Informatics 1

Lecture 9 Davis Putnam et al

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Clausal Form

Clausal form is a set of sets of literals

$\big\{$ {¬A,C}, {¬B,D}, {¬E,B}, {¬E,A}, {A,E}, {E,B},{¬B, ¬C, ¬D} $\big\}$

A (partial) truth assignment makes a clause true
iff it makes at least one of its literals true
(so it can never make the empty clause {} true)

A (partial) truth assignment makes a clausal form true
iff it makes all of its clauses true
(so the empty clausal form $\{\}$ is always true).

Clausal form is a set of sets of literals

$$\{\ \mathbf{X}_0,\ \mathbf{X}_1,\ \ldots\ ,\ \mathbf{X}_{n-1}\ \}$$

Resolution rule for clauses

$$\frac{\mathbf{X} \qquad\qquad \mathbf{Y}}{(\mathbf{X} \cup \mathbf{Y}) \setminus \{\ \neg A,\ A\ \}} \qquad \text{where } \neg A \in \mathbf{X},\ A \in \mathbf{Y}$$

$$A \longrightarrow B \longrightarrow C \longrightarrow D \longrightarrow E$$

$$\dfrac{\dfrac{\neg A \lor B \qquad \neg B \lor C}{\neg A \lor C} \qquad \dfrac{\neg C \lor D \qquad \neg D \lor E}{\neg C \lor E}}{\neg A \lor E}$$

$$\dfrac{\neg B \lor C \qquad \neg C \lor E}{\neg B \lor E} \qquad\qquad \dfrac{\neg B \lor C \qquad \neg C \lor E}{\neg B \lor E}$$

$$\dfrac{\neg B \lor C \qquad \neg C \lor D}{\neg B \lor D}$$

$$A \longrightarrow B \longrightarrow C \longrightarrow D \longrightarrow E$$

B C D

$\neg A \vee B$	$\neg A \vee C$	$\neg A \vee D$	$\neg A \vee E$
$\neg B \vee C$		$\neg B \vee D$	$\neg B \vee E$
$\neg C \vee D$			$\neg C \vee E$
$\neg D \vee E$			

Other orders for resolution will give the same results.

Davis Putnam

Take a collection $\mathcal{C}$ of clauses.

For each propositional letter, A

For each pair $(X, Y) \mid X \in \mathcal{C} \wedge Y \in \mathcal{C} \wedge \mathbf{A} \in X \wedge \neg\mathbf{A} \in Y$

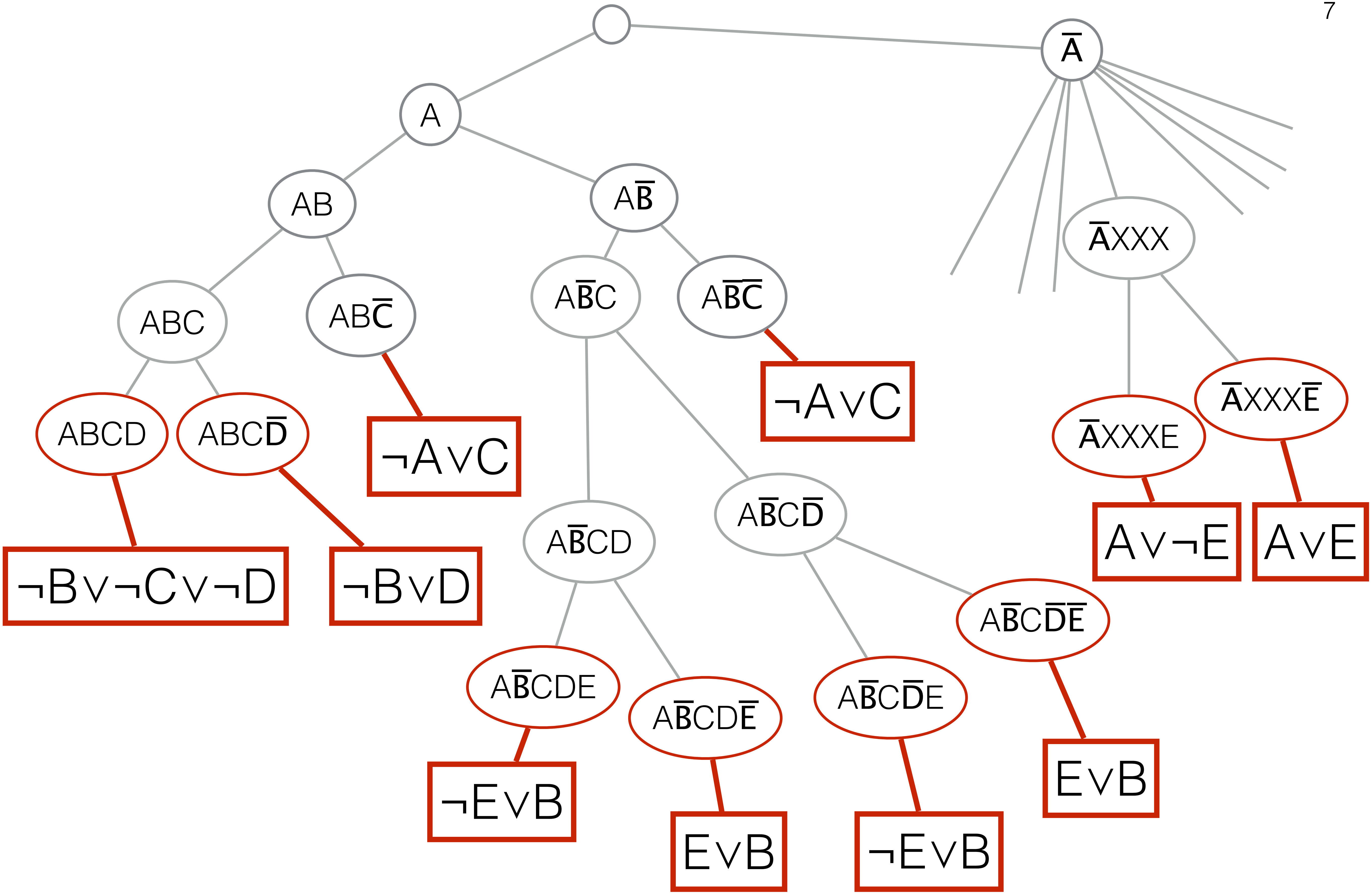
if $\mathcal{R}(X, Y, \mathbf{A}) = \{\}$ return UNSAT

if $\mathcal{R}(X, Y, \mathbf{A})$ is consistent $\mathcal{C} := \mathcal{C} \cup \{ \mathcal{R}(X, Y) \}$

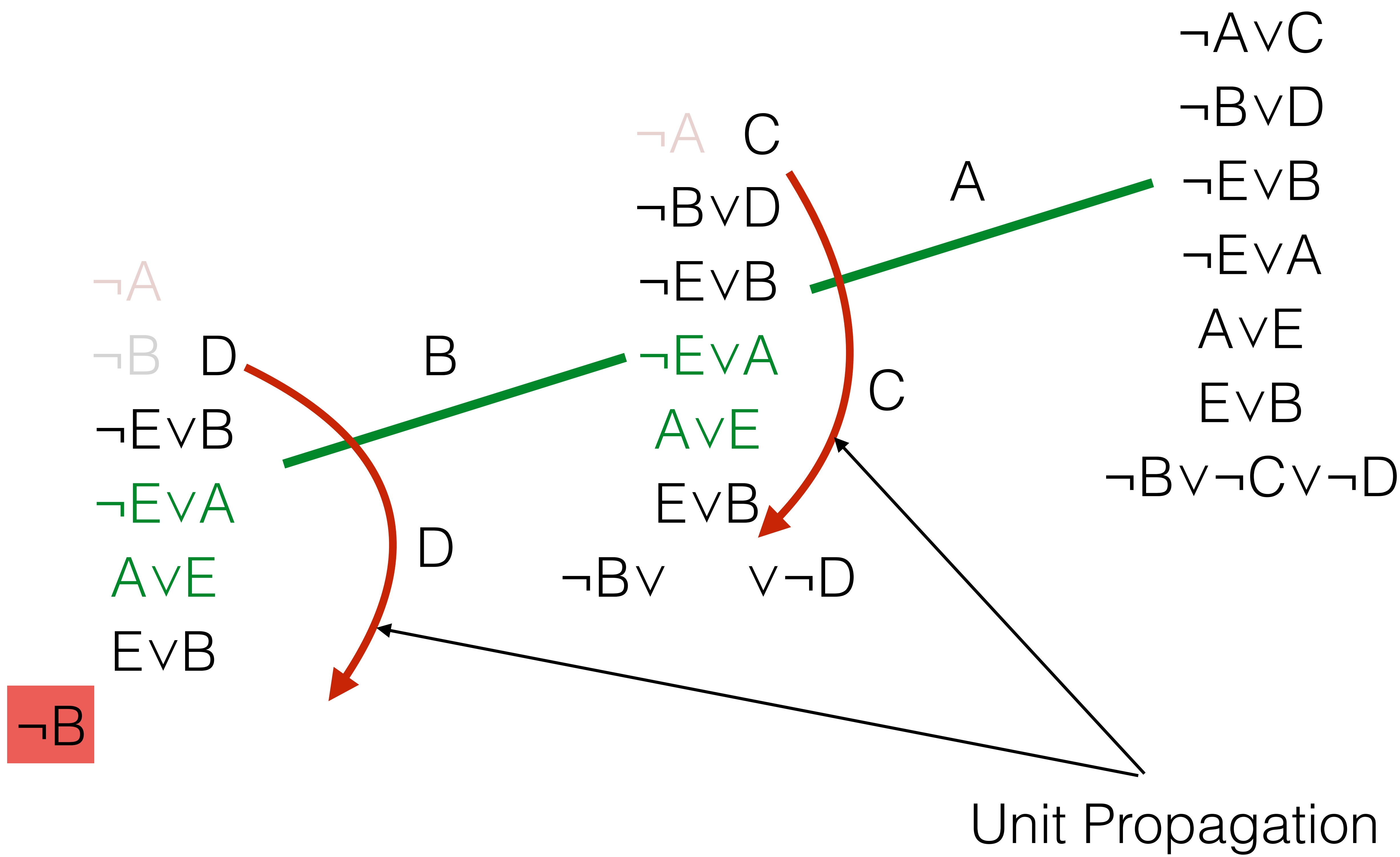
return SAT

Where $\mathcal{R}(X, Y, \mathbf{A}) = X \cup Y \setminus \{ \mathbf{A}, \neg\mathbf{A} \}$

Heuristic: start with variables that occur seldom.


$\bar{A}$
A
AB
$A\bar{B}$
ABC
$AB\bar{C}$
$A\bar{B}C$
$A\bar{B}\bar{C}$
ABCD
$ABC\bar{D}$
$\neg A \vee C$
$\neg A \vee C$
$\neg B \vee \neg C \vee \neg D$
$\neg B \vee D$
$A\bar{B}CD$
$A\bar{B}C\bar{D}$
$A\bar{B}CDE$
$A\bar{B}CD\bar{E}$
$A\bar{B}C\bar{D}E$
$A\bar{B}C\bar{D}\bar{E}$
$\neg E \vee B$
$E \vee B$
$\neg E \vee B$
$E \vee B$
$\bar{A}XXX$
$\bar{A}XXXE$
$\bar{A}XXX\bar{E}$
$A \vee \neg E$
$A \vee E$

Idea! Use the problem to simplify the search



Davis Putnam Logemann Loveland (DPLL)

```
function DPLL(Φ)
   if Φ is a consistent set of literals
      then return true;
   if Φ contains an empty clause
      then return false;
   for every unit clause l in Φ
     Φ ← unit-propagate(l, Φ);
   l ← choose-literal(Φ);
   return DPLL(Φ ∪ {l}) or DPLL(Φ ∪ {not(l)});
```

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Choose, at random, a clause with 5 literals

¬A	B	C	¬D	E	F	¬G	H

Watched Literals

For each clause, we watch two literals

¬A		C	¬D		F	¬G	H	¬I	J		L

Unless I negate one of your watched literals do nothing.
When I do negate one of your watched literals
Check your position
If you still have a free literal
– one that is not negated, or true –
watch that one.
Otherwise, shout “UNIT”,
as you have only one literal left unnegated
I will then make your literal true.