Informatics 1 - Computation & Logic: Tutorial 2

Propositional Logic: Truth Tables

Week 4: 7-11 October 2013

Please attempt the entire worksheet in advance of the tutorial, and bring with you all work, including (if a computer is involved) printouts of code and test results. Tutorials cannot function properly unless you do the work in advance.

You may work with others, but you must understand the work; you can't phone a friend during the exam.

Assessment is formative, meaning that marks from coursework do not contribute to the final mark. But coursework is not optional. If you do not do the coursework you are unlikely to pass the exams.

Attendance at tutorials is **obligatory**; please let your tutor know if you cannot attend.

1. In words, describe when an expression in propositional logic is:

   (a) Contingent:

   (b) Tautologous:

(c) Inconsistent:

2. Construct truth tables for the following expressions of propositional logic, and use these to decide whether the expressions are contingent, tautologous or inconsistent:

   (a) $(A \to B) \lor (\neg A \lor \neg B)$

   Draw the truth table here:

   This expression is CONTINGENT/TAUTOLOGOUS/INCONSISTENT

   (b) $\neg(A \land \neg B) \leftrightarrow \neg(\neg A \lor B)$

   Draw the truth table here:

   This expression is CONTINGENT/TAUTOLOGOUS/INCONSISTENT

(c) $A \to (B \land (A \lor B))$

Draw the truth table here:

This expression is CONTINGENT/TAUTOLOGOUS/INCONSISTENT

(d) $(\neg A \land B) \lor C \leftrightarrow ((A \lor \neg B) \to C)$

Draw the truth table here:

This expression is CONTINGENT/TAUTOLOGOUS/INCONSISTENT

3. (a) How many rows will a truth table for an expression in propositional logic with n atomic propositions have? Why?

   (b) In general, is this a limitation? Why?

4. An *argument* of propositional logic is of the form

$$\phi_1, \ldots, \phi_n \vdash \psi$$

where ϕ_i, ψ are all expressions of propositional logic. The ϕ_i expressions are the *premises* of the argument and ψ is the *conclusion*. An argument is *valid* if and only if there is no possible assignment of truth values to atomic propositional symbols such that the premises are all true and the conclusion false.

Using a truth table, determine whether the following arguments are valid or invalid:

   (a) $(A \wedge B) \rightarrow A,\ B \vee \neg A \ \vdash\ A \vee B$

This expression is VALID/INVALID

(b) $\neg A \vee (B \to C),\ B \wedge C,\ C \to A\ \vdash\ A$

This expression is VALID/INVALID

This tutorial exercise sheet was written by Mark McConville, revised by Paolo Besana, Thomas French and Areti Manataki. Send comments to `A.Manataki@ed.ac.uk`

Summary of useful symbols

Capital	Lowercase	Name
A	α	alpha
B	β	beta
Γ	γ	gamma
Δ	δ	delta
E	ϵ	epsilon
Z	ζ	zeta
H	η	eta
Θ	θ	theta
I	ι	iota
K	κ	kappa
Λ	λ	lambda
M	μ	mu
N	ν	nu
Ξ	ξ	xi
Π	π	pi
P	ρ	rho
Σ	σ	sigma
T	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
X	χ	chi
Ψ	ψ	psi
Ω	ω	omega

Symbol	Meaning	Example
$\neg$	not	$\neg A$
$\wedge$	and	$A \wedge B$
$\vee$	or	$A \vee B$
$\rightarrow$	implies	$A \rightarrow B$
$\leftrightarrow$	equivalent	$A \leftrightarrow B$
$\vdash$	can be proved	$[\beta_1, ..., \beta_n] \vdash \alpha$