Informatics 1 - Computation & Logic: Tutorial 9 Solutions

Revision

Week 11: 25 - 29 November 2013

Truth tables

1. Construct a truth table for the following expression of propositional logic, and use these to decide whether the expressions are contingent, tautologous or inconsistent:

$\neg(A \wedge B) \wedge (B \vee (C \rightarrow \neg A))$

A	B	C	$\neg A$	$A \wedge B$	$\neg(A \wedge B)$	$C \rightarrow \neg A$	$B \vee (C \rightarrow \neg A)$	expr
T	T	T	F	T	F	F	T	F
T	T	F	F	T	F	T	T	F
T	F	T	F	F	T	F	F	F
F	T	T	T	F	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	F	T	T	T	T
T	F	F	F	F	T	T	T	T
F	F	F	T	F	T	T	T	T

This expression is <u>CONTINGENT</u>/TAUTOLOGOUS/INCONSISTENT

Sequent Calculus

2. Assume the following proof rules from Tutorial Exercise 4, known respectively as '*immediate*', '$\wedge$ *introduction*', '$\rightarrow$ *introduction*', '$\wedge$ *elimination*' and '$\rightarrow$ *elimination*':

$$\frac{\mathcal{A}, X \vdash X}{} \qquad \frac{\mathcal{A} \vdash X \wedge Y}{\begin{array}{c}\mathcal{A} \vdash X \\ \mathcal{A} \vdash Y\end{array}} \qquad \frac{\mathcal{A} \vdash X \rightarrow Y}{\mathcal{A}, X \vdash Y}$$

$$\frac{\mathcal{A}, X \wedge Y \vdash Z}{\mathcal{A}, X, Y \vdash Z} \qquad \frac{\mathcal{A}, X \rightarrow Z \vdash Z}{\mathcal{A} \vdash X}$$

Prove that the following argument is valid using this method:

$[\mathsf{A}, (\mathsf{A} \wedge \mathsf{B}) \rightarrow \mathsf{C}] \vdash \mathsf{B} \rightarrow \mathsf{C}$

Applying $\rightarrow$ *intro* we obtain:
$[\mathsf{A}, (\mathsf{A} \wedge \mathsf{B}) \rightarrow \mathsf{C}, \mathsf{B}] \vdash \mathsf{C}$
Applying $\rightarrow$ *elim* we obtain:
$[\mathsf{A}, \mathsf{B}] \vdash \mathsf{A} \wedge \mathsf{B}$
Applying $\wedge$*intro* we obtain two branches:

- $[\mathsf{A}, \mathsf{B}] \vdash \mathsf{A}$
  That can be proved immediatly
- $[\mathsf{A}, \mathsf{B}] \vdash \mathsf{B}$
  That can be proved immediatly

3. Here are some more proof rules, called respectively '$\vee$*introduction left*', '$\vee$*introduction right*', and '$\vee$*elimination':*

$$\frac{\mathcal{A} \vdash X \vee Y}{\mathcal{A} \vdash X} \qquad \frac{\mathcal{A} \vdash X \vee Y}{\mathcal{A} \vdash Y} \qquad \frac{\mathcal{A},\, X \vee Y \vdash Z}{\begin{array}{c}\mathcal{A}, X \vdash Z \\ \mathcal{A}, Y \vdash Z\end{array}}$$

Prove that the following argument is valid:

$[(\mathsf{A} \vee \mathsf{B}) \rightarrow \mathsf{C},\ \mathsf{C} \rightarrow \mathsf{A}] \vdash \mathsf{B} \rightarrow \mathsf{C}$

Applying $\rightarrow$ *intro* we obtain:
$[(\mathsf{A} \vee \mathsf{B}) \rightarrow \mathsf{C},\ \mathsf{C} \rightarrow \mathsf{A},\ \mathsf{B}] \vdash \mathsf{C}$
Applying $\rightarrow$ *elim* we obtain:
$[\mathsf{C} \rightarrow \mathsf{A},\ \mathsf{B}] \vdash \mathsf{A} \vee \mathsf{B}$
Applying $\vee introduction\, left$ we obtain a first branch to prove. If it can be proved, we have finished.

- $[\mathsf{C} \rightarrow \mathsf{A},\ \mathsf{B}] \vdash \mathsf{A}$
  We can try to apply $\rightarrow$ *elim* and obtain:
  $[\mathsf{B}] \vdash \mathsf{C}$
  That cannot be proved. We have to backtrack and try the second branch.

Applying $\vee introduction\, right$ we obtain the second branch:

- $[\mathsf{C} \rightarrow \mathsf{A},\ \mathsf{B}] \vdash \mathsf{B}$
  That can be proved immediatly

4. Using resolution, prove whether the following argument is valid:
   $(P \wedge Q) \rightarrow R, \neg R \vdash P \rightarrow \neg Q$

   (a) Convert into conjunctions:

   $((P \wedge Q) \rightarrow R) \wedge \neg R \wedge \neg\,(P \rightarrow \neg Q)$

   (b) Convert to conjunctive normal form:

   $((P \wedge Q) \rightarrow R) \wedge \neg R \wedge \neg\,(P \rightarrow \neg Q)$
   $(\neg(P \wedge Q) \vee R) \wedge \neg R \wedge \neg\,(\neg P \vee \neg Q)$
   $(\neg P \vee \neg Q \vee R) \wedge \neg R \wedge P \wedge Q$

   (c) Convert into clausal form:

   $[[\neg P, \neg Q, R]\,, [\neg R]\,, [P]\,, [Q]]$

   (d) Apply the resolution rule:

   $[[\underline{\neg P}, \neg Q, R]\,, [\neg R]\,, [\underline{P}]\,, [Q]]$
   $\left[[\underline{\neg Q}, R]\,, [\neg R]\,, [\underline{Q}]\right]$
   $[[\underline{R}]\,, [\underline{\neg R}]]$
   $[]$
   This means that the argument is valid.

5. Using resolution, prove whether the following argument is valid:

$P \to Q, P \to R \vdash P \to (Q \wedge R)$

(a) Convert into conjunctions:

$(P \to Q) \wedge (P \to R) \wedge \neg (P \to (Q \wedge R))$

(b) Convert to conjunctive normal form:

$(P \to Q) \wedge (P \to R) \wedge \neg (P \to (Q \wedge R))$
$(\neg P \vee Q) \wedge (\neg P \vee R) \wedge \neg (\neg P \vee (Q \wedge R))$
$(\neg P \vee Q) \wedge (\neg P \vee R) \wedge P \wedge \neg(Q \wedge R)$
$(\neg P \vee Q) \wedge (\neg P \vee R) \wedge P \wedge (\neg Q \vee \neg R)$

(c) Convert into clausal form:

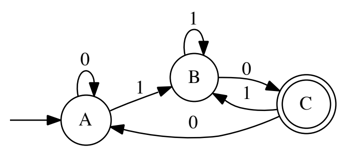
$[[\neg P, Q], [\neg P, R], [P], [\neg Q, \neg R]]$

(d) Apply the resolution rule:

$[[\neg P, \underline{Q}], [\neg P, R], [P], [\underline{\neg Q}, \neg R]]$
$[[\neg P, \underline{\neg R}], [\neg P, \underline{R}], [P]]$
$[[\underline{\neg P}], [\underline{P}]]$
$[]$
This means that the argument is not valid.

Finite State Machines

6. Consider the following finite state acceptor over the alphabet $\{0, 1\}$:



(a) Draw the transition table for this machine:

	0	**1**
A	A	B
B	C	B
C	A	B

(b) For each of the following strings, say whether they are *accepted* or *rejected* by this machine and indicate in which state the machine ends. Note that ϵ denotes the empty string.

STRING	ACCEPT/REJECT	FINAL STATE
110	A	C
1010	A	C
101	R	B
0011010	A	C
10011	R	B
110110	A	C
0110	A	C
00	R	A
00111	R	B
ϵ	R	A

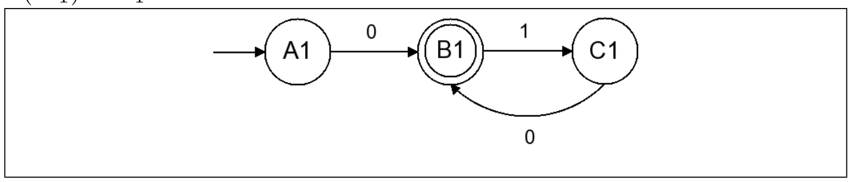
(c) Give the language for the above FSM:

$0^*11^*0(11^*0)^*(00^*11^*0(11^*0)^*)^*$
(any string ending in 10)

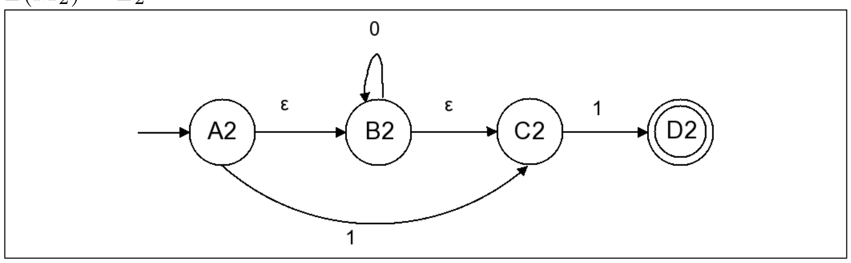
7. Given the following languages $L_1(0(10)^*)$, $L_2((1|0^*)1)$ and $L_3((11)^*0)$,

(a) Draw deterministic or non-deterministic machines M_i for each of the languages L_i, such that:

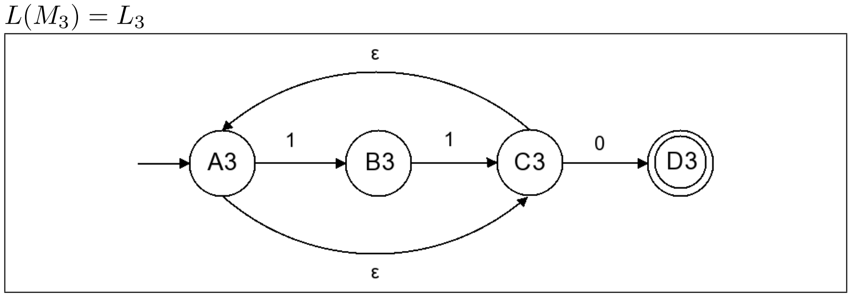
i. $L(M_1) = L_1$



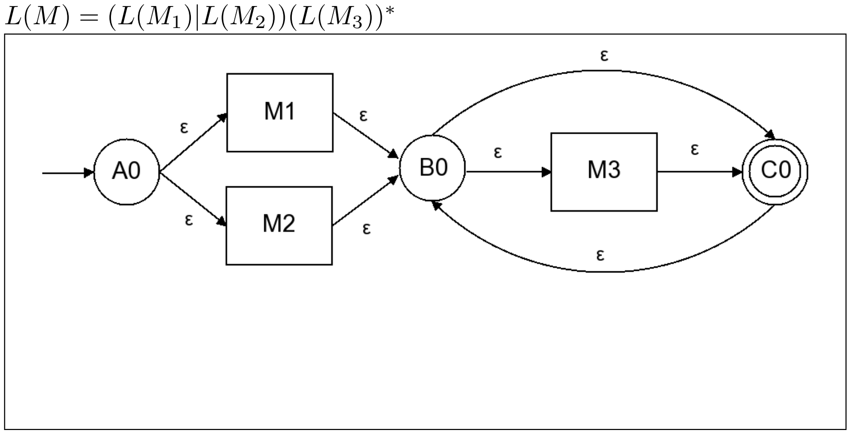
ii. $L(M_2) = L_2$



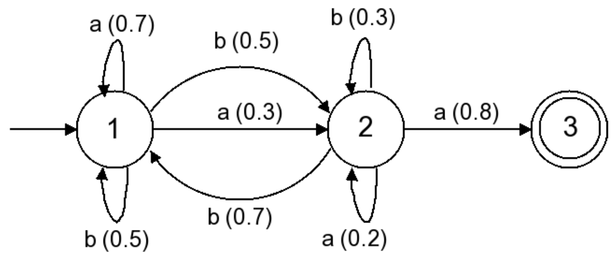
iii. $L(M_3) = L_3$



(b) Compose the machines together to create a machine, M, such that $L(M) = (L(M_1)|L(M_2))(L(M_3))^*$



8. Consider the following Probabilistic Finite State Machine that accepts strings over the alphabet `{a, b}`.



(a) Identify all possible traces that would accept string `abaa`.

There are 4 possible traces: `1a1b1a2a3`, `1a1b2a2a3`, `1a2b2a2a3` and `1a2b1a2a3`.

(b) Calculate the probability of each trace identified.

The probability of `1a1b1a2a3` is: $0.7 * 0.5 * 0.3 * 0.8 = 0.084$
The probability of `1a1b2a2a3` is: $0.7 * 0.5 * 0.2 * 0.8 = 0.056$
The probability of `1a2b2a2a3` is: $0.3 * 0.3 * 0.2 * 0.8 = 0.0144$
The probability of `1a2b1a2a3` is: $0.3 * 0.7 * 0.3 * 0.8 = 0.0504$

(c) Finally, calculate the probability that the string `abaa` would be accepted by this FSM.

It is the sum of the probabilities per trace, thus:
$0.084 + 0.056 + 0.0144 + 0.0504 = 0.2048$

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