Informatics 1 - Computation & Logic: Tutorial 7 Solutions

Computation: Regular Expressions

Week 9: 11-15 November 2013

Regular expressions

1. Which language is defined by the following regular expression?

$$(a|ab)(c|bc)$$

$\{ac, abc, abbc\}$

2. Write FOUR other regular expressions which define the same language.

$ac|abc|abbc$, $(a|ab|abb)c$, $ac|(ab(bc|c))$, $a(\epsilon|b|bb)c$
Other solutions are possible.

3. Which languages are defined by the following regular expressions?

   (a) $(a|b)^*$

   any string over $\{a, b\}$

   (b) $a|b^*$

   ϵ, a, b, bb, bbb, $bbbb$, $\ldots$

(c) $(ab)^*$

ϵ, ab, $abab$, $ababab$, $abababab$, ...

(d) ab^*

a, ab, abb, $abbb$, $abbbb$, ...

(e) $((ba^*b)|a^*)^*$

any string over $\{a, b\}$ which contains an even number of b's

4. Write regular expressions for the following languages:

(a) the set of strings over $\{a, b\}$ which contain no more than two a's

$b^* \mid b^*ab^* \mid b^*ab^*ab^*$ [others possible]

(b) the set of strings over $\{a, b\}$ which both start and end with a

$a \mid a(a\|b)^*a$ [others possible]

(c) the set of binary numbers which are multiples of four

$1(0\|1)^*00$ [others possible]

5. Verify if the following regular expressions are equivalent, using the following algebraic laws:

$$
\begin{aligned}
&L(\emptyset|R) = L(R) = L(R|\emptyset) && (1)\\
&L(R|R) = L(R) && (2)\\
&L(R|S) = L(R) \cup L(S) = L(S|R) && (3)\\
&L((R|S)|T) = L(R|(S|T)) && (4)\\
&L(\epsilon R) = L(R) = L(R\epsilon) && (5)\\
&L(\emptyset R) = L(\emptyset)L(R) = L(\emptyset) = L(R\emptyset) && (6)\\
&L((RS)T)) = L(R(ST)) && (7)\\
&L(R(S|T)) = L(R)L(S|T) = L(RS|RT) && (8)\\
&L((R|S)T) = L(R|S)L(T) = (L(R) \cup L(S))L(T) = L(RT|ST) && (9)\\
&L(\emptyset^*) = L(\emptyset)^* = \{\epsilon\} = L(\epsilon) && (10)\\
&L(RR^*) = L(R)L(R^*) = L(R^*R) && (11)\\
&L(RR^*|\epsilon) = L(R^*) && (12)\\
&L((R|S)^*) = L((R^*S^*)^*) && (13)\\
&L((RS)^*R) = L(R(SR)^*) && (14)
\end{aligned}
$$

(a) $((aa^* \mid \epsilon)\, c) \mid ((b \mid b)\, c)$ equivalent to $(a^* \mid b)\, c$

YES: $(aa^* \mid \epsilon)$ is equivalent to a^*, $(b \mid b)$ is equivalent to b, so we obtain:
$a^*c \mid bc$, that is equivalent to $(a^* \mid b)\, c$

(b) $\left(a\,(ba)^*\, b \mid (ab)^*\right)$ equivalent to (ab)

NO: $a\,(ba)^*\, b$ is equivalent to $(ab)\,(ab)^*$
$(ab)^*$ becomes $(ab)\,(ab)^* \mid \epsilon$, yielding
$(ab)\,(ab)^* \mid (ab)\,(ab)^* \mid \epsilon$
that can be transformed into: $(ab)\,(ab)^* \mid \epsilon$
that is equivalent to: $(ab)^*$, and not (ab)

(c) $\left((\epsilon\,(a \mid \emptyset))^*\, b^*\right)^*$ equivalent to $(a \mid b)^*$

YES: $(\epsilon\,(a \mid \emptyset))$ is equivalent to $(a \mid \emptyset)$, that in turn is equivalent to a, yielding
$(a^*b^*)^*$ that is equivalent to $(a \mid b)^*$

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