Informatics 1

Lecture 8 Searching for Satisfaction

Michael Fourman

$\neg A \lor C$

$\neg B \lor D$

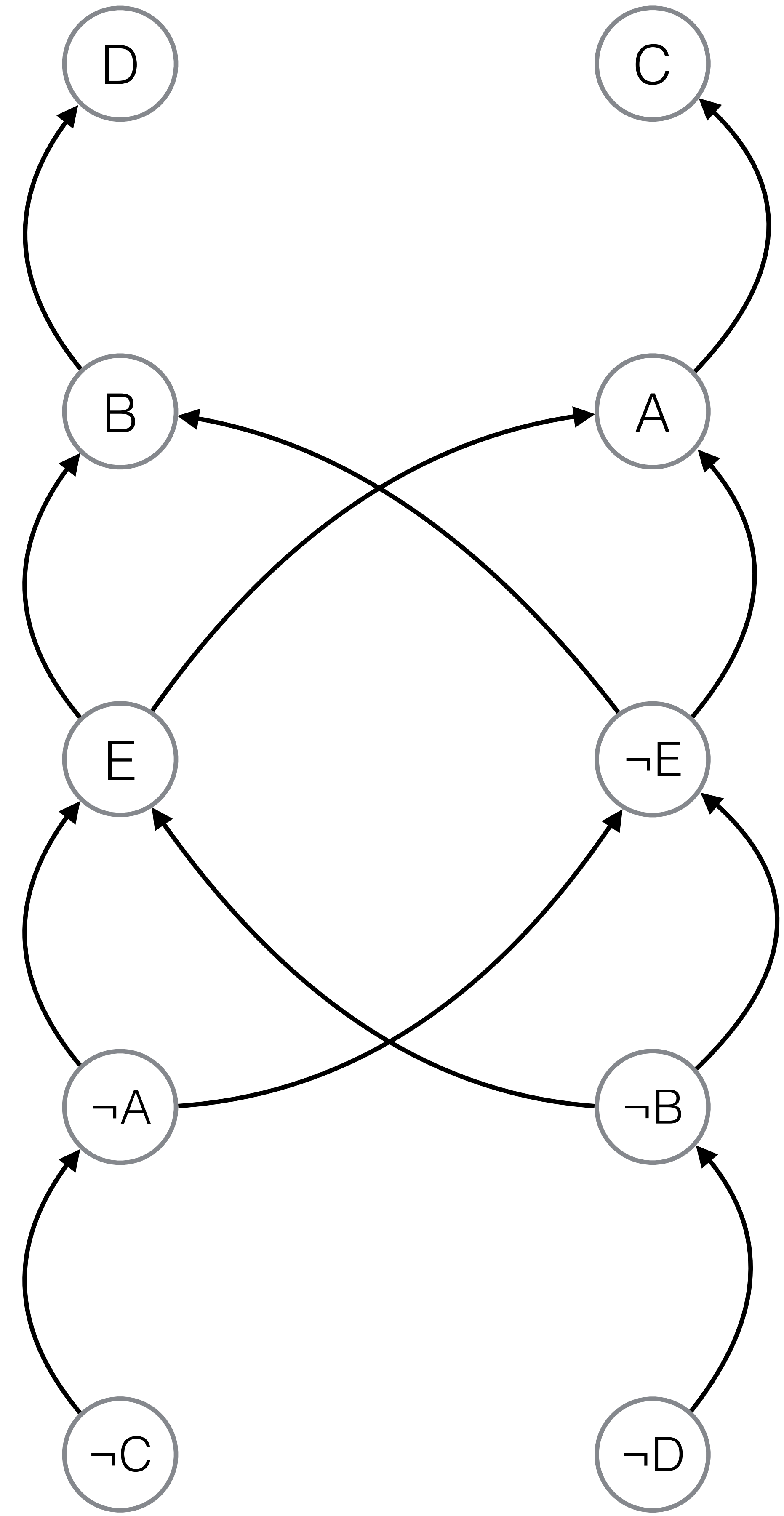
$\neg E \lor B$

$\neg E \lor A$

$A \lor E$

$E \lor B$

$\neg B \lor \neg C \lor \neg D$



$\neg B \lor \neg C \lor \neg D$

$\neg A \lor C$

$\neg B \lor D$

$\neg E \lor B$

$\neg E \lor A$

$A \lor E$

$E \lor B$

$B \to D$

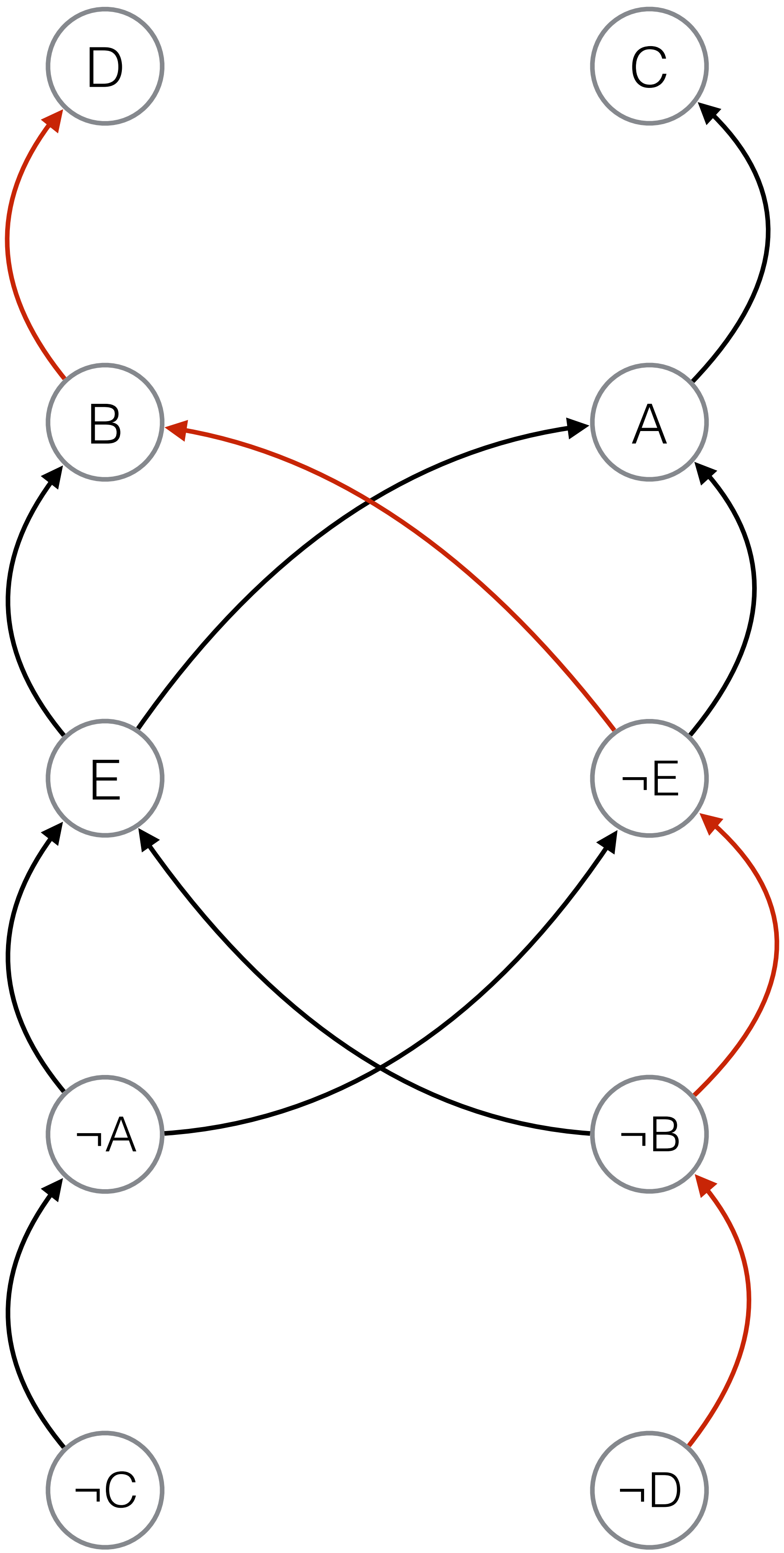
$\neg D \to \neg B$

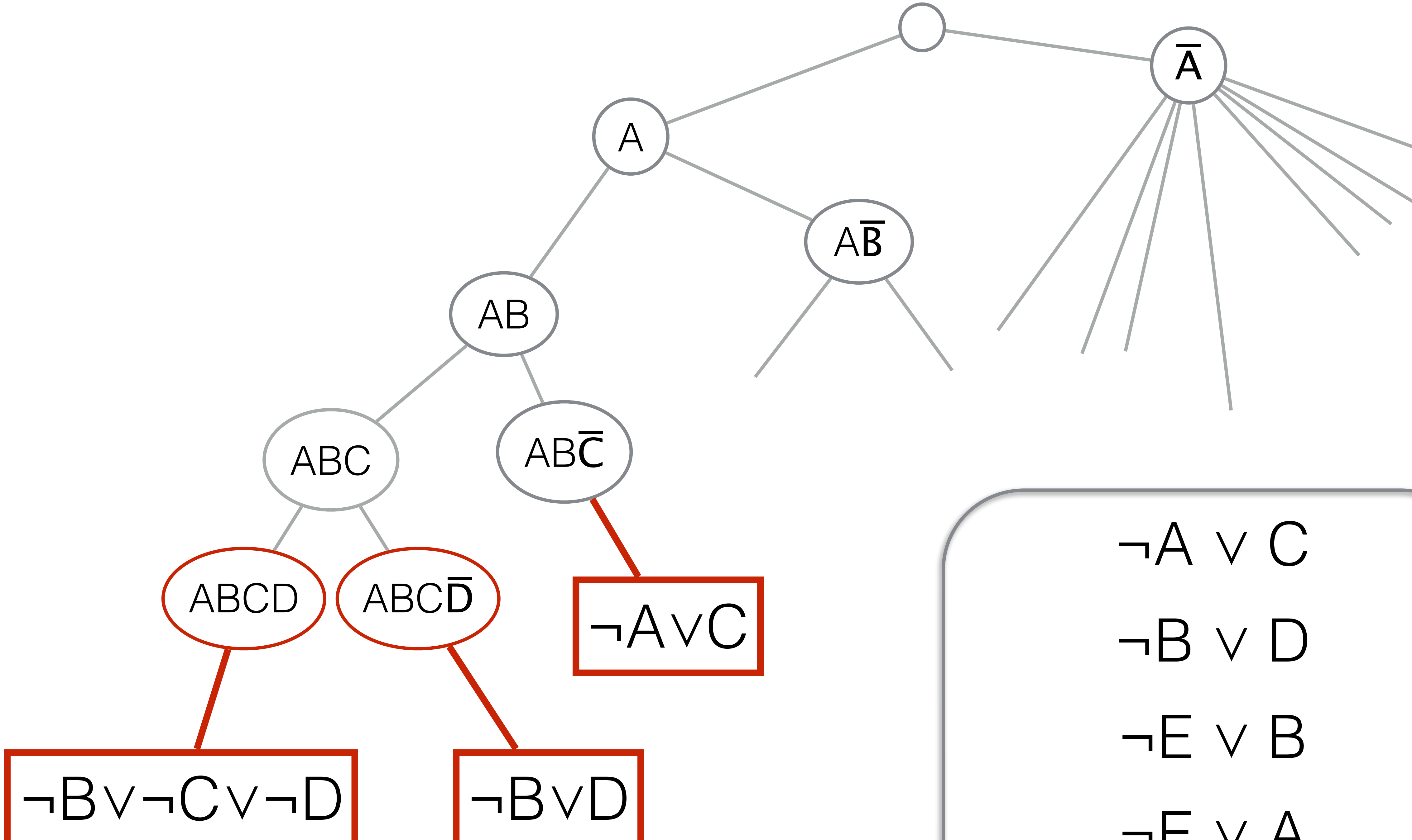
$\neg B \to \neg E$

$\neg E \to B$

$$\frac{\neg E \to B \qquad B \to D}{\neg E \to D}$$

$$\frac{E \lor B \qquad \neg B \lor D}{E \lor D}$$





¬A ∨ C

¬B ∨ D

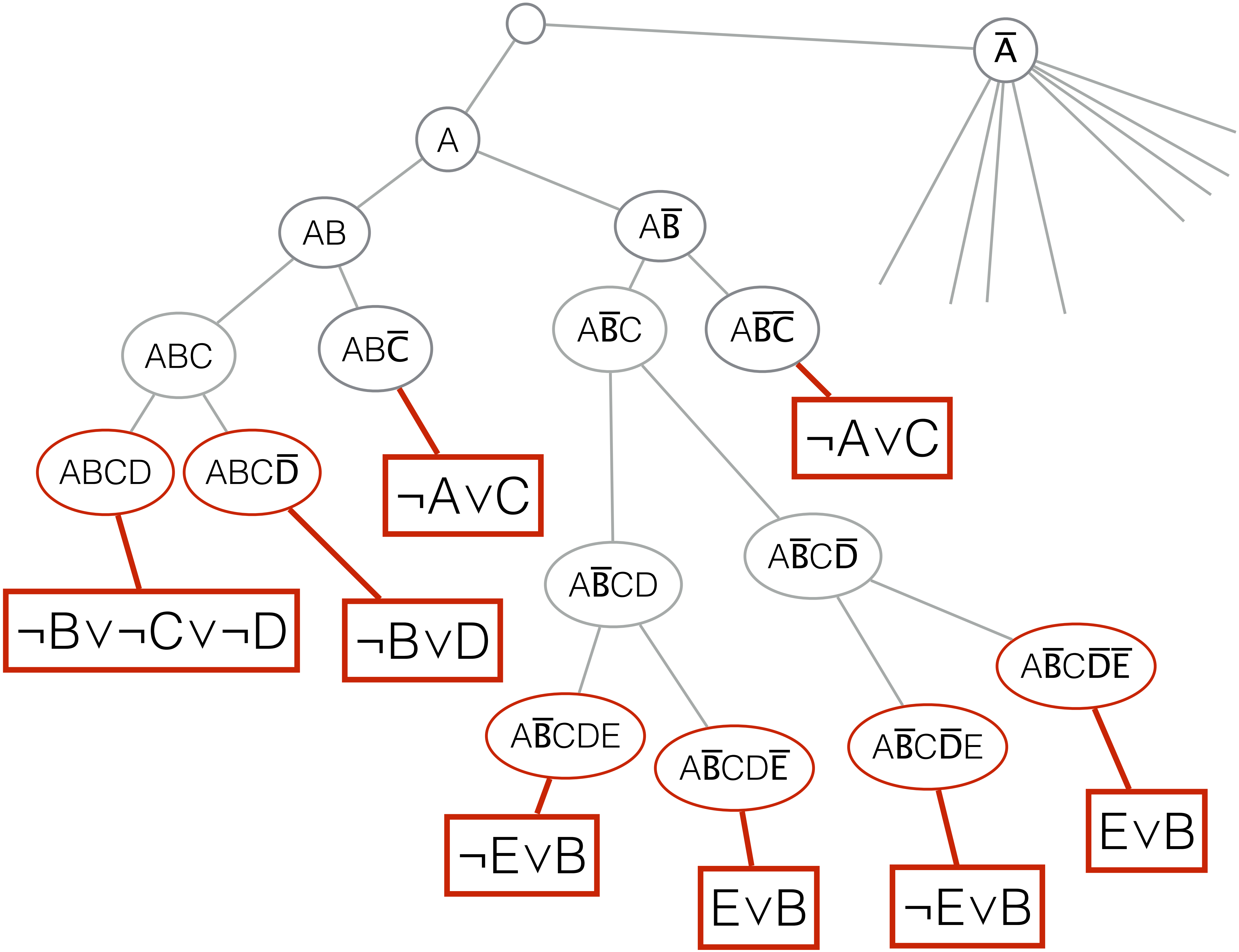
¬E ∨ B

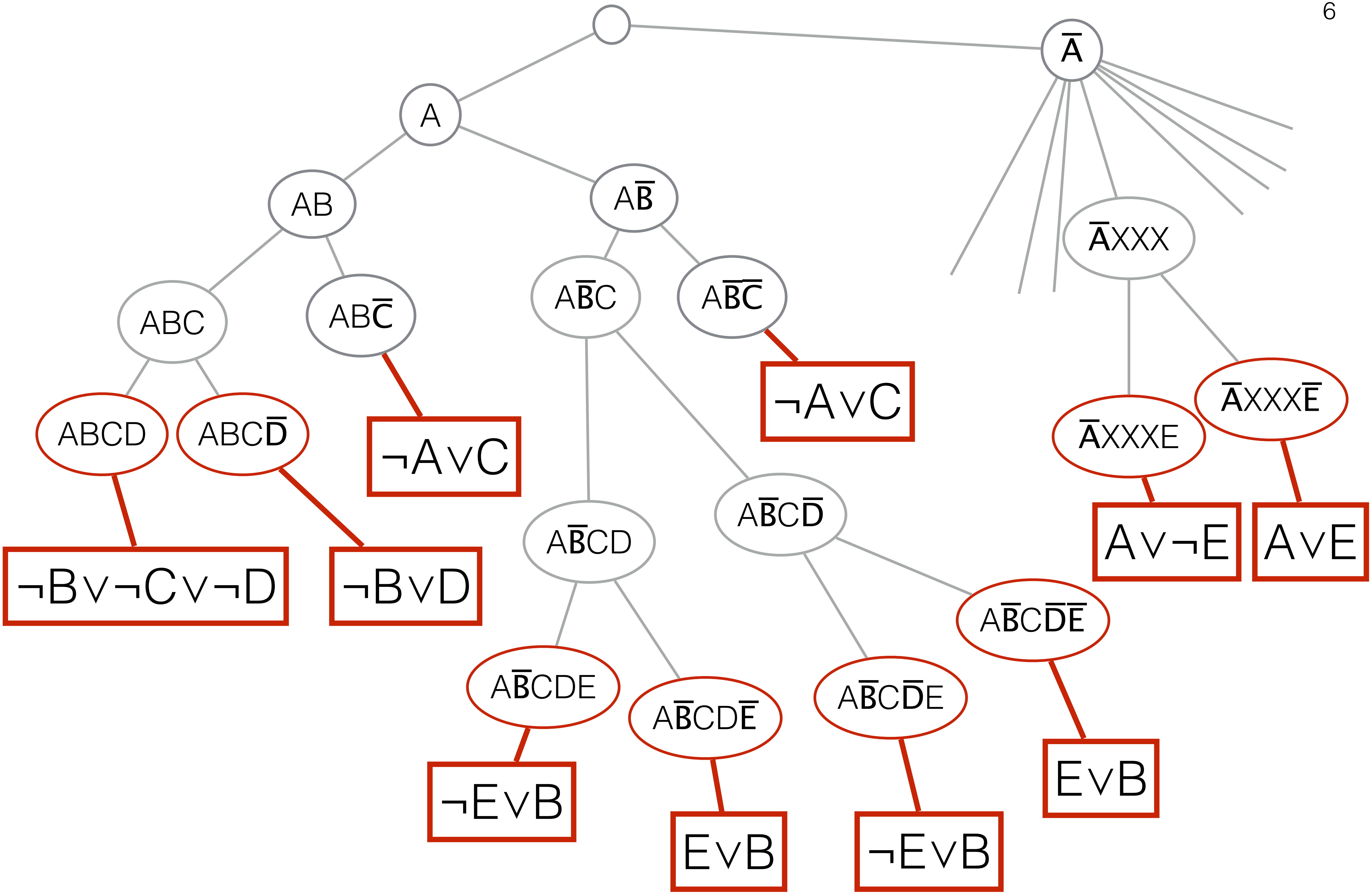
¬E ∨ A

A ∨ E

E ∨ B

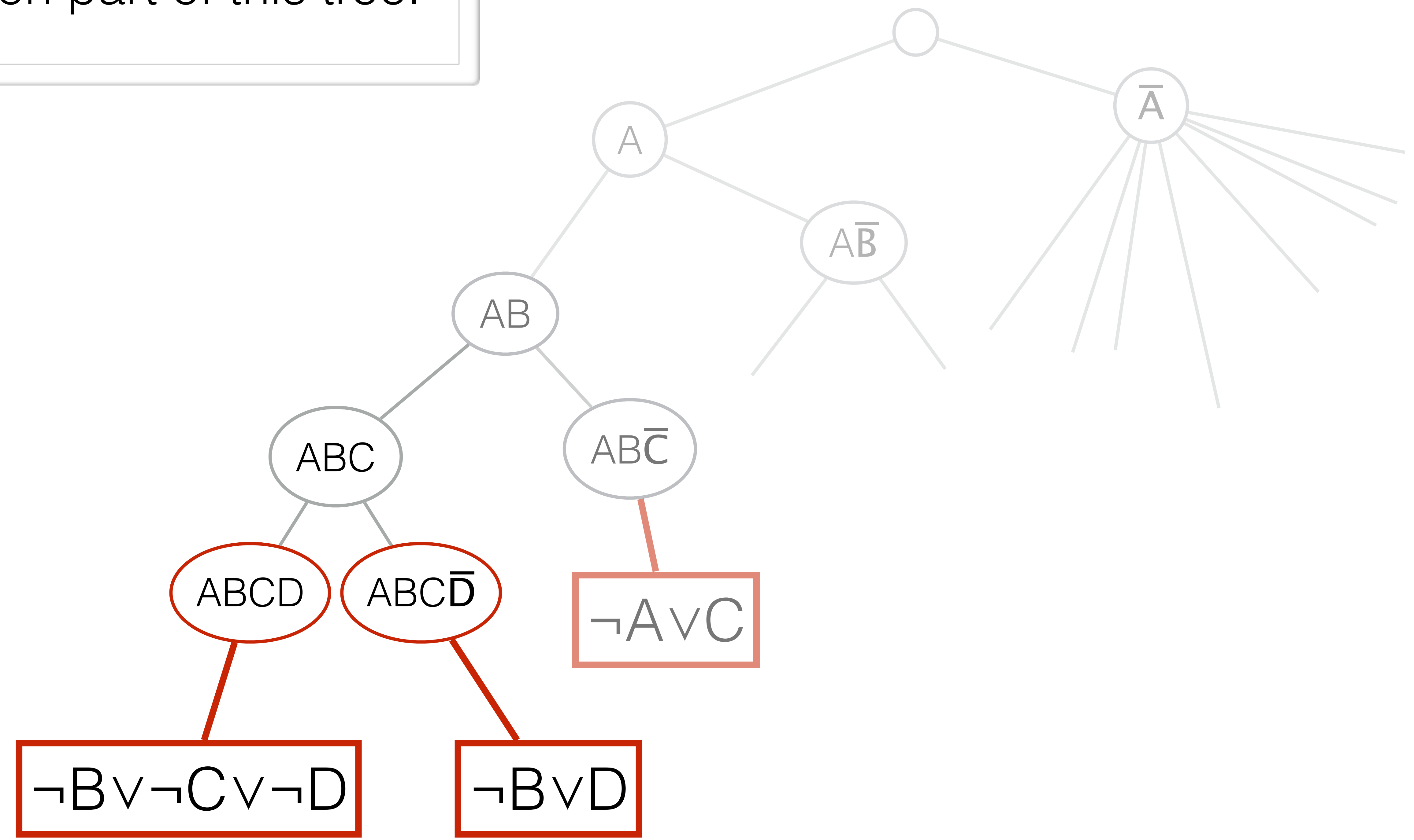
¬B ∨ ¬C ∨ ¬D

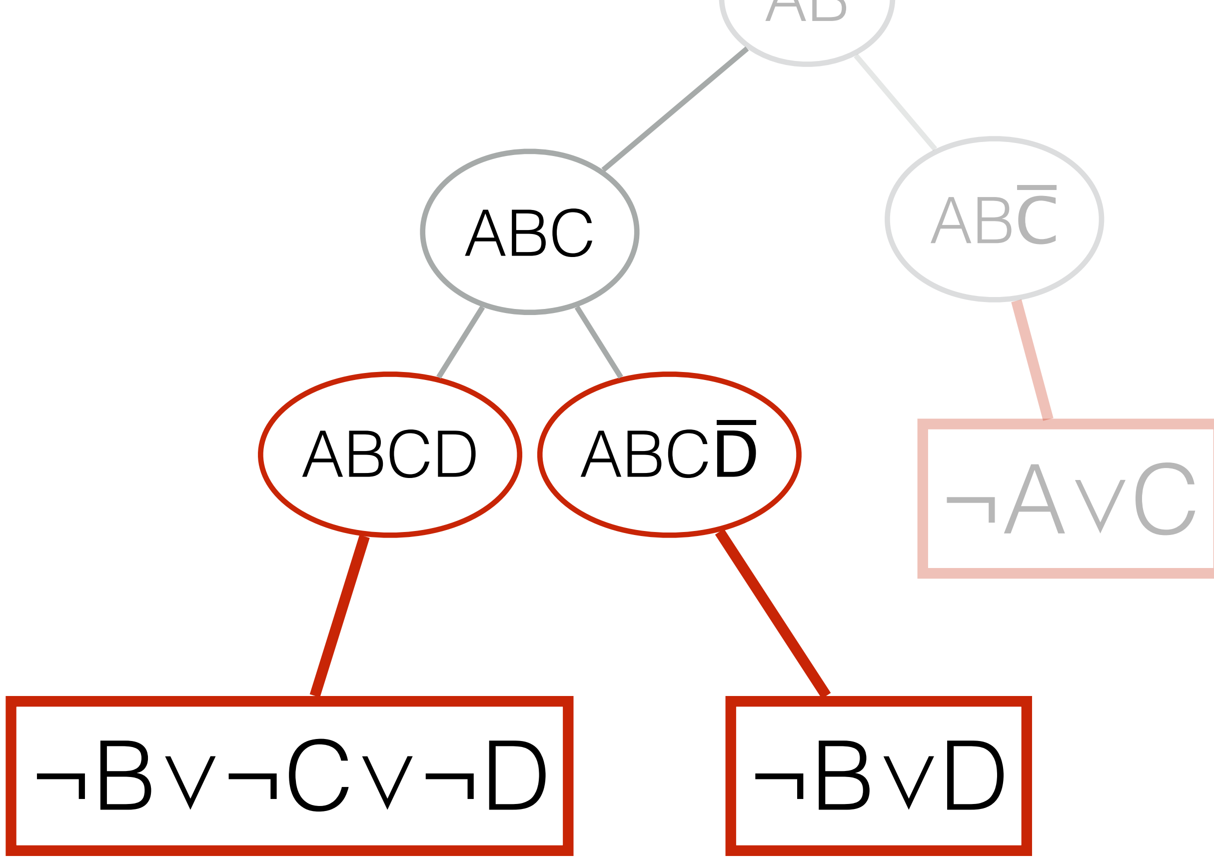

$\overline{A}$
A
AB
$A\overline{B}$
ABC
$AB\overline{C}$
$A\overline{B}C$
$A\overline{B}\overline{C}$
ABCD
$ABC\overline{D}$
¬A∨C
¬A∨C
¬B∨¬C∨¬D
¬B∨D
$A\overline{B}C\overline{D}$
$A\overline{B}CD$
$A\overline{B}C\overline{D}\overline{E}$
$A\overline{B}CDE$
$A\overline{B}CD\overline{E}$
$A\overline{B}C\overline{D}E$
¬E∨B
E∨B
¬E∨B
E∨B



$\bar{A}$
A
AB
$A\bar{B}$
ABC
$AB\bar{C}$
$A\bar{B}C$
$A\bar{B}\bar{C}$
¬A∨C
$\bar{A}XXX$
ABCD
$ABC\bar{D}$
¬A∨C
$\bar{A}XXX\bar{E}$
$\bar{A}XXXE$
$A\bar{B}C\bar{D}$
$A\bar{B}CD$
A∨¬E
A∨E
¬B∨¬C∨¬D
¬B∨D
$A\bar{B}C\bar{D}\bar{E}$
$A\bar{B}CDE$
$A\bar{B}CD\bar{E}$
$A\bar{B}C\bar{D}E$
E∨B
¬E∨B
E∨B
¬E∨B

focus on part of this tree.





Premises

¬B∨¬C∨**¬D**   ¬B∨**D**

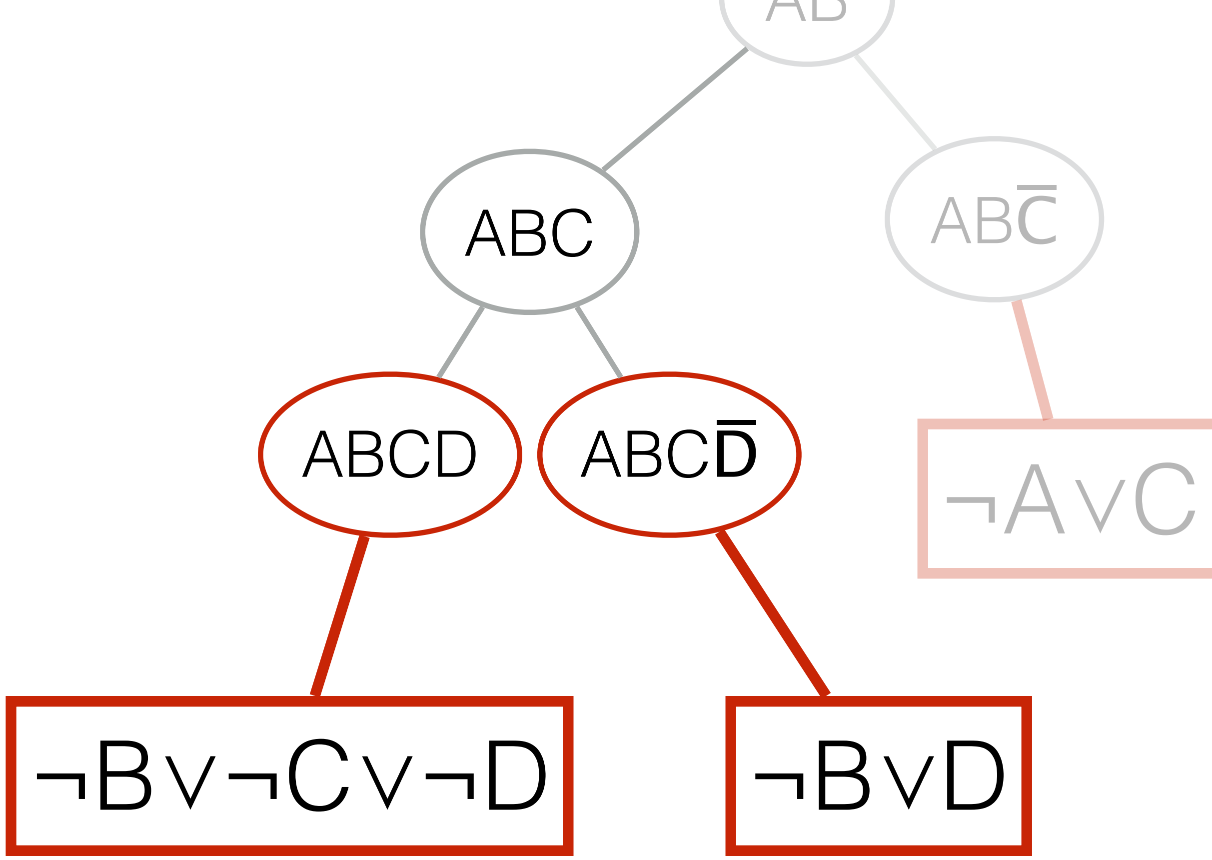
¬B∨**¬C**

Conclusion

A valid inference

Any assignment of truth values that makes all the premises true will make the conclusion true.

The conclusion follows from the premises



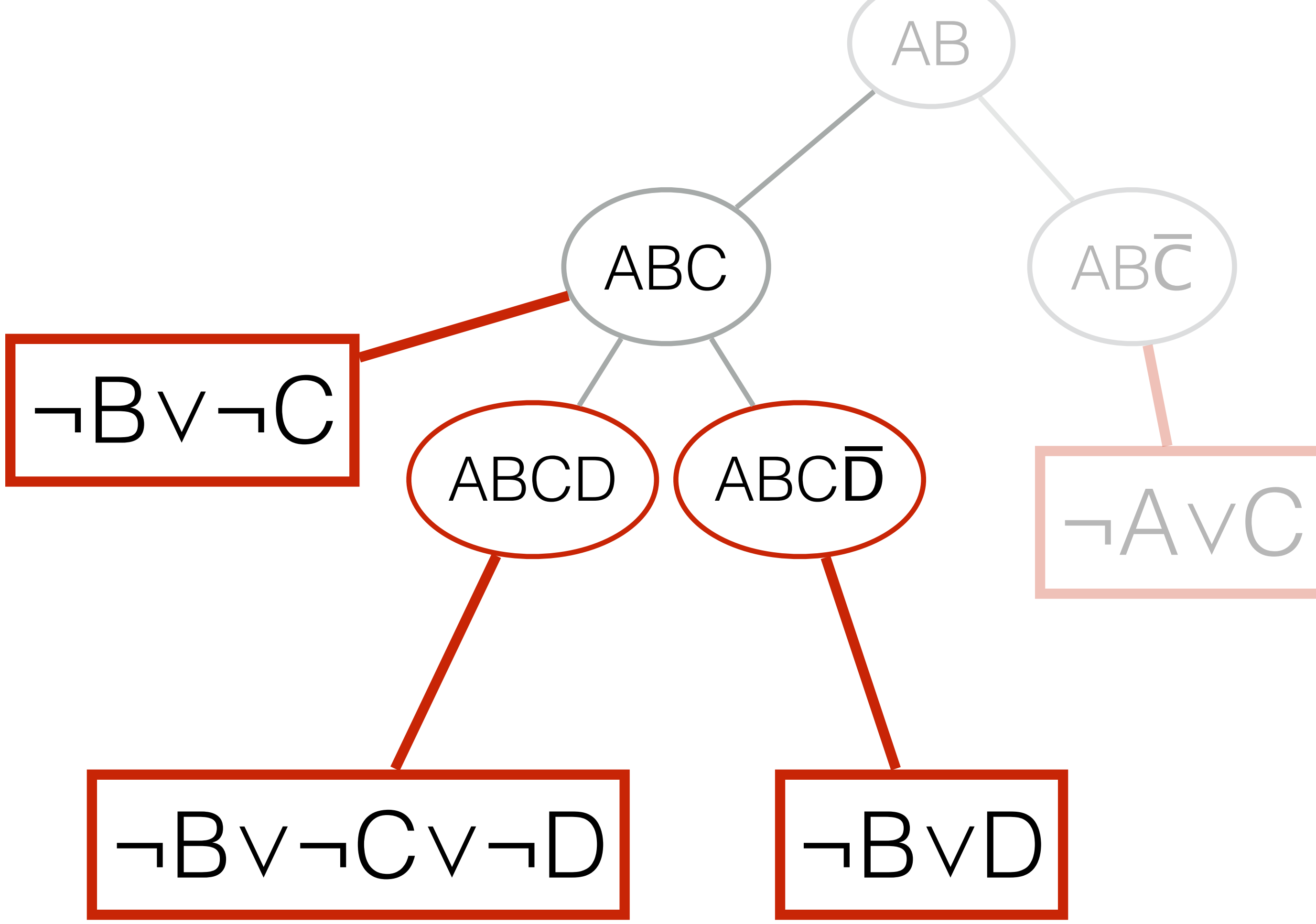
Premises

¬B∨¬C∨**¬D** ¬B∨**D**

¬B∨¬C

Conclusion

For any valid inference	Any assignment of truth values that makes the conclusion false will make at least one of the premises false.



Premises

¬B∨¬C∨**¬D**   ¬B∨**D**

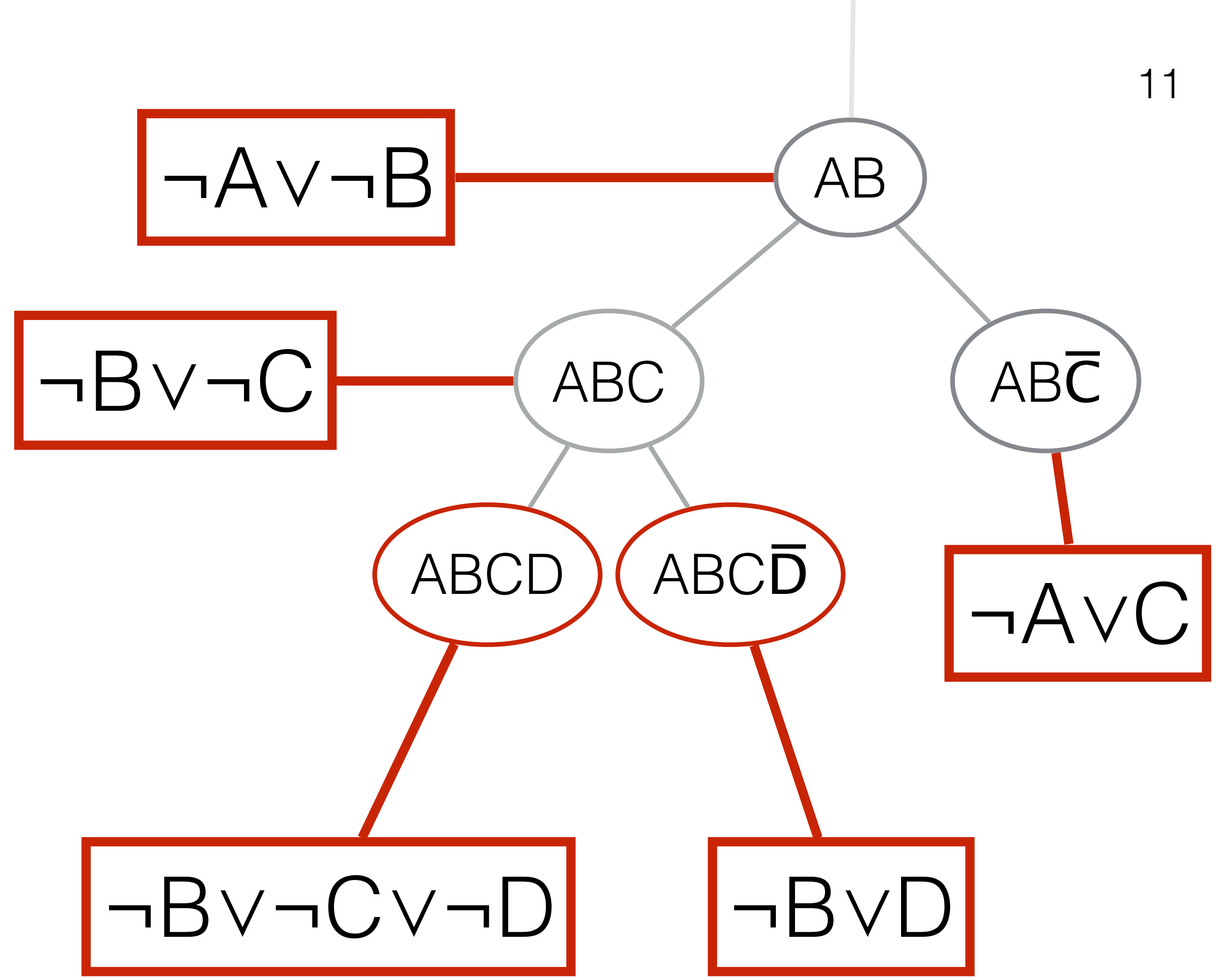
¬B∨¬C

Conclusion

*A **special property** of this inference*

If some assignment XYZ of values for ABC makes the conclusion false then the assignments XYZ**D** and XYZ**$\bar{D}$** each make one or other of the two premises false.

Resolution



$$\dfrac{\neg A \lor \mathbf{C} \qquad \dfrac{\neg B \lor \neg C \lor \neg D \qquad \neg B \lor D}{\neg B \lor \mathbf{\neg C}}}{\neg B \lor \neg A}$$

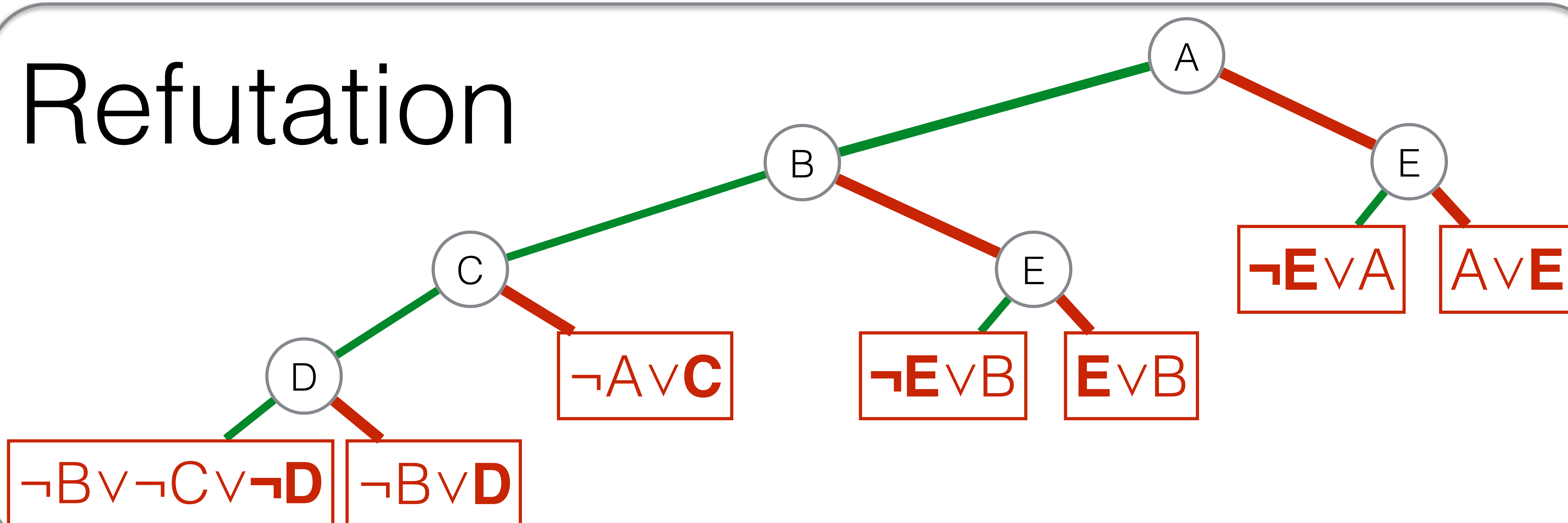
Resolution

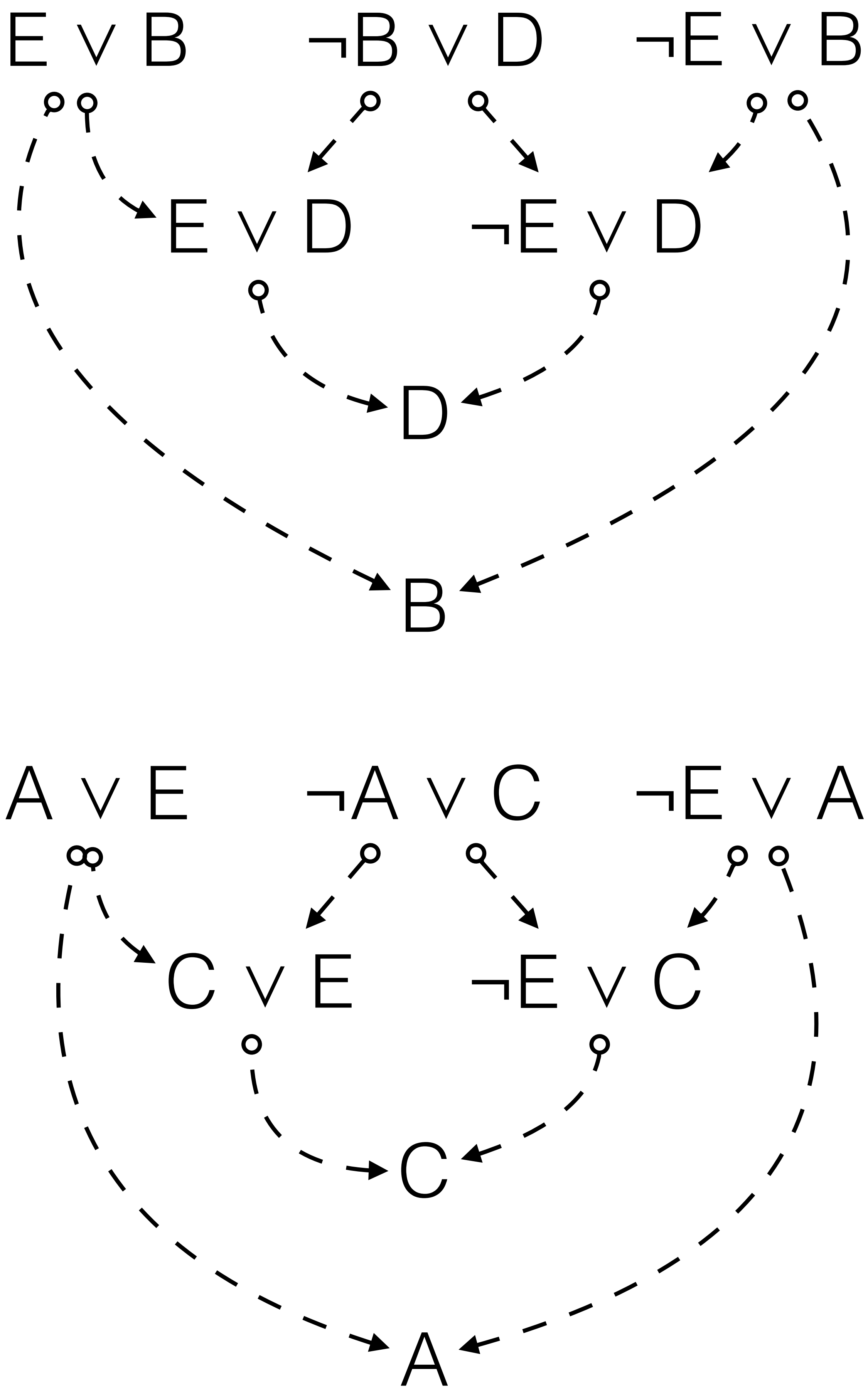
$$\frac{U \vee \mathbf{V} \vee \mathbf{W} \vee \mathbf{X} \vee \neg\mathbf{C} \qquad X \vee \mathbf{Y} \vee \mathbf{Z} \vee \mathbf{C}}{U \vee \mathbf{V} \vee \mathbf{W} \vee X \vee \mathbf{Y} \vee \mathbf{Z}}$$

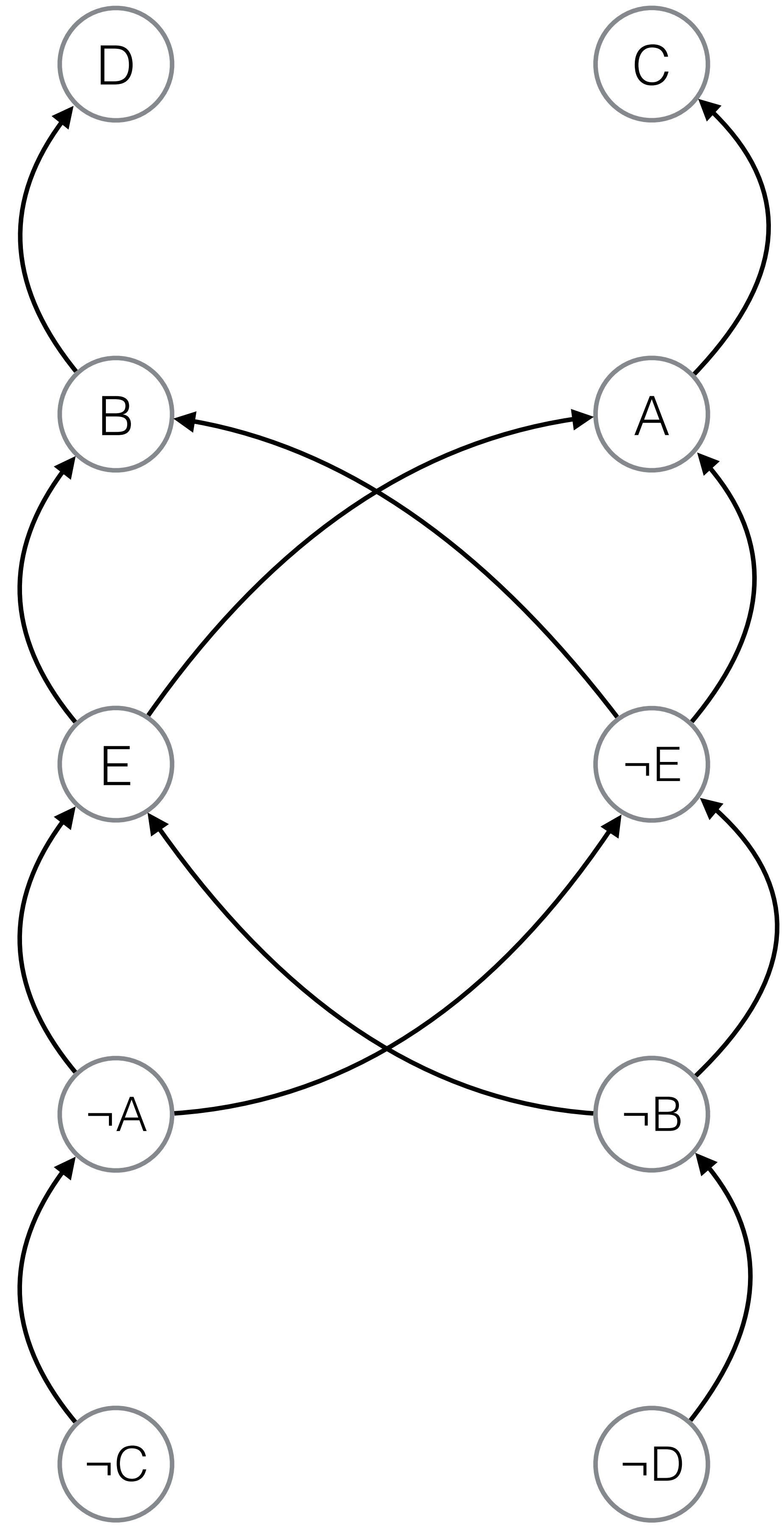
Resolution

$$
\cfrac{
  \cfrac{
    \cfrac{\neg B \lor \neg C \lor \mathbf{\neg D} \qquad \neg B \lor \mathbf{D}}{\neg B \lor \mathbf{\neg C}}
    \qquad \neg A \lor \mathbf{C}
  }{\mathbf{\neg B} \lor \neg A}
  \qquad
  \cfrac{\mathbf{\neg E} \lor B \qquad \mathbf{E} \lor B}{\mathbf{B}}
  \qquad\qquad
  \cfrac{}{}
}{\mathbf{\neg A}}
\qquad
\cfrac{\mathbf{\neg E} \lor A \qquad A \lor \mathbf{E}}{\mathbf{A}}
\qquad\Longrightarrow\qquad
\cfrac{\mathbf{\neg A} \qquad \mathbf{A}}{\perp}
$$

Refutation




E ∨ B
¬B ∨ D
¬E ∨ B
E ∨ D
¬E ∨ D
D
B
A ∨ E
¬A ∨ C
¬E ∨ A
C ∨ E
¬E ∨ C
C
A


D
C
B
A
E
¬E
¬A
¬B
¬C
¬D

Clausal Form

Resolution uses CNF
a conjunction of disjunctions of literals

$(\neg A \lor C) \land (\neg B \lor D) \land (\neg E \lor B) \land (\neg E \lor A) \land (A \lor E) \land (E \lor B) \land (\neg B \lor \neg C \lor \neg D)$

Clausal form is a set of sets of literals

$\{\{\neg A, C\}, \{\neg B, D\}, \{\neg E, B\}, \{\neg E, A\}, \{A, E\}, \{E, B\}, \{\neg B, \neg C, \neg D\}\}$

Each set of literals represents the disjunction of its literals.

An empty set of literals {} represents false $\perp$.

The clausal form represents the conjunction of these disjunctions

Clausal Form

Clausal form is a set of sets of literals

$\{$ {¬A,C}, {¬B,D}, {¬E,B}, {¬E,A}, {A,E}, {E,B},{¬B, ¬C, ¬D} $\}$

A (partial) truth assignment makes a clause true
iff it makes at least one of its literals true
(so it can never make the empty clause {} true)

A (partial) truth assignment makes a clausal form true
iff it makes all of its clauses true
(so the empty clausal form $\{\}$ is always true).

Clausal form is a set of sets of literals

$$\{\ \mathbf{X}_0, \mathbf{X}_1, \ldots, \mathbf{X}_{n-1}\ \}$$

Resolution rule for clauses

$$\frac{\mathbf{X} \qquad\qquad \mathbf{Y}}{(\mathbf{X} \cup \mathbf{Y}) \setminus \{\neg A, A\}} \quad \text{where } \neg A \in \mathbf{X},\ A \in \mathbf{Y}$$

(A?B:C)