UNIVERSITY OF EDINBURGH
COLLEGE OF SCIENCE AND ENGINEERING
SCHOOL OF INFORMATICS

INFORMATICS 1 — COMPUTATION & LOGIC

November 6 2014

INSTRUCTIONS

This is a takehome paper designed to give you experience of exam-style questions. You should attempt all questions.

YOUR ANSWERS SHOULD BE
LABELLED WITH YOUR STUDENT NUMBER AND RETURNED TO
THE ITO
BY NOON ON FRIDAY 14th NOVEMBER
THEY WILL BE MARKED AND RETURNED FOR FEEDBACK

1. (a) Explain what it means for an entailment to be valid.

   (b) For each of the following entailments, use a truth table to determine whether it is valid.

      i. $A \models ((A \to B) \to B)$
      ii. $(\neg(A \to \neg B) \to \neg A) \models B$
      iii. $(A \to \neg B) \models (\neg C \to B) \to (A \to C)$

   (c) This part concerns the 256 possible truth valuations of the following eight propositional letters A, B, C, D, E, F, G, H. For each of the following expressions, say how many of the 256 valuations satisfy the expression, and briefly explain your reasoning. For example, the expression D is satisfied by half of the valuations, that is 128 of the 256, since for each valuation that makes D true there is a matching valuation that makes D false.

      i. $C \vee A \vee B$
      ii. $(E \wedge F) \vee C$
      iii. $(B \to D) \to A$
      iv. $B \to (D \to A)$
      v. $(A \to H) \vee B$
      vi. $(A \vee H) \to B$
      vii. $(A \vee H) \wedge B$
      viii.
      $$(A \to B) \wedge (B \to C) \wedge (D \to E) \wedge (F \to G) \wedge (G \to H)$$
      ix.
      $$\begin{aligned}(A \to B) \wedge (C \to D) \wedge (D \to A)\\ \wedge (E \to F) \wedge (F \to E) \wedge (G \to H) \wedge (H \to E)\end{aligned}$$
      x.
      $$(A \to (B \wedge C \wedge D)) \wedge ((B \vee C \vee D) \to E) \wedge (E \to (F \wedge G)) \wedge ((F \vee G) \to H)$$

2. You are given the following inference rules: (Γ, Δ vary over finite sets of expressions; A, B vary over expressions):

$$\frac{}{\Gamma, A, B \vdash \Delta, A}\ (I)$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R)$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ (\wedge R)$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \to B \vdash \Delta}\ (\to L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta}\ (\to R)$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ (\neg R)$$

(Where A and B are propositional expressions, Γ, Δ are sets of expressions, and Γ, A refers to $\Gamma \cup \{A\}$.)

(a) Use these rules to determine whether each of the following entailments is valid. In each case, either give a proof or use a failed proof attempt to produce a counter-example.

i.
$$P \to Q \wedge (R \to \neg Q) \vdash R \to \neg P$$

ii.
$$(P \to Q) \to R \vdash P \to (R \vee \neg Q)$$

(b) Show that the rule $(\neg L)$ is sound.

(c) Show that the rule $(\wedge R)$ has the following property:
A counterexample to either of its antecedents (above the line) is a counterexample to its conclusion.

3. It is claimed that the proposition U follows from the following three assumptions:

$$\neg(\neg T \wedge Q) \qquad (\neg U \rightarrow T) \wedge (\neg S \rightarrow \neg P) \qquad \neg U \rightarrow (T \rightarrow (\neg S \wedge P))$$

This question concerns the resolution of this claim.

(a) Express each of the assumptions in clausal form.

i. $\neg(\neg T \wedge Q)$

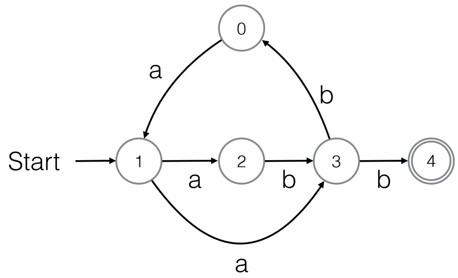
ii. $(\neg U \rightarrow T) \wedge (\neg S \rightarrow \neg P)$

iii. $\neg U \rightarrow (T \rightarrow (\neg S \wedge P))$

(b) Explain how you would use resolution to determine whether the claim is correct.

(c) Use resolution to determine whether the claim is correct. (Show your working.)

4. (a) Which of the following strings are accepted by the NFA in the diagram? (The start state is indicated by an arrow and the accepting state by a double border.)



i. abbaabb
ii. aabb
iii. abaaabab

(b) Draw a DFA that accepts the same language. Label the states of your DFA to make clear their relationship to the states of the original NFA.

(c) Write a regular expression for the language accepted by this NFA.

(d) For each of the following regular expressions, draw a non-deterministic finite state machine that accepts the language described by the regular expression.

i. $(x|xy)$
ii. x^*y
iii. $(x|y)(xy)^*$