Informatics 1 - Computation & Logic: Tutorial 1 Solutions

Propositional Logic: An Introduction

Week 3: 30 September - 4 October 2013

1. Here are some propositional symbols, together with the English sentences they stand for:

A	*Sam Mendes directed Skyfall*	TRUE
B	*Leonardo DiCaprio starred in Skyfall*	FALSE
C	*Leonardo DiCaprio starred in Django Unchained*	TRUE
D	*Quentin Tarantino is a film director*	TRUE
E	*Leonardo DiCaprio is an actress*	FALSE
F	*Judi Dench is an actress*	TRUE
G	*Judi Dench acted in Skyfall*	TRUE
H	*Leonardo DiCaprio was married to Judi Dench*	FALSE
J	*Django Unchained was released in 2012*	TRUE
K	*Skyfall is set in 16th century Scotland*	FALSE
L	*Leonardo DiCaprio is a woman*	FALSE
M	*Judi Dench used to be married*	TRUE
N	*Leonardo DiCaprio is an actor*	TRUE

   Every expression of propositional logic is either **true** or **false**, and no expression can be both true and false. Based on the relationship between propositional symbols and English sentences above, your own general knowledge, and if need be the Internet Movie Database, decide whether each propositional symbol is true or false.

2. Assume the propositional symbols in question 1. Assume also that: (a) the symbol $\neg$ represents the negation operator 'not'; (b) the symbol $\wedge$ represents the conjunction connective 'and'; (c) the symbol $\vee$ represents the disjunction connective 'or'; (d) the symbol $\rightarrow$ represents the implication connective; and (e) the symbol $\leftrightarrow$ represents the equivalence connective.

   Some of the following are well-formed expressions of propositional logic and the others are symbol soup. Decide which is which.

(a) $A \ \wedge \ \neg C$ WFF

(b) $\neg (F \rightarrow D)$ WFF

(c) $\leftrightarrow (N \ \neg \ B)$ SOUP

(d) $(G \vee \neg L) \leftrightarrow \neg \neg E$ WFF

(e) $A \vee \neg (C \rightarrow H)$ WFF

(f) $\vee (K \rightarrow \neg \neg B)$ SOUP

(g) $F \vee D \wedge$ SOUP

(h) $H \wedge \neg (A \leftrightarrow \neg C)$ WFF

3. Translate the following expressions of propositional logic into reasonably natural English, assuming the key in question 1:

   (a) $E \wedge B$
   Leonardo DiCaprio is an actress who starred in Skyfall

   (b) $J \vee \neg K$
   Django Unchained was released in 2012, or Skyfall was not set in 16th century Scotland

   (c) $E \rightarrow L$
   If Leonardo DiCaprio is an actress then Leonardo DiCaprio is a woman

   (d) $(C \ \wedge \ \neg \ L) \rightarrow N$
   If Leonardo DiCaprio starred in Django Unchained and Leonardo DiCaprio isn't a woman, then he is an actor

4. Translate the following English sentences into propositional logic, using the appropriate propositional symbols from question 1):

   (a) *Leonardo DiCaprio and Judi Dench both starred in Skyfall*
   $B \wedge G$

   (b) *If Leonardo DiCaprio was Judi Dench's husband, then Judi Dench was married and Leonardo DiCaprio is not an actress*
   $H \rightarrow (M \ \wedge \neg E)$

(c) *Skyfall did not star either Judi Dench or Leonardo DiCaprio*
$\neg(B \vee G)$, or, $\neg B \wedge \neg G$

(d) *If Leonardo DiCaprio is a woman, then he isn't an actor and wasn't married to Judi Dench*
$L \rightarrow (\neg N \wedge \neg H)$

5. The truth or falsity of a complex expression of propositional logic is a function of the truth/falsity of the propositional symbols it is made out of. Based on the answers you gave in question 1, and your knowledge of the truth tables for negation, conjunction, disjunction and implication, work out whether the following expressions are true or false:

(a) $E \wedge B$ FALSE

(b) $(C \wedge \neg L) \rightarrow N$ TRUE

(c) $(H \rightarrow \neg M) \wedge (M \rightarrow \neg H)$ TRUE

(d) $(K \wedge \neg L) \rightarrow N$ TRUE

6. Consider the expression $(H \rightarrow \neg M)$. Work out whether this expression is true or false. Can you explain why this expression is true/false, considering its English translation and the true/false values of the literals?

Based on the answers given in question 1, the expression is true.
Its English translation is: If Leonardo DiCaprio was Judi Dench's husband, then Judi Dench was not married.
This English expression might intuitively seem false. One could say "If Leonardo DiCaprio was indeed Judi Dench's husband, then this must mean that Judi Dench was married." But what if he wasn't? Then she could be or not be married, we simply don't know. Hence, in the case where Leonardo DiCaprio wasn't married to Judi Dench, we would accept any of the two cases for Judi Dench (i.e. she was married or she wasn't married).
Remember that given our answers in Question 1, Leonardo DiCaprio wasn't actually married to Judi Dench. Hence, we can accept that Judi Dench was not married, even though in reality she was. In other words, we are accepting the given expression as true.

This difference between our initial, intuitive answer and the final answer can be somewhat confusing. This is also known as the paradox of material implication. This paradox lies in a mismatch between the interpretation of the validity of logical implication in natural language, and its formal interpretation in classical logic.

Another way to think about it is the following: The truth or falsity of the expression $(p \rightarrow q)$ depends not on any relationship between the meanings of the propositions but only on the truth-values of p and q. So since H is false, the expression given is true, no matter what the literals H and M mean.

One last point worth mentioning: In classical logic, a false statement implies anything. Therefore, the following expressions are true:

- *If you score at least 85% in this course, then you will get an A.*
- *If elephants can fly, then Andy Murray will be the Wimbledon champion this year.*
- *If elephants can fly, then Andy Murray will not be the Wimbledon champion this year.*
- *If Leonardo DiCaprio was Judi Dench's husband, then you will get an A.*

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