Informatics 1 - Computation and Logic: Tutorial 8 Solutions

Computation: Regular Expressions

Week 10: 19 - 23 November 2012

Regular expressions

1. Which language is defined by the following regular expression?

$$(a|ab)(c|bc)$$

$\{ac, abc, abbc\}$

2. Write FOUR other regular expressions which define the same language.

$ac|abc|abbc$, $(a|ab|abb)c$, $ac|(ab(bc|c))$, $a(\epsilon|b|bb)c$
Other solutions are possible.

3. Which languages are defined by the following regular expressions?

   (a) $(a|b)^*$

   any string over $\{a, b\}$

   (b) $a|b^*$

   ϵ, a, b, bb, bbb, $bbbb$, ...

(c) $(ab)^*$

ϵ, ab, $abab$, $ababab$, $abababab$, ...

(d) ab^*

a, ab, abb, $abbb$, $abbbb$, ...

(e) $((ba^*b)|a^*)^*$

any string over $\{a, b\}$ which contains an even number of b's

4. Write regular expressions for the following languages:

   (a) the set of strings over $\{a, b\}$ which contain no more than two a's

$b^* \mid b^*ab^* \mid b^*ab^*ab^*$ [others possible]

   (b) the set of strings over $\{a, b\}$ which both start and end with a

$a \mid a(a\|b)^*a$ [others possible]

   (c) the set of binary numbers which are multiples of four

$1(0\|1)^*00$ [others possible]

5. Verify if the following regular expressions are equivalent:

(a) $((aa^* \mid \epsilon)\, c) \mid ((b \mid b)\, c)$ equivalent to $(a^* \mid b)\, c$

YES: $(aa^* \mid \epsilon)$ is equivalent to a^*, $(b \mid b)$ is equivalent to b, so we obtain:
$a^*c \mid bc$, that is equivalent to $(a^* \mid b)\, c$

(b) $\left(a\,(ba)^*\, b \mid (ab)^*\right)$ equivalent to (ab)

NO: $a\,(ba)^*\, b$ is equivalent to $(ab)\,(ab)^*$
$(ab)^*$ becomes $(ab)\,(ab)^* \mid \epsilon$, yielding
$(ab)\,(ab)^* \mid (ab)\,(ab)^* \mid \epsilon$
that can be transformed into: $(ab)\,(ab)^* \mid \epsilon$
that is equivalent to: $(ab)^*$, and not (ab)

(c) $\left((\epsilon\,(a \mid \emptyset))^*\, b^*\right)^*$ equivalent to $(a \mid b)^*$

YES: $(\epsilon\,(a \mid \emptyset))$ is equivalent to $(a \mid \emptyset)$, that in turn is equivalent to a, yielding
$(a^*b^*)^*$ that is equivalent to $(a \mid b)^*$

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