Informatics 1 - Computation and Logic: Tutorial 9 Solutions

Revision

Week 11: 26 - 30 November 2012

Truth tables

1. Construct a truth table for the following expression of propositional logic, and use these to decide whether the expressions are contingent, tautologous or inconsistent:

   $\neg(A \wedge B) \wedge (B \vee (C \rightarrow \neg A))$

A	B	C	$\neg A$	$A \wedge B$	$\neg(A \wedge B)$	$C \rightarrow \neg A$	$B \vee (C \rightarrow \neg A)$	expr
T	T	T	F	T	F	F	T	F
T	T	F	F	T	F	T	T	F
T	F	T	F	F	T	F	F	F
F	T	T	T	F	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	F	T	T	T	T
T	F	F	F	F	T	T	T	T
F	F	F	T	F	T	T	T	T

This expression is CONTINGENT/TAUTOLOGOUS/INCONSISTENT

Sequent Calculus

2. Assume the following proof rules from Tutorial Exercise 4, known respectively as '*immediate*', '∧ *introduction*', '→ *introduction*', '∧ *elimination*' and '→ *elimination*':

$$\frac{\mathcal{A}, X \vdash X}{} \qquad \frac{\mathcal{A} \vdash X \wedge Y}{\begin{array}{c}\mathcal{A} \vdash X \\ \mathcal{A} \vdash Y\end{array}} \qquad \frac{\mathcal{A} \vdash X \rightarrow Y}{\mathcal{A}, X \vdash Y}$$

$$\frac{\mathcal{A}, X \wedge Y \vdash Z}{\mathcal{A}, X, Y \vdash Z} \qquad \frac{\mathcal{A}, X \rightarrow Z \vdash Z}{\mathcal{A} \vdash X}$$

Prove that the following argument is valid using this method:

$[\mathsf{A}, \mathsf{A} \wedge \mathsf{B} \rightarrow \mathsf{C}] \vdash \mathsf{B} \rightarrow \mathsf{C}$

Applying → *intro* we obtain:
$[\mathsf{A}, \mathsf{A} \wedge \mathsf{B} \rightarrow \mathsf{C}, \mathsf{B}] \vdash \mathsf{C}$
Applying → *elim* we obtain:
$[\mathsf{A}, \mathsf{B}] \vdash \mathsf{A} \wedge \mathsf{B}$
Applying ∧*intro* we obtain two branches:

- $[\mathsf{A}, \mathsf{B}] \vdash \mathsf{A}$
  That can be proved immediatly
- $[\mathsf{A}, \mathsf{B}] \vdash \mathsf{B}$
  That can be proved immediatly

3. Here are some more proof rules, called respectively '$\vee$*introduction left*', '$\vee$*introduction right*', and '$\vee$*elimination':*

$$\frac{\mathcal{A} \vdash X \vee Y}{\mathcal{A} \vdash X} \qquad \frac{\mathcal{A} \vdash X \vee Y}{\mathcal{A} \vdash Y} \qquad \frac{\mathcal{A},\, X \vee Y \vdash Z}{\begin{array}{c}\mathcal{A}, X \vdash Z \\ \mathcal{A}, Y \vdash Z\end{array}}$$

Prove that the following argument is valid:

$[\mathsf{A} \vee \mathsf{B} \rightarrow \mathsf{C},\ \mathsf{C} \rightarrow \mathsf{A}] \vdash \mathsf{B} \rightarrow \mathsf{C}$

Applying $\rightarrow$ *intro* we obtain:
$[\mathsf{A} \vee \mathsf{B} \rightarrow \mathsf{C},\ \mathsf{C} \rightarrow \mathsf{A},\ \mathsf{B}] \vdash \mathsf{C}$
Applying $\rightarrow$ *elim* we obtain:
$[\mathsf{C} \rightarrow \mathsf{A},\ \mathsf{B}] \vdash \mathsf{A} \vee \mathsf{B}$
Applying $\vee introduction\, left$ we obtain a first branch to prove. If it can be proved, we have finished.

- $[\mathsf{C} \rightarrow \mathsf{A},\ \mathsf{B}] \vdash \mathsf{A}$
  We can try to apply $\rightarrow$ *elim* and obtain:
  $[\mathsf{B}] \vdash \mathsf{C}$
  That cannot be proved. We have to backtrack and try the second branch.

Applying $\vee introduction\, right$ we obtain the second branch:

- $[\mathsf{C} \rightarrow \mathsf{A},\ \mathsf{B}] \vdash \mathsf{B}$
  That can be proved immediatly

4. Using resolution, prove whether the following argument is valid:

   $(P \wedge Q) \rightarrow R, \neg R \vdash P \rightarrow \neg Q$

   (a) Convert into conjunctions:

   $((P \wedge Q) \rightarrow R) \wedge \neg R \wedge \neg (P \rightarrow \neg Q)$

   (b) Convert to conjunctive normal form:

   $((P \wedge Q) \rightarrow R) \wedge \neg R \wedge \neg (P \rightarrow \neg Q)$

   $(\neg(P \wedge Q) \vee R) \wedge \neg R \wedge \neg (\neg P \vee \neg Q)$

   $(\neg P \vee \neg Q \vee R) \wedge \neg R \wedge P \wedge Q$

   (c) Convert into clausal form:

   $[[\neg P, \neg Q, R], [\neg R], [P], [Q]]$

   (d) Apply the resolution rule:

   $[[\underline{\neg P}, \neg Q, R], [\neg R], [\underline{P}], [Q]]$

   $\left[\left[\underline{\neg Q}, R\right], [\neg R], \left[\underline{Q}\right]\right]$

   $[[\underline{R}], [\underline{\neg R}]]$

   $[]$

   This means that the argument is valid.

5. Using resolution, prove whether the following argument is valid:

$P \to Q, P \to R \vdash P \to (Q \wedge R)$

(a) Convert into conjunctions:

$(P \to Q) \wedge (P \to R) \wedge \neg (P \to (Q \wedge R))$

(b) Convert to conjunctive normal form:

$(P \to Q) \wedge (P \to R) \wedge \neg (P \to (Q \wedge R))$
$(\neg P \vee Q) \wedge (\neg P \vee R) \wedge \neg (\neg P \vee (Q \wedge R))$
$(\neg P \vee Q) \wedge (\neg P \vee R) \wedge P \wedge \neg (Q \wedge R)$
$(\neg P \vee Q) \wedge (\neg P \vee R) \wedge P \wedge (\neg Q \vee \neg R)$

(c) Convert into clausal form:

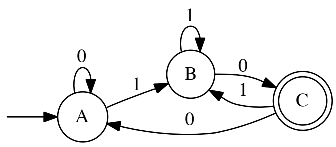
$[[\neg P, Q], [\neg P, R], [P], [\neg Q, \neg R]]$

(d) Apply the resolution rule:

$\left[[\neg P, \underline{Q}], [\neg P, R], [P], [\underline{\neg Q}, \neg R]\right]$
$[[\neg P, \underline{\neg R}], [\neg P, \underline{R}], [P]]$
$[[\underline{\neg P}], [\underline{P}]]$
$[]$
This means that the argument is not valid.

Finite State Machines

6. Consider the following finite state acceptor over the alphabet $\{0, 1\}$:



(a) Draw the transition table for this machine:

	0	**1**
A	A	B
B	C	B
C	A	B

(b) For each of the following strings, say whether they are *accepted* or *rejected* by this machine and indicate in which state the machine ends. Note that ϵ denotes the empty string.

STRING	ACCEPT/REJECT	FINAL STATE
110	A	C
1010	A	C
101	R	B
0011010	A	C
10011	R	B
110110	A	C
0110	A	C
00	R	A
00111	R	B
ϵ	R	A

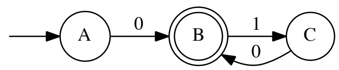
(c) Give the language for the above FSM:

0*11*0(11*0)*(00*11*0(11*0)*)*
(any string ending in 10)

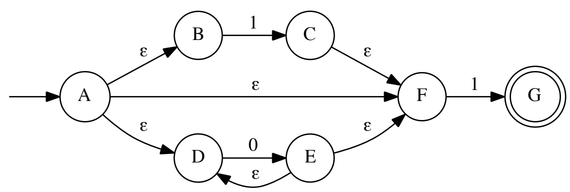
7. Given the following languages $L_1(0(10)^*)$, $L_2((1|0^*)1)$ and $L_3((11)^*0)$,

   (a) Draw machines M_i for each of the languages L_i, such that:

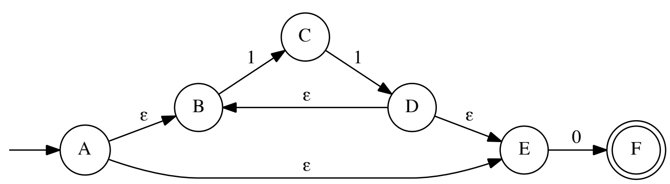
   i. $L(M_1) = L_1$



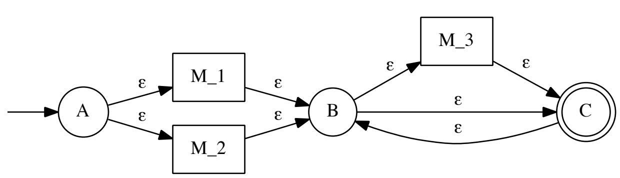
   ii. $L(M_2) = L_2$



   iii. $L(M_3) = L_3$



   (b) Compose the machines together to create a machine, M, such that $L(M) = (L(M_1)|L(M_2))(L(M_3))^*$



8. Using the subset procedure, convert the following non-deterministic machine, M_N, to a deterministic FSM, M_D, such that $L(M_N) = L(M_D)$. The machine M_N is defined by the tuple:

$$M_N = (\{A, B, C\}, \{a, b\}, A, \{C\}, \delta_N)$$

where $\delta_N = \{\langle A, a, A\rangle, \langle A, b, A\rangle, \langle A, b, B\rangle, \langle B, a, B\rangle, \langle B, b, B\rangle, \langle B, b, C\rangle\}$

(a) Give the complete set of states, Q_D, of M_D.

$Q_D = \{\emptyset, \{A\}, \{B\}, \{C\}, \{A, B\}, \{A, C\}, \{B, C\}, \{A, B, C\}\}$

(b) What is the start state, s_0, of M_D?

$s_0 = \{A\}$

(c) Give the set of transitions of M_D, δ_D:

$\delta_D = \{\langle\{A\}, a, \{A\}\rangle, \langle\{A\}, b, \{A, B\}\rangle, \langle\{A, B\}, a, \{A, B\}\rangle,$
$\langle\{A, B\}, b, \{A, B, C\}\rangle, \langle\{A, B, C\}, a, \{A, B\}\rangle, \langle\{A, B, C\}, b, \{A, B, C\}\rangle\}$

(d) What are the set of accepting states, F_D, of M_D?

$F_D = \{\{A, B, C\}\}$

(e) Finally, give the definition of M_D:

$M_D = (Q_D, \{a, b\}, \{A\}, \{\{A, B, C\}\}, \delta_D)$, where
$Q_D = \{\{A\}, \{A, B\}, \{A, B, C\}\}$, and
$\delta_D = \{\langle\{A\}, a, \{A\}\rangle, \langle\{A\}, b, \{A, B\}\rangle, \langle\{A, B\}, a, \{A, B\}\rangle,$
$\langle\{A, B\}, b, \{A, B, C\}\rangle, \langle\{A, B, C\}, a, \{A, B\}\rangle, \langle\{A, B, C\}, b, \{A, B, C\}\rangle\}$

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