UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFORMATICS 1 — COMPUTATION & LOGIC

Tuesday 1st April 2014

00:00 to 00:00

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**
3. **CALCULATORS MAY NOT BE USED IN THIS EXAMINATION.**

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) Use truth tables to determine which of the following entailments are valid. [*4 marks*]

   i. $A \to (B \vee C) \models A \to C$

   ii. $A \to (B \wedge C) \models A \to C$

   iii. $(A \vee B) \to C \models A \to C$

   iv. $(A \wedge B) \to C \models A \to C$

   (b) This part concerns the 256 possible truth valuations of the following eight propositional letters A, B, C, D, E, F, G, H. For each of the following expressions, say how many of the 256 valuations satisfy the expression, and briefly explain your reasoning. For example, the expression D is satisfied by half of the valuations, that is 128 of the 256, since for each valuation that makes D true there is a matching valuation that make D false. [*6 marks*]

   i. $A \wedge B$

   ii. $(A \vee B) \wedge C$

   iii. $(A \to B) \to C$

   iv.

$$(A \to B) \wedge (B \to A) \wedge (C \to D) \wedge (D \to E) \wedge (E \to F) \wedge (F \to G) \wedge (G \to H)$$

   v.

$$\begin{aligned} &(A \to B) \wedge (B \to A) \wedge (C \to D) \wedge (D \to C) \\ &\qquad \wedge (E \to F) \wedge (F \to G) \wedge (G \to H) \end{aligned}$$

   vi.

$$(H \to A) \wedge (A \to B \wedge C) \wedge (B \vee C \to D) \wedge (A \to E) \wedge (E \to F) \wedge (F \to G) \wedge (G \to H)$$

2. You are given the following inference rules: (Γ, Δ vary over finite sets of expressions; A, B vary over expressions):

$$\frac{}{\Gamma, A \vdash \Delta, A}\ (I)$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R)$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ (\wedge R)$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \to B \vdash \Delta}\ (\to L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta}\ (\to R)$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ (\neg R)$$

(Where A and B are propositional expressions, Γ, Δ are sets of expressions, and Γ, A refers to $\Gamma \cup \{A\}$.)

(a) Explain what it means to claim that these rules are: [*4 marks*]

i. **sound**,

ii. and **complete**.

(b) Use these rules to show that [*10 marks*]

$$(A \to B) \to A \vdash A$$

(c) Use these rules to build an attempted proof of

$$P \to (Q \vee R), (Q \wedge R) \to S \vdash P \to S$$

[*7 marks*]

(d) Explain how you can derive a counter-example from your attempted proof. [*2 marks*]

(e) Is this a complete set of rules? Briefly justify your answer. [*2 marks*]

3. It is claimed that

$$P \to (Q \vee R), Q \to \neg S, S \vee R, R \to Q, (Q \wedge R) \to S \vdash P \to S$$

This question concerns the resolution of this claim.

(a) Express each of the assumptions in clausal form. *[6 marks]*

i. $P \to (Q \vee R)$

ii. $Q \to \neg S$

iii. $S \vee R$

iv. $R \to Q$

v. $(Q \wedge R) \to S$

(b) Explain how you would use resolution to determine whether the claim is correct. *[4 marks]*

(c) Use resolution to determine whether the claim is correct. (Show your working.) *[5 marks]*

4. Consider the following clauses:

- $\{\neg A, \neg B, \neg C, \neg D\}$
- $\{\neg B, \neg C, D\}$
- $\{\neg A, \neg B, C\}$
- $\{\neg A, B, \neg C\}$

(a) Use the table overleaf to record the search for a satisfying valuation for these clauses, using the one watched literal algorithm.

Start with the empty valuation, watching the first literal in each clause. At each step of the search use one column of the table provided to record in the first four squares the truth values assigned to the atoms, A, B, C, D and use one of remaining rows for each clause, to record the new position of the watched literals at the conclusion of each step. When you need to find a new literal to watch always choose the first one available (reading each expression left-to-right). Place an $\times$ in the watched literal square when no suitable literal is available. *[6 marks]*

(b) What is the invariant for the one watched literal algorithm? *[2 marks]*

(c) When must the search backtrack? *[2 marks]*

(d) Use table below to record the search for a satisfying valuation for these clauses, using the one watched literal algorithm.

A											
B											
C											
D											

5. An ATM requires the user to enter a four-digit PIN.

When the user inserts her card, the machine requests the PIN. The user enters a string of four decimal digits, one digit at a time.

If the user enters the correct four digits of the password, the ATM displays a welcome string, the user can then request a balance or withdraw £100. Their card is then returned.

When the user enters an incorrect string of four digits, the ATM displays a screen that informs the user that an incorrect password was entered.

If the user enters the incorrect password three times, the account is locked, and a message is displayed.

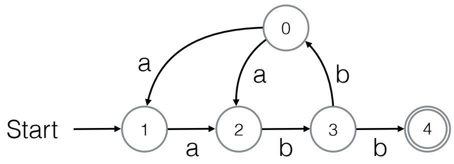
Here, assume that the correct password is 7442.

(a) Design a transducer-style finite state machine to model this. Describe the machine and explain clearly what the input and output alphabets mean, and what the set of states and transition function are. (You do not need to give a diagram including all the states and transitions, but your answer should make it clear what states there are and how the transition relation is constructed. It may be helpful to sketch parts of the FSM.) *[15 marks]*

Your design should not let the user know whether the pin is correct until four digits have been entered. The user's card should be returned before any cash is dispensed.

(b) Give the trace of the machine corresponding to an interaction with a user who fails to enter the correct PIN on the first attempt, but then succeeds and makes a withdrawal. *[5 marks]*

6. (a) Which of the following strings are accepted by the NFA in the diagram? (The start state is indicated by an arrow and the accepting state by a double border.)



i. abb
ii. abbabbabbaaabb
iii. abbabbaabbabbabb
iv. abbabaabbabbabb [*3 marks*]

(b) Write a regular expression for the language accepted by this NFA. [*3 marks*]

(c) Draw a DFA that accepts the same language. Label the states of your DFA to make clear their relationship to the states of the original NFA. [*10 marks*]

(d) For each of the following regular expressions, draw a non-deterministic finite state machine that accepts the language described by the regular expression.

i. x^*y
ii. $(x^*|y)$
iii. $(x^*y)^*$ [*9 marks*]