Informatics 1 - Computation and Logic: Tutorial 2 Solutions

Propositional Logic: Truth Tables

Week 4: 8 - 12 October 2012

1. In words, describe when an expression in propositional logic is:

   (a) Contingent:

   > When the expression is sometimes true and sometimes false.

   (b) Tautologous:

   > When the expression is always true regardless of the values of the propositions that it contains.

   (c) Inconsistent:

   > When the expression is always false regardless of the values of the propositions. Also known as a contradiction.

2. Construct truth tables for the following expressions of propositional logic, and use these to decide whether the expressions are contingent, tautologous or inconsistent:

   (a) $(A \to B) \vee (\neg A \vee \neg B)$

   Draw the truth table here:

A	B	$\neg A$	$\neg B$	$A \to B$	$\neg A \vee \neg B$	expr
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

This expression is CONTINGENT/<u>TAUTOLOGOUS</u>/INCONSISTENT

(b) $\neg(A \wedge \neg B) \leftrightarrow \neg(\neg A \vee B)$

Draw the truth table here:

A	B	$\neg A$	$\neg B$	$A \wedge \neg B$	$\neg(A \wedge \neg B)$	$\neg A \vee B$	$\neg(\neg A \vee B)$	expr
T	T	F	F	F	T	T	F	F
T	F	F	T	T	F	F	T	F
F	T	T	F	F	T	T	F	F
F	F	T	T	F	T	T	F	F

This expression is CONTINGENT/TAUTOLOGOUS/<u>INCONSISTENT</u>

(c) $A \to (B \wedge (A \vee B))$

Draw the truth table here:

A	B	$A \vee B$	$B \wedge (A \vee B)$	expr
T	T	T	T	T
T	F	T	F	F
F	T	T	T	T
F	F	F	F	T

This expression is <u>CONTINGENT</u>/TAUTOLOGOUS/INCONSISTENT

(d) $(\neg A \wedge B) \vee C \leftrightarrow ((A \vee \neg B) \to C)$

Draw the truth table here:

A	B	C	$\neg A$	$\neg B$	$(\neg A \wedge B)$	$(\neg A \wedge B) \vee C$	$(A \vee \neg B)$	$((A \vee \neg B) \rightarrow C)$	expr
T	T	T	F	F	F	T	T	T	T
T	T	F	F	F	F	F	T	F	T
T	F	T	F	T	F	T	T	T	T
F	T	T	T	F	T	T	F	T	T
F	T	F	T	F	T	T	F	T	T
F	F	T	T	T	F	T	T	T	T
T	F	F	F	T	F	F	T	F	T
F	F	F	T	T	F	F	T	F	T

This expression is CONTINGENT/<u>TAUTOLOGOUS</u>/INCONSISTENT

3. (a) How many rows will a truth table for an expression in propositional logic with n atomic propositions have? Why?

The table will have 2^n rows because we need a row for every possible assignment of true or false to an expression's atomic propositions.

(b) In general, is this a limitation? If so, why?

Yes, because we have an exponential number of rows, which for any practical purposes quickly becomes unmanageable or intractable to compute.

4. An *argument* of propositional logic is of the form

$$\phi_1, \ldots, \phi_n \vdash \psi$$

where ϕ_i, ψ are all expressions of propositional logic. The ϕ_i expressions are the *premises* of the argument and ψ is the *conclusion*. An argument is *valid* if and only if there is no possible assignment of truth values to atomic propositional symbols such that the premises are all true and the conclusion false.

Using a truth table, determine whether the following arguments are valid or invalid:

(a) $(A \wedge B) \rightarrow A,\ B \vee \neg A \ \vdash\ A \vee B$

A	B	$\neg A$	$A \wedge B$	$(A \wedge B) \rightarrow A$	$B \vee \neg A$	$A \vee B$
T	T	F	T	**T**	**T**	**T**
T	F	F	F	**T**	**F**	**T**
F	T	T	F	**T**	**T**	**T**
F	F	T	F	**T**	**T**	**F***

This expression is VALID/<u>INVALID</u>

(b) $\neg A \vee (B \rightarrow C),\ B \wedge C,\ C \rightarrow A \ \vdash\ A$

A	B	C	$\neg A$	$B \rightarrow C$	$\neg A \vee (B \rightarrow C)$	$B \wedge C$	$C \rightarrow A$
T	T	T	F	T	***T***	***T***	***T***
T	T	F	F	F	**F**	**F**	**T**
T	F	T	F	T	**T**	**F**	**T**
F	T	T	T	T	**T**	**T**	**F**
F	T	F	T	F	**T**	**F**	**T**
F	F	T	T	T	**T**	**F**	**F**
T	F	F	F	T	**T**	**F**	**T**
F	F	F	T	T	**T**	**F**	**T**

This expression is <u>VALID</u>/INVALID

This tutorial exercise sheet was written by Mark McConville, revised by Paolo Besana, and further extended by Thomas French. Send comments to `s.bijani@ed.ac.uk`

Summary of useful symbols

Capital	Lowercase	Name
A	α	alpha
B	β	beta
Γ	γ	gamma
Δ	δ	delta
E	ϵ	epsilon
Z	ζ	zeta
H	η	eta
Θ	θ	theta
I	ι	iota
K	κ	kappa
Λ	λ	lambda
M	μ	mu
N	ν	nu
Ξ	ξ	xi
Π	π	pi
P	ρ	rho
Σ	σ	sigma
T	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
X	χ	chi
Ψ	ψ	psi
Ω	ω	omega

Symbol	Meaning	Example
$\neg$	not	$\neg A$
$\wedge$	and	$A \wedge B$
$\vee$	or	$A \vee B$
$\rightarrow$	implies	$A \rightarrow B$
$\leftrightarrow$	equivalent	$A \leftrightarrow B$
$\vdash$	can be proved	$\beta_1, ..., \beta_n \vdash \alpha$