Informatics 1

Lecture 7 Logic by colouring

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Computational Logic

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Chapter 1
Introduction

§1.1 Human Logic

Humans are, among other things, information processors. We acquire information about the world and use this information to further our ends. One of the strengths of human information processing is our ability to represent and manipulate logical information, not just simple facts but also more complex forms of information, such as negations, alternatives, constraints, and so forth.

To illustrate this ability, consider a simple puzzle in the world of children's blocks. We are given some facts about the arrangement of five blocks in a stack, and we are asked to determine their exact arrangement.

The sentences shown below constitute the premises of the problem. The first sentence tells us the exact location of the red block. The second sentence is not so exact, giving us only a constraint on the relative locations of the green and the blue block. The third sentence tells us what is not true, without saying what is true. The fourth sentence says that one condition holds or another but does not say which. The fifth sentence assures us that an object exists but does not give its identity.

The red block is on the green block.
The green block is somewhere ***above*** *the blue block.*
The green block is ***not*** *on the blue block.*
The yellow block is on the green block ***or*** *the blue block.*
There is ***some*** *block on the black block.*

Even though the information we need is not literally present in what we are given, it is possible to derive that information. In particular, the conclusions shown below all follow from the premises above.

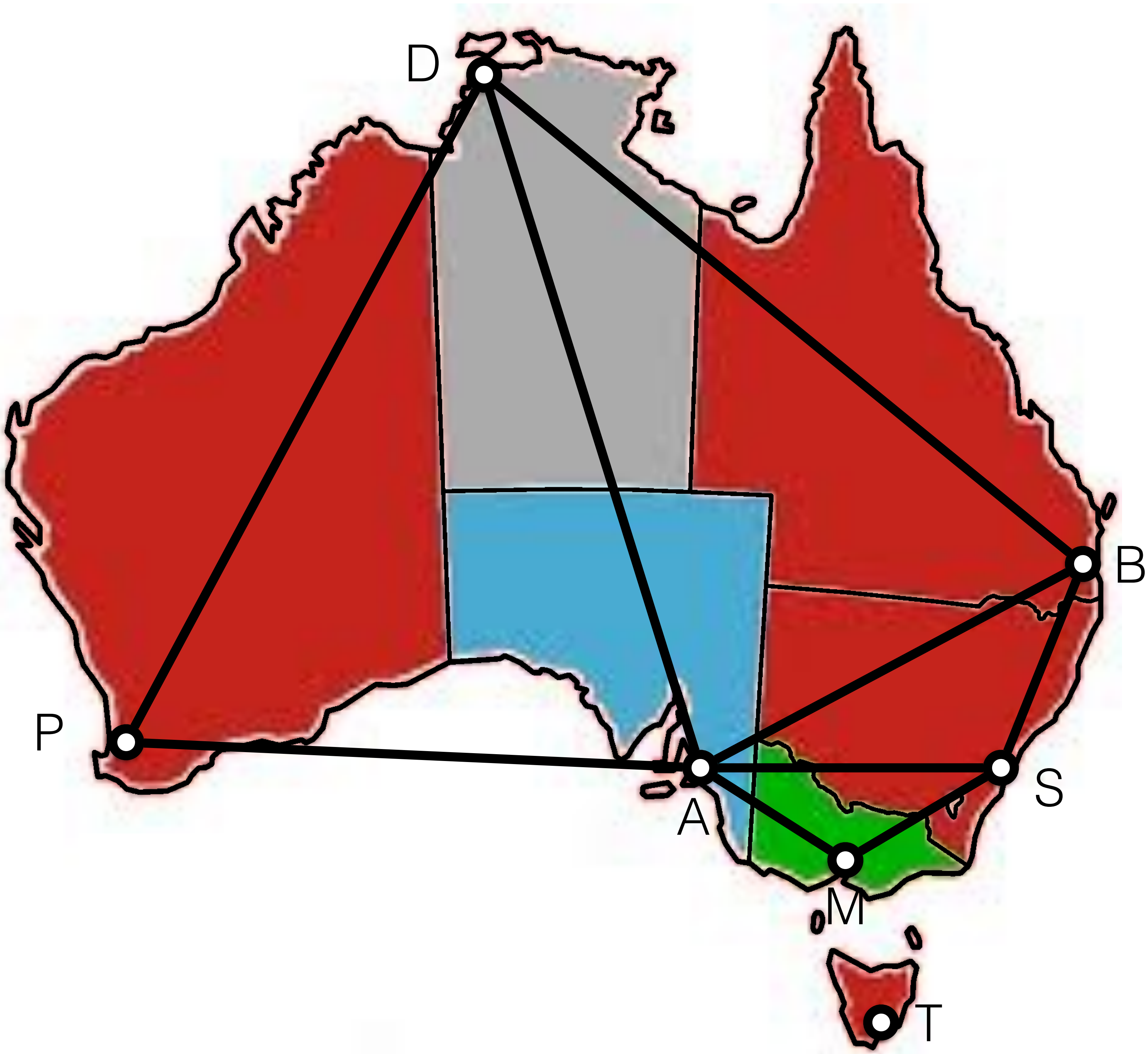
The red block is on the green block.
The green block is on the yellow block.
The yellow block is on the blue block.
The blue block is on the black block.
The black block is directly on the table.

Unfortunately, it is not always apparent which conclusions may be safely drawn from a given set of premises. What's more, even when we are given the conclusions, as in this example, their correctness may not be immediately obvious.

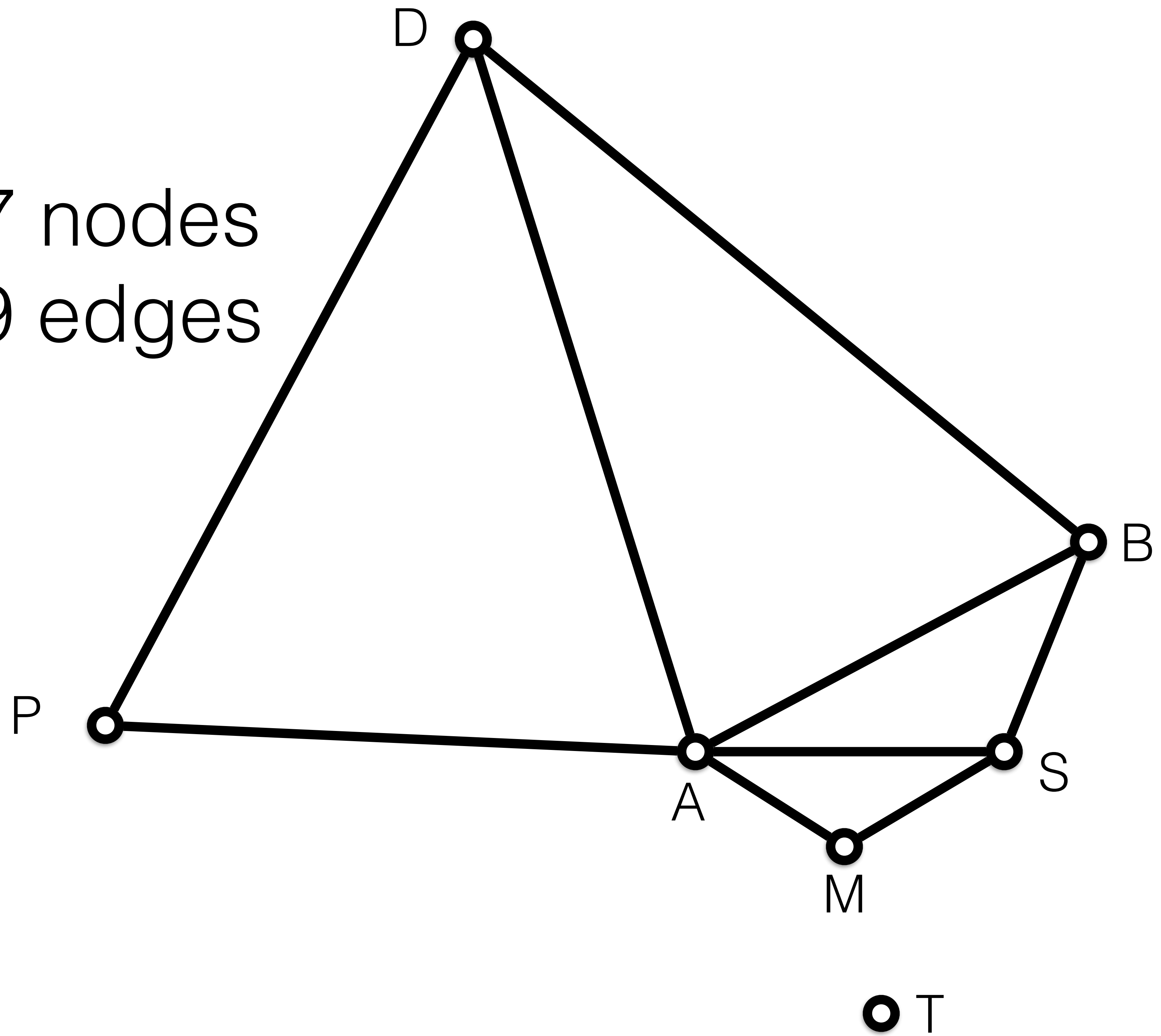
In order to persuade others of a conclusion that we have drawn, as well as to convince ourselves, it is useful to write down a *proof*, i.e. a series of intermediate conclusions in which each step is immediately obvious.

As an example, consider the following informal proof that, given the premises shown above, the yellow block is on the blue block.


Darwin
Brisbane
Perth
Adelaide
Sydney
Melbourne
Hobart

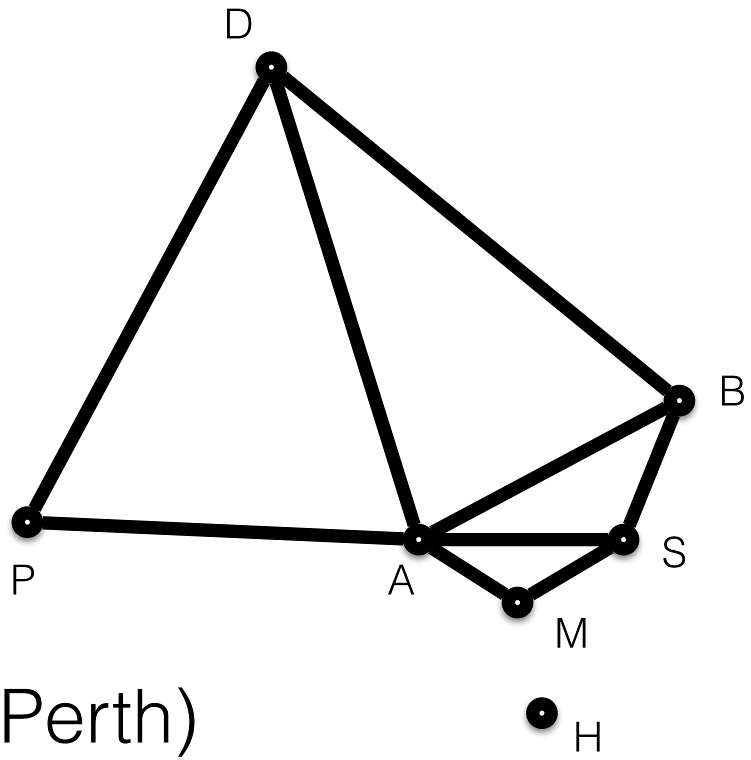

D
B
P
A
S
M
T

7 nodes
9 edges



21 atoms

		Melbourne	Sydney	Hobart	Darwin	Perth	Adelaide	Brisbane
red	🔴	Mr	Sr	Hr	Dr	Pr	Ar	Br
green	🟢	Mg	Sg	Hg	Dg	Pg	Ag	Bg
amber	🟠	Ma	Sa	Ha	Da	Pa	Aa	Ba



eg:
Pr ≡ red(Perth)

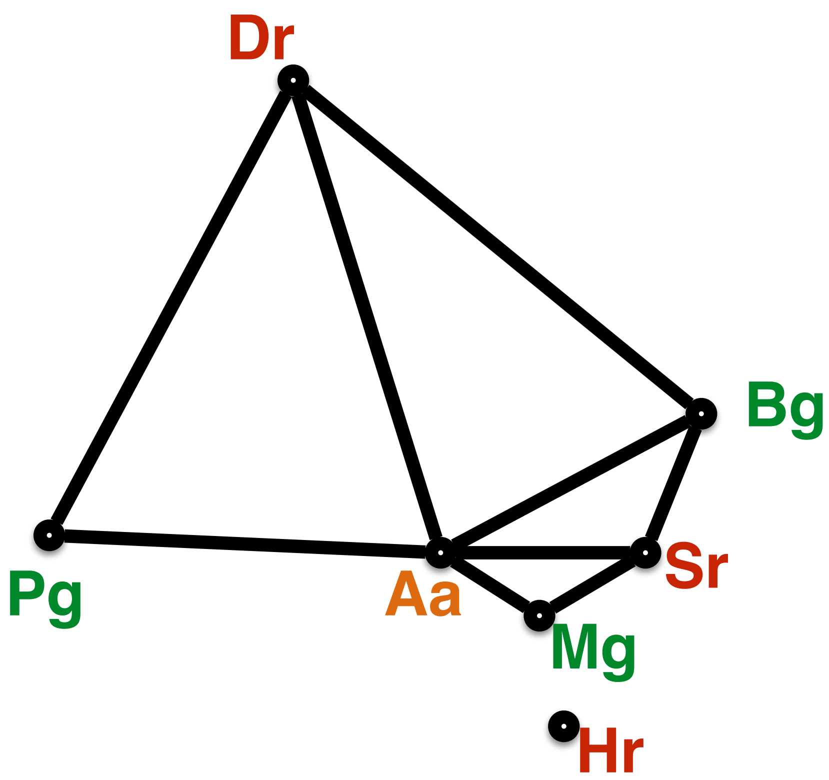
34 clauses

1 for each node (eg D)
Dr ∨ Dg ∨Da

3 for each edge (eg D–B)
¬Dr ∨ ¬Br
¬Dg ∨ ¬Bg
¬Da ∨ ¬Ba

21 atoms

	Melbourne	Sydney	Hobart	Darwin	Perth	Adelaide	Brisbane
red	Mr	Sr	Hr	Dr	Pr	Ar	Br
green	Mg	Sg	Hg	Dg	Pg	Ag	Bg
amber	Ma	Sa	Ha	Da	Pa	Aa	Ba



34 clauses

1 for each node (eg D)

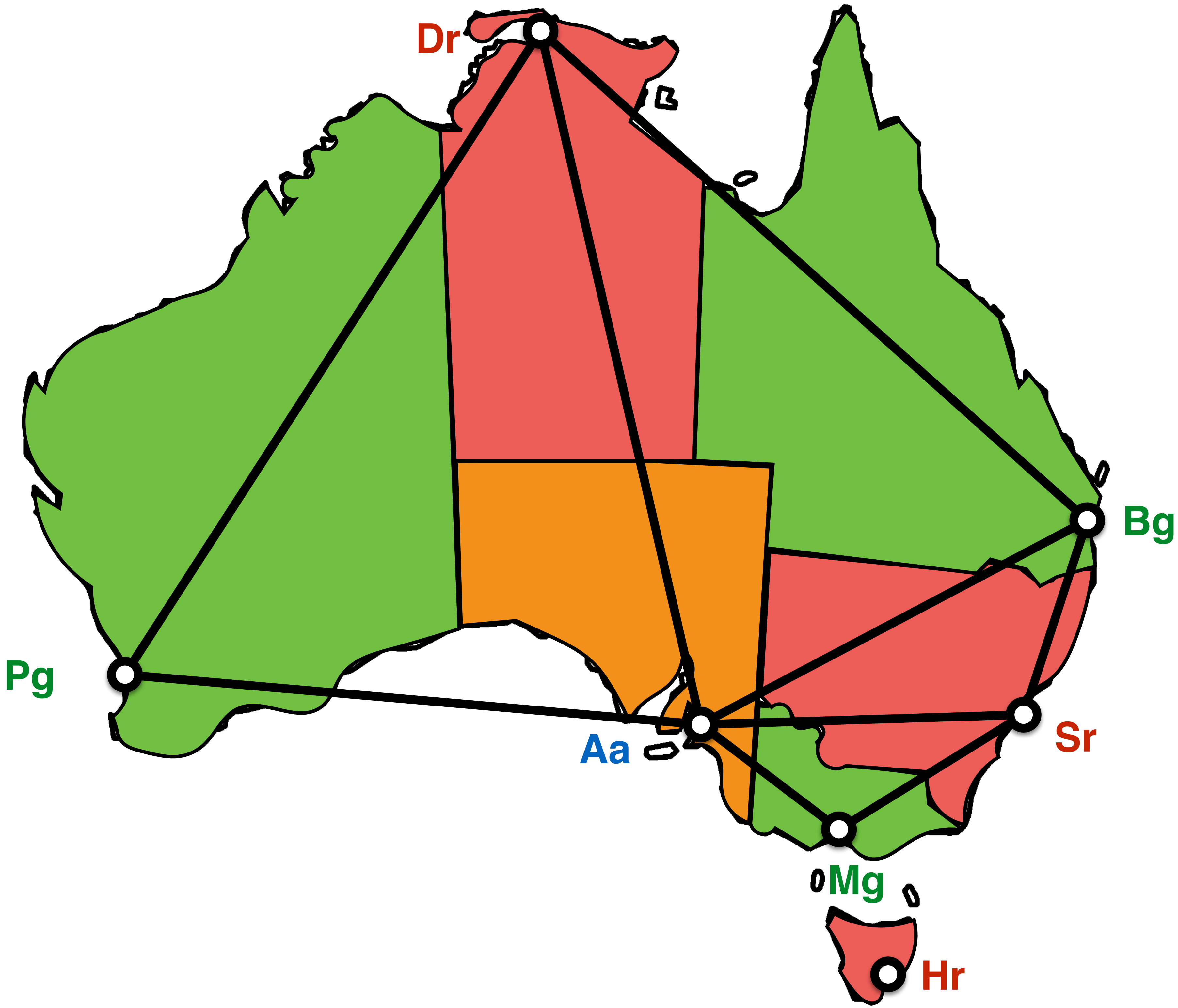
Dr ∨ Dg ∨Da

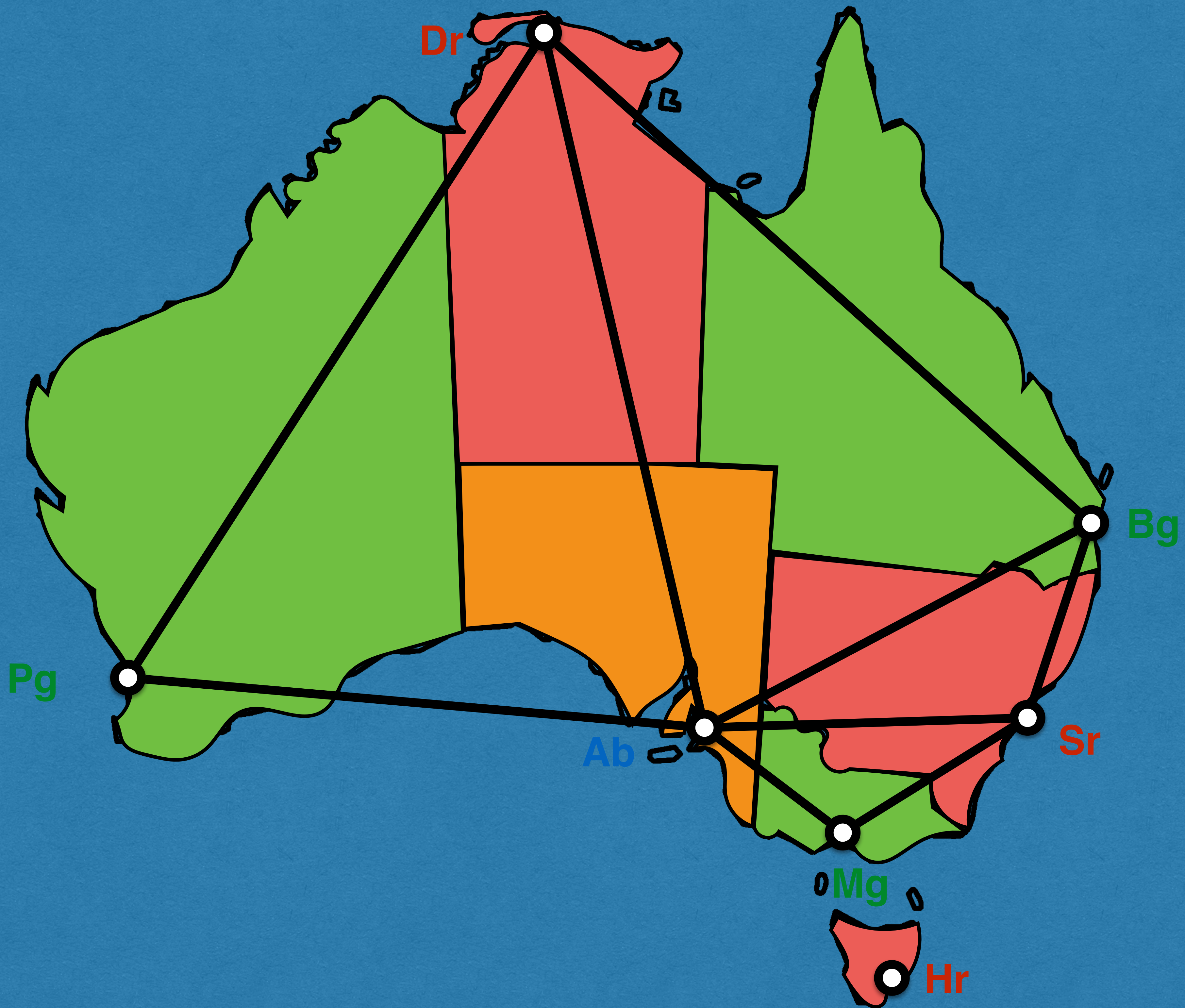
3 for each edge (eg D–B)

¬Dr ∨ ¬Br

¬Dg ∨ ¬Bg

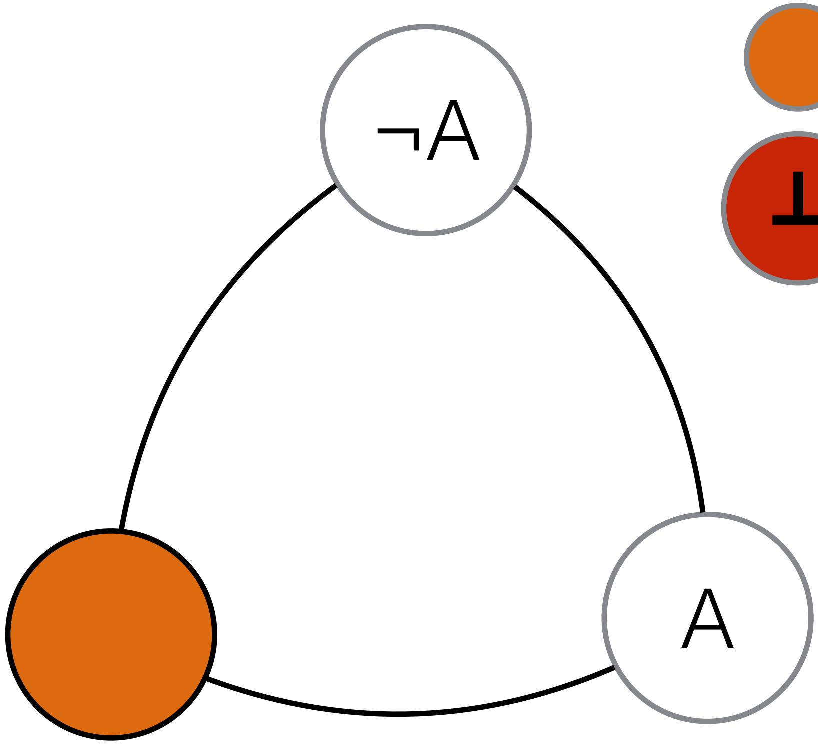
¬Da ∨ ¬Ba

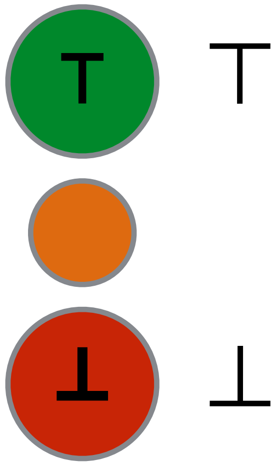

Dr
Bg
Pg
Aa
Sr
Mg
Hr


Dr
Bg
Pg
Ab
Sr
Mg
Hr

Colouring by numbers

Logic by 3-colouring





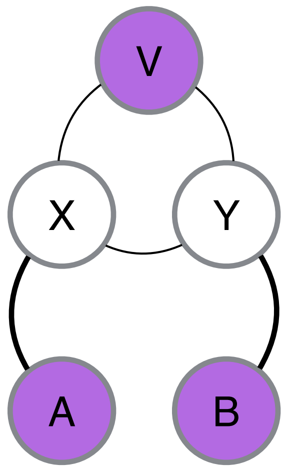
Red and Green represent False and True.

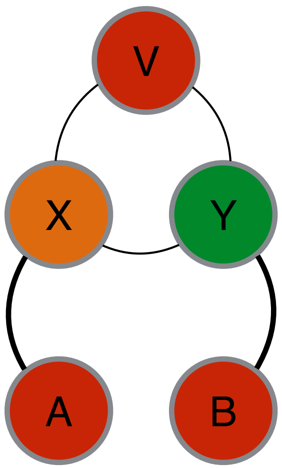
Amber is used to control the colouring of other nodes.

Any node connected to an Amber node must have a logical value.

If two logical nodes are connected they must take complementary values.

If A and B take the same value,
what can we say about V?



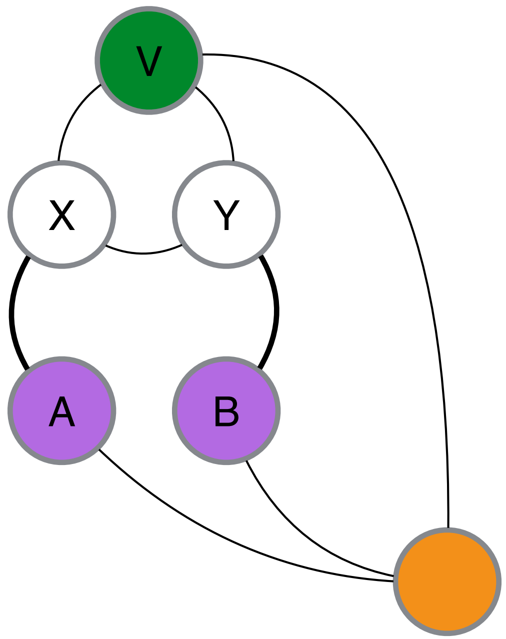


If both A and B both have the same colour then V must also have that colour

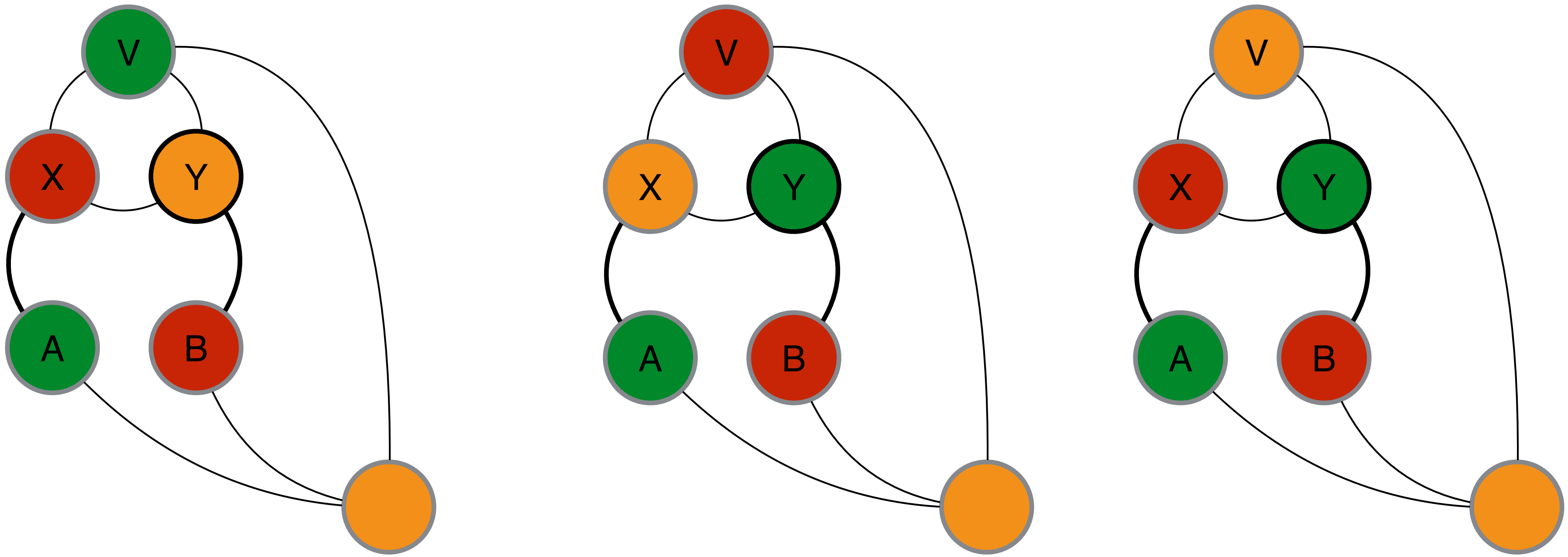
in particular …

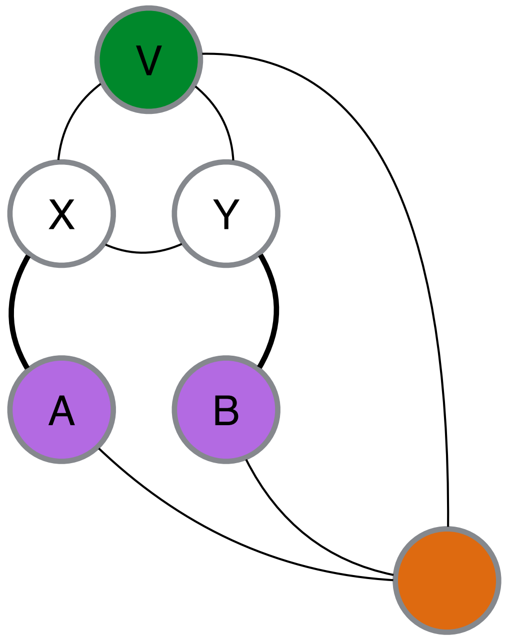
If both A and B are coloured false then V must be coloured false

If V is coloured green (true)
and the graph can be coloured,
what can we say about A and B?

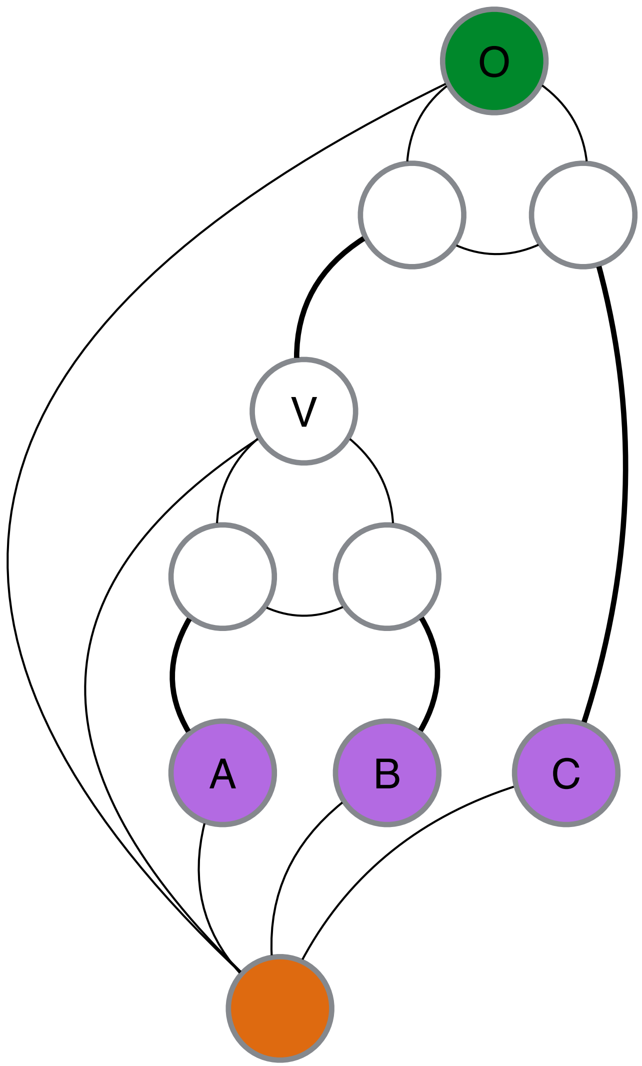


If A ≠ B what can we say about V?





If V is coloured green (true) we can colour this graph iff at least one of A and B is true

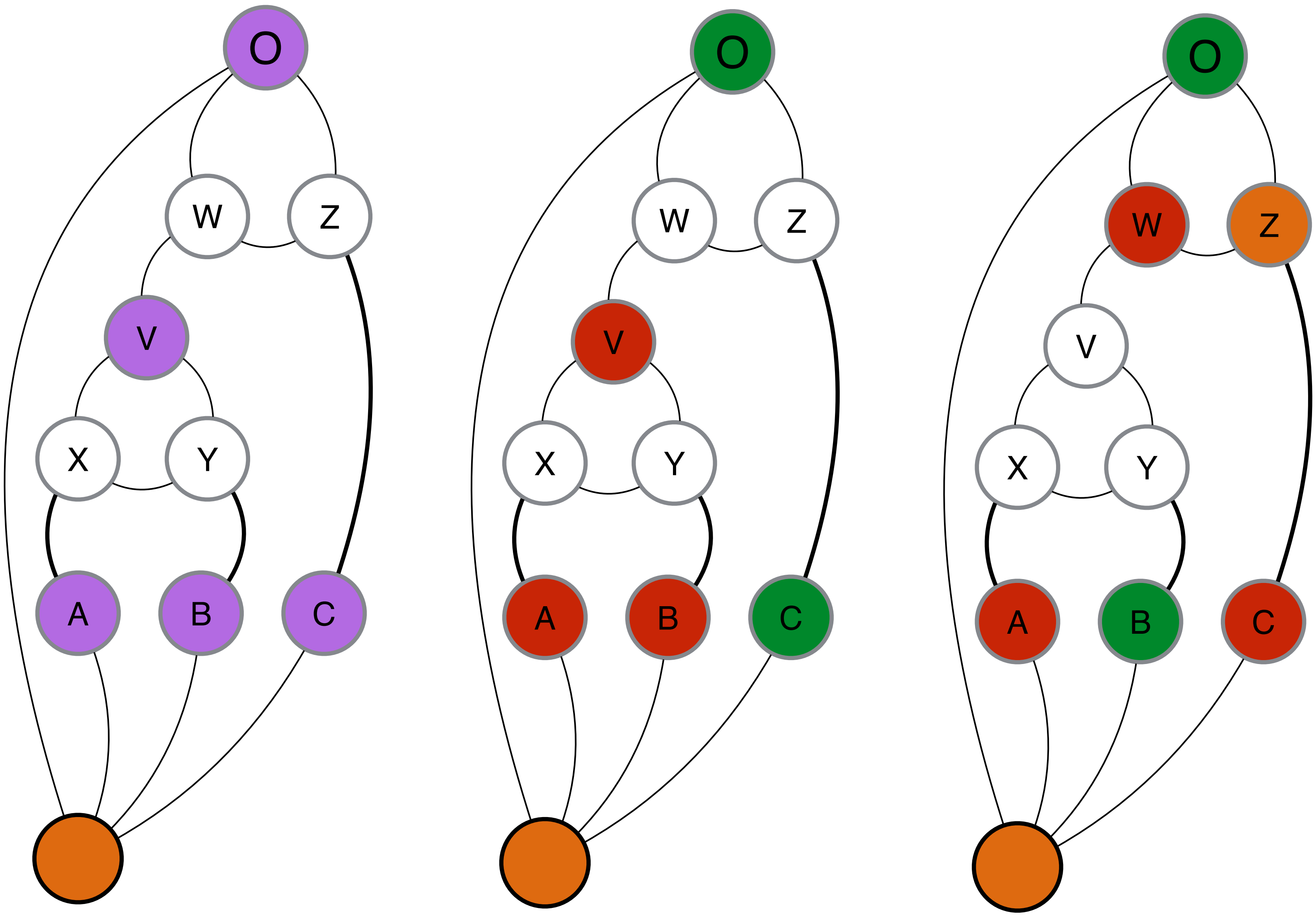


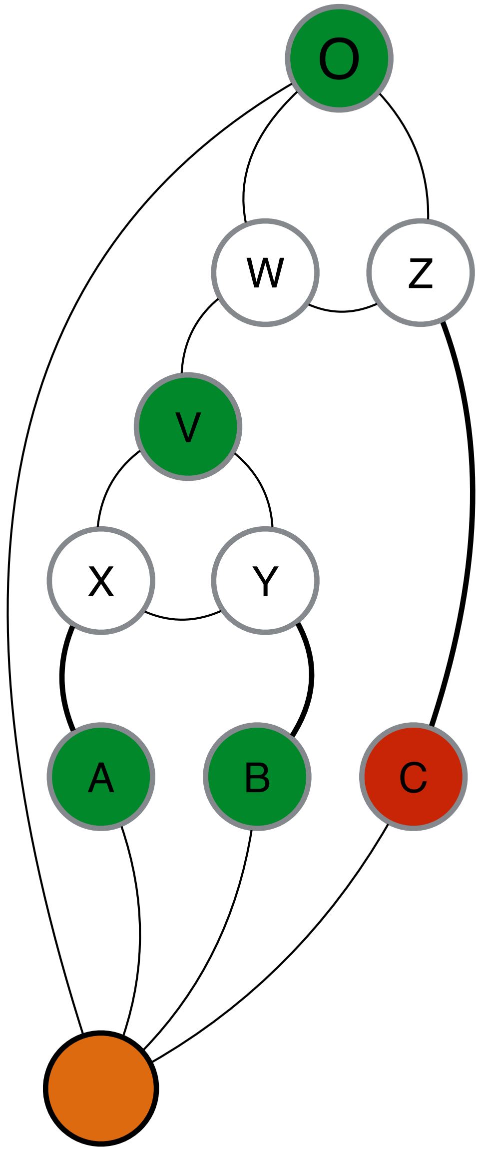
If O is coloured green (true)
we can colour this graph

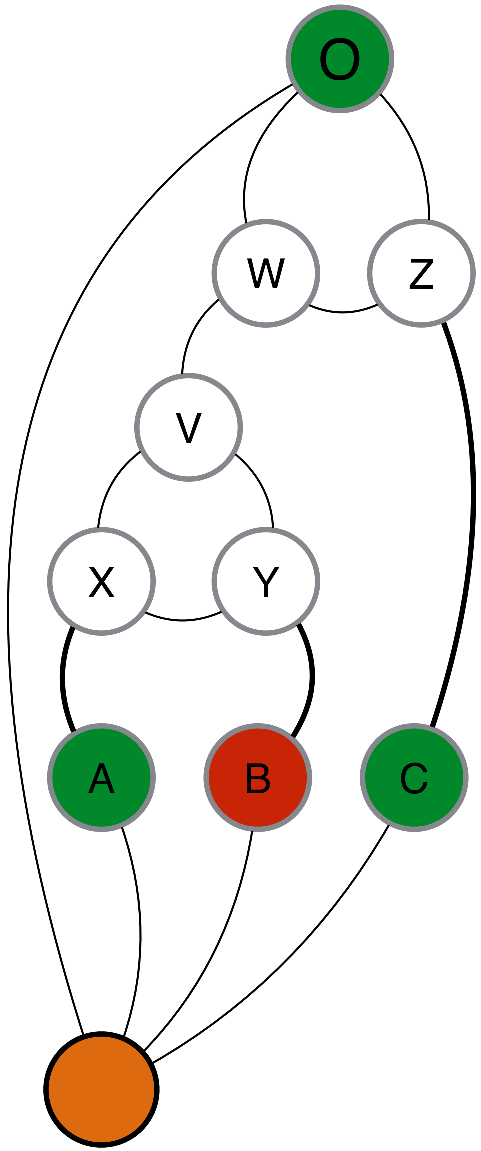
iff at least one of V and C is true

iff

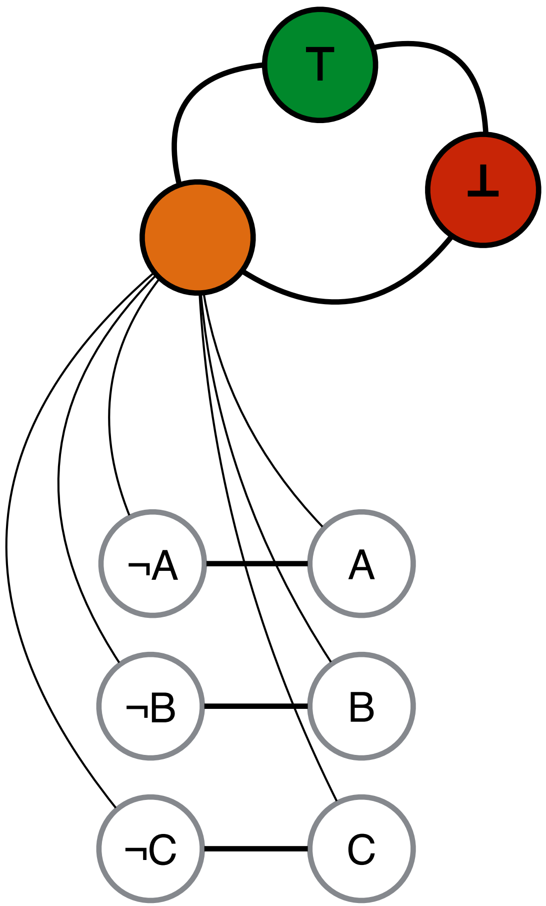
at least one of A, B, C is true


O
W
Z
V
X
Y
A
B
C
O
W
Z
V
X
Y
A
B
C
O
W
Z
V
X
Y
A
B
C

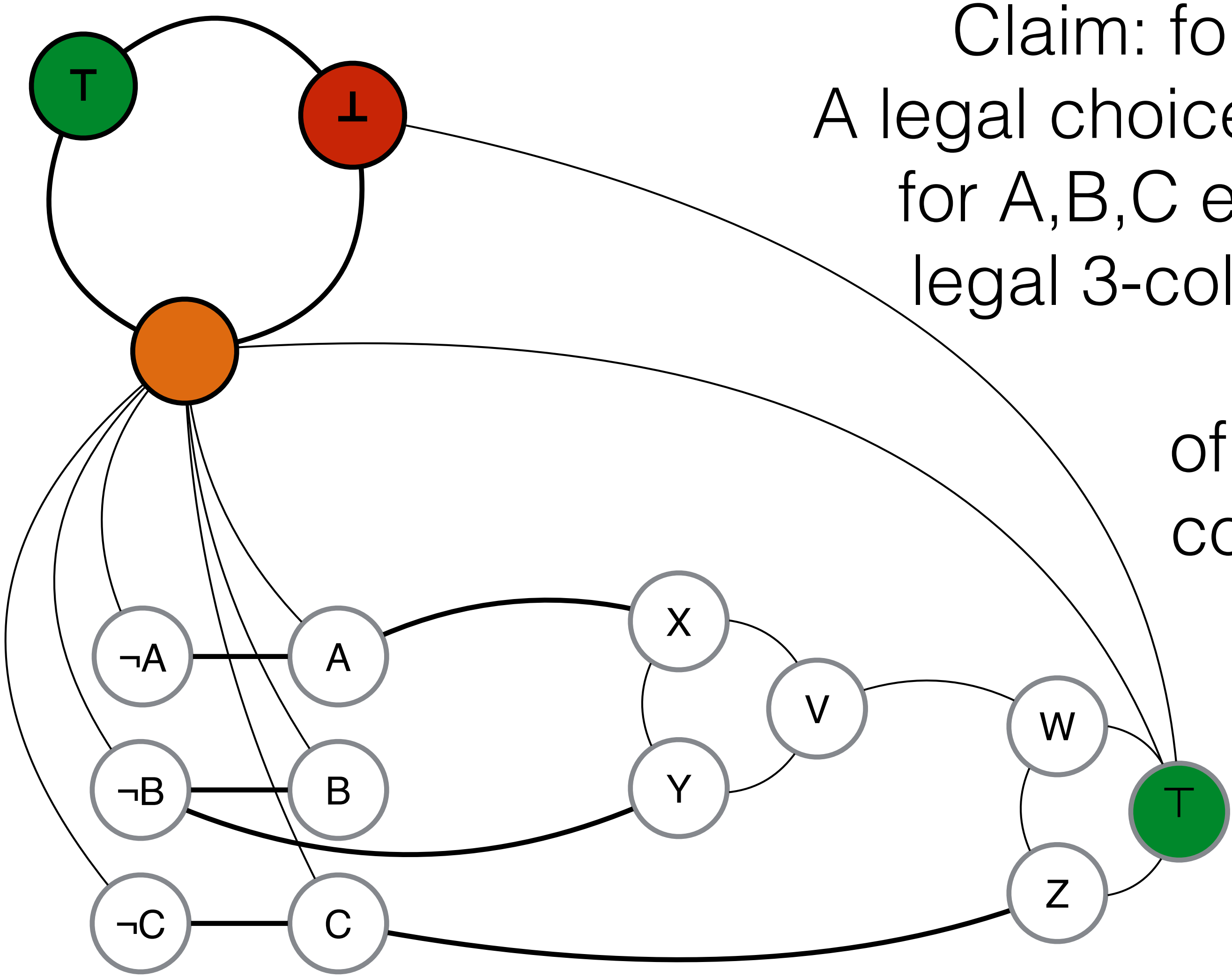

O
W
Z
V
X
Y
A
B
C


O
W
Z
V
X
Y
A
B
C

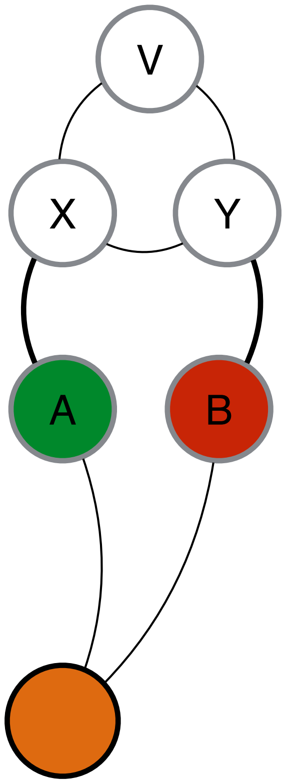
Computing by 3-colouring



Computing by 3-colouring



Claim: for this graph A legal choice of colours for A,B,C extends to a legal 3-colouring iff at least one of A, ¬B, C is coloured true


V
X
Y
A
B

(A?B:C)