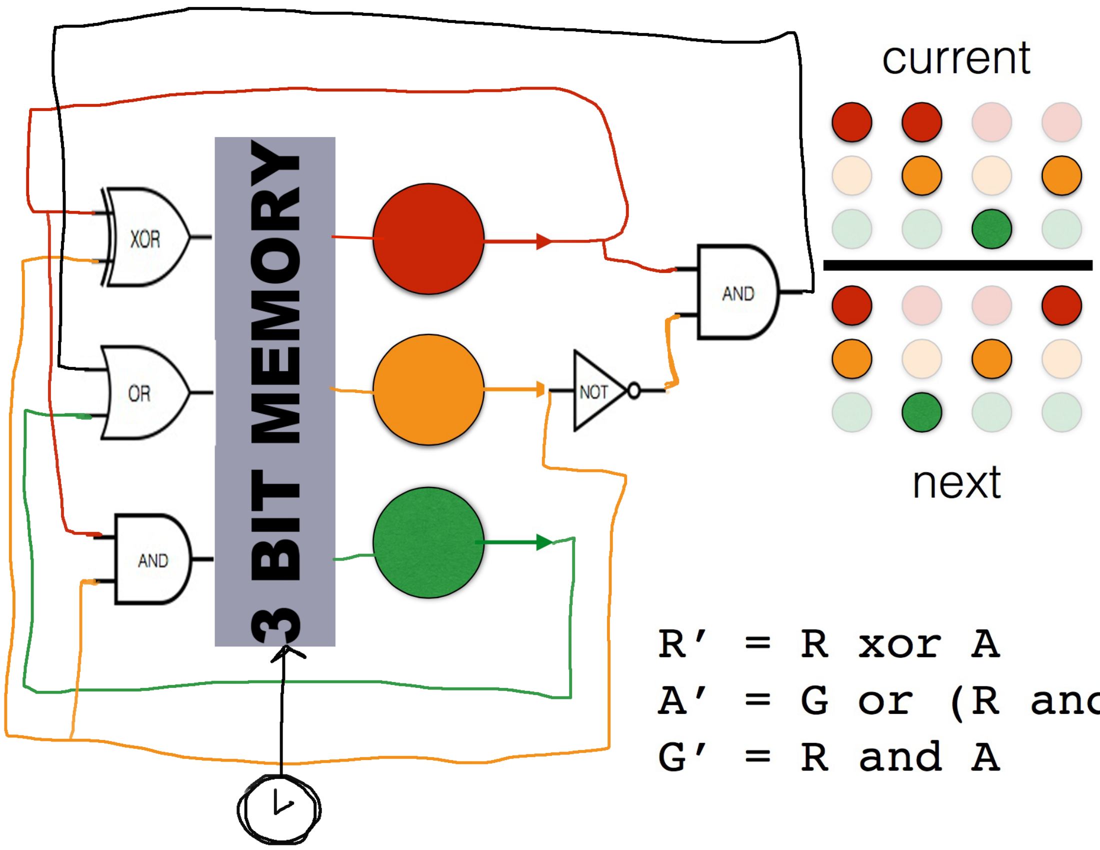
For example, letting

A′ = G or (R and not A).

Draw the circuit for this implementation.

Is this a correct implementation of the controller? Explain your answer.

Exercise 1.2

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R′ = R xor A
A′ = G or (R and not A)
G′ = R and A
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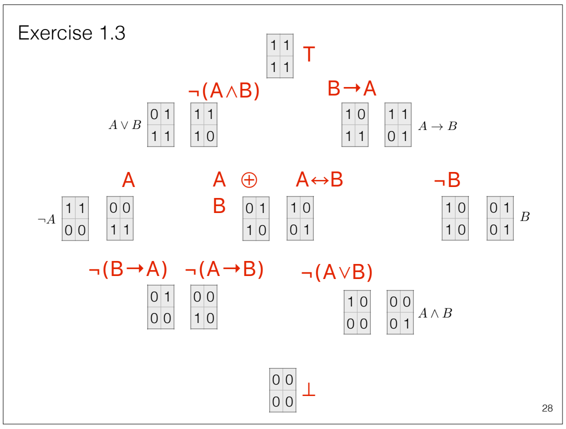
This implementation is based on a logical characterisation, for each light, of the set of current states such that the light is on in the next state. Check that the expression for A′ gives the correct answer in each of the four cases.

In tutorial you should discuss the fact that the expression for A′ is NOT logically equivalent to that given in class (A′ = not A). However, on the assumption that we start in one of the legal states (for example, (R or A) xor G), they are equivalent.

Check that
((R or A) xor G) implies ((not A) equiv (G or (R and not A))
is a tautology.



3.

Each of the 16 2x2 tables above represents the truth table of a binary boolean operation.

Label each table with a boolean expression for which it is the truth table (five tables are already labelled – begin by checking whether these are correct).

How many of the binary operations actually depend on both variables? 10

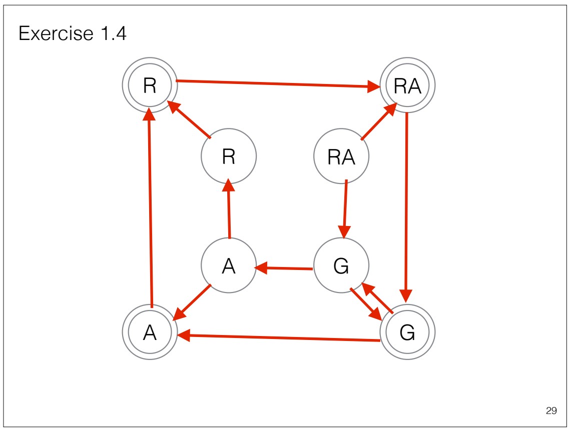
How many depend on only one variable? 4

How many depend on no variables? 2

Note the symmetries and duality apparent in the diagram.

If we think of each truth table as a set of valuations, then the sets are arranged by size (number of 1s in the truth table).

Of course we can use de Morgan, and expansions of → and ⊕, etc., to give other equivalent expressions.



4.

As discussed in the lecture, the diagram represents the beginnings of a refinement of our description of the traffic light controller. We model a sensor that detects a car ready to pass the light. For each state of the lights, (R, RA, G, A) we have two states, one (with a double circle) where there is a car, and the other, without a car, as before.

Draw arrows to indicate state changes that still obey the correct sequence for the lights, but also respect the following two rules.

1. A car can only pass the light if it is green.
2. The light only changes from red to red-amber when a car is detected

Optional: You may also design the logic for the controller.

Use a new boolean variable C to represent the presence of a car, and give equations for R′ A′ and G′.

Should we also give an equation for C′ ?

This machine responds to an external event, the arrival of a car. We shouldn't give an equation.

Discuss what may happen if a car breaks the rules, and runs a red light.

5. Here are some propositional symbols, together with the English sentences they represent:

A	*Sam Mendes directed Skyfall*	T
B	*Leonardo DiCaprio starred in Skyfall*	F
C	*Leonardo DiCaprio starred in Django Unchained*	T
D	*Quentin Tarantino is a director*	T
E	*Leonardo DiCaprio is an actress*	F
F	*Judi Dench is an actress*	T
G	*Judi Dench acted in Skyfall*	T
H	*Leonardo DiCaprio was married to Judi Dench*	F
J	*Django Unchained was released in 2012*	T
K	*Skyfall is set in 16th century Scotland*	F
L	*Leonardo DiCaprio is a woman*	F
M	*Judi Dench used to be married*	T
N	*Leonardo DiCaprio is an actor*	T

Every expression of propositional logic is either **true** or **false**, and no expression can be both true and false. Based on the relationship between propositional symbols and English sentences above, your own general knowledge, and, if need be, the Internet Movie Database, decide whether each proposition is true or false.

6. Assume the propositional symbols in Question 5. Assume also that: (a) the symbol $\neg$ represents the negation operator 'not'; (b) the symbol $\wedge$ represents the conjunction connective 'and'; (c) the symbol $\vee$ represents the disjunction connective 'or'; (d) the symbol $\rightarrow$ represents the implication connective; and (e) the symbol $\leftrightarrow$ represents the equivalence connective.

   Some of the following are well-formed expressions of propositional logic and the others are symbol soup. Decide which is which.

   (a) $A \wedge \neg C$
   (b) $\neg(F \rightarrow D)$
   (c) $\cancel{\leftrightarrow (N \neg B)}$
   (d) $(G \vee \neg L) \leftrightarrow \neg\neg E$ Note: ¬¬E = E
   (e) $A \vee \neg(C \rightarrow H)$
   (f) $\cancel{\vee (K \rightarrow \neg\neg B)}$
   (g) $\cancel{F \vee D \wedge}$
   (h) $H \wedge \neg(A \leftrightarrow \neg C)$

7. Translate the following expressions of propositional logic into reasonably natural English, assuming the key in Question 5:

(a) $E \wedge B$

Leonardo DiCaprio is an actress AND Leonardo DiCaprio starred in Skyfall

(b) $J \vee \neg K$

Django Unchained was released in 2012 OR Skyfall is NOT set in 16th century Scotland

(c) $E \rightarrow L$

IF Leonardo DiCaprio is an actress THEN Leonardo DiCaprio is a woman

(d) $(C \wedge \neg L) \rightarrow N$

IF Leonardo DiCaprio starred in Django Unchained
AND Leonardo DiCaprio is NOT a woman,
THEN Leonardo DiCaprio is an actor

8. Translate the following English sentences into propositional logic, using the appropriate propositional symbols from Question 5:

(a) *Leonardo DiCaprio and Judi Dench both starred in Skyfall*

$B \wedge G$

(b) *If Leonardo DiCaprio was Judi Dench's husband, then Judi Dench was married and Leonardo DiCaprio is not an actress*

$H \rightarrow (M \wedge \neg E)$

(c) *Skyfall did not star either Judi Dench or Leonardo DiCaprio*

$\neg(B \vee G)$

(d) *If Leonardo DiCaprio is a woman, then he isn't an actor and isn't married to Judi Dench*

$L \rightarrow \neg(N \vee H)$

9. The truth or falsity of a complex expression of propositional logic is a function of the truth/falsity of the propositional symbols it is consists of. Based on the answers you gave in Question 5, and your knowledge of the truth tables for negation, conjunction, disjunction, implication and equivalence, work out whether the following expressions are true or false:

(a) $E \wedge B$ False
F F F

(b) $(C \wedge \neg L) \rightarrow N$ True
T T T F T T

(c) $(H \rightarrow \neg M) \wedge (M \rightarrow \neg H)$ True
F T F T T T T T F

(d) $(K \wedge \neg L) \rightarrow N$ True
F F T F T T

10. Consider the expression $(H \rightarrow \neg M)$. Work out whether this expression is true or false. Can you explain why this expression is true/false, considering its English translation and the true/false values of the literals?

A conditional is always evaluated to be true if the consequent if false. Also, note that propositional logic is simply insensitive to the meanings assigned to variables; to express what is contradictory about the claim that JD was married to LdC, and was never married, we need a more powerful logic; predicate logic.

Truth tables of the basic operators

A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

We can also express the truth tables for binary operations in a different style, which makes the symmetries more immediately apparent.

$x \wedge y$		y = 0	y = 1
x	0	0	0
	1	0	1

$x \vee y$		y = 0	y = 1
x	0	0	1
	1	1	1

$x \to y$		y = 0	y = 1
x	0	1	1
	1	0	1

$x \leftrightarrow y$		y = 0	y = 1
x	0	1	0
	1	0	1

$x \oplus y$		y = 0	y = 1
x	0	0	1
	1	1	0