Informatics 1

Lecture 4
Finite Situations

Michael Fourman

No crowded shops are comfortable.

The expression $\quad \text{Crowded}(s) \to \text{Comfortable}(s)$

means $\quad \{s \mid \text{Crowded}(s) \to \text{Comfortable}(s)\}\,.$

To make the universal statement that *all* crowded shops are uncomfortable,

we write, $\quad \forall s.\, \text{Crowded}(s) \to \text{Comfortable}(s)\,,$

which means, $\quad \{s \mid \text{Crowded}(s) \to \text{Comfortable}(s)\} = S\,,$

where S is the set of all shops.

No crowded shops are comfortable.

To make the existential statement that *some* crowded shops are comfortable, we introduce a third expression:

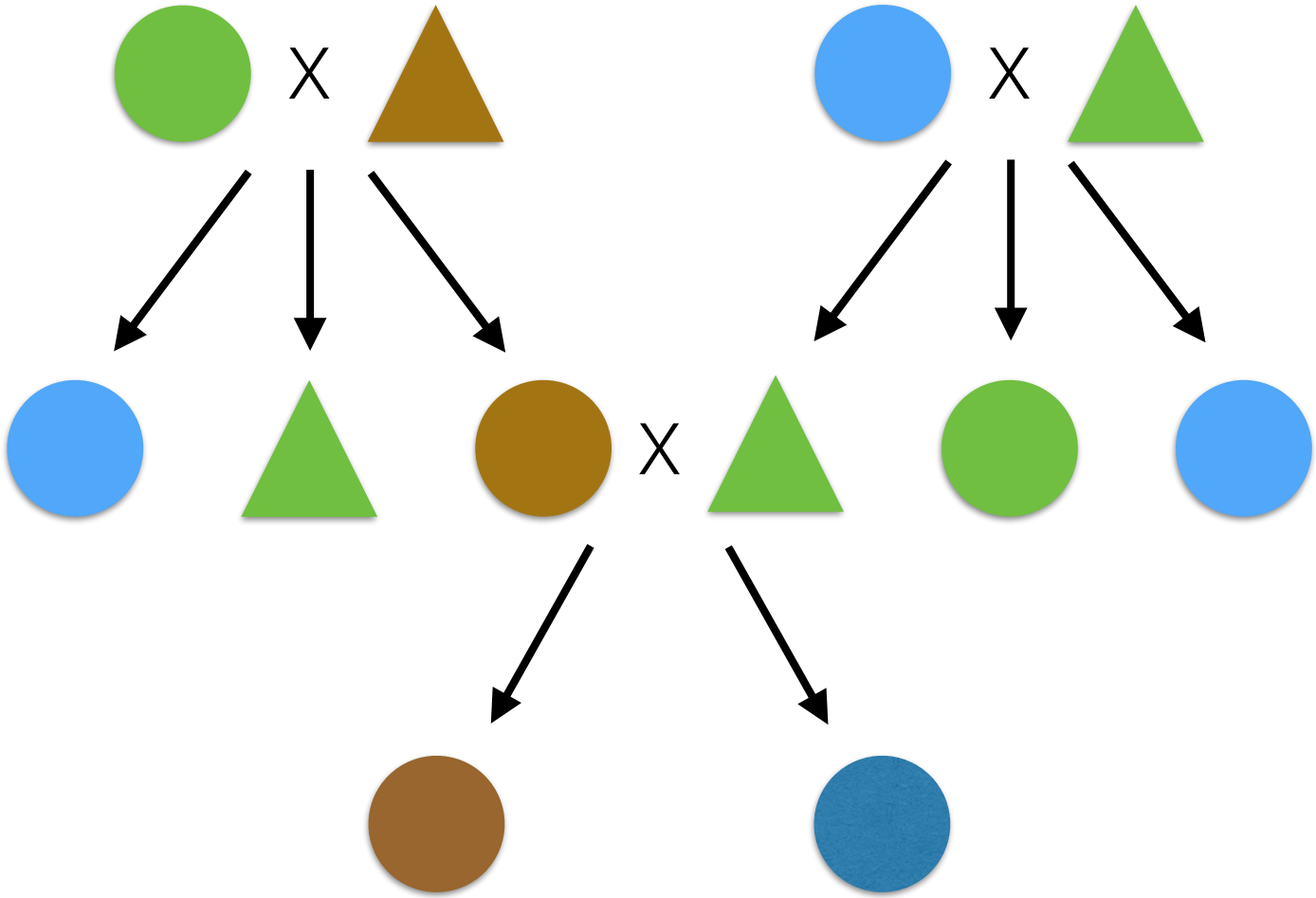
we write,	$\exists s.\,\big(\text{Crowded}(s) \wedge \text{Comfortable}(s)\big)$
which means,	$\big\{s \mid \text{Crowded}(s) \wedge \text{Comfortable}(s)\big\} \neq \emptyset$

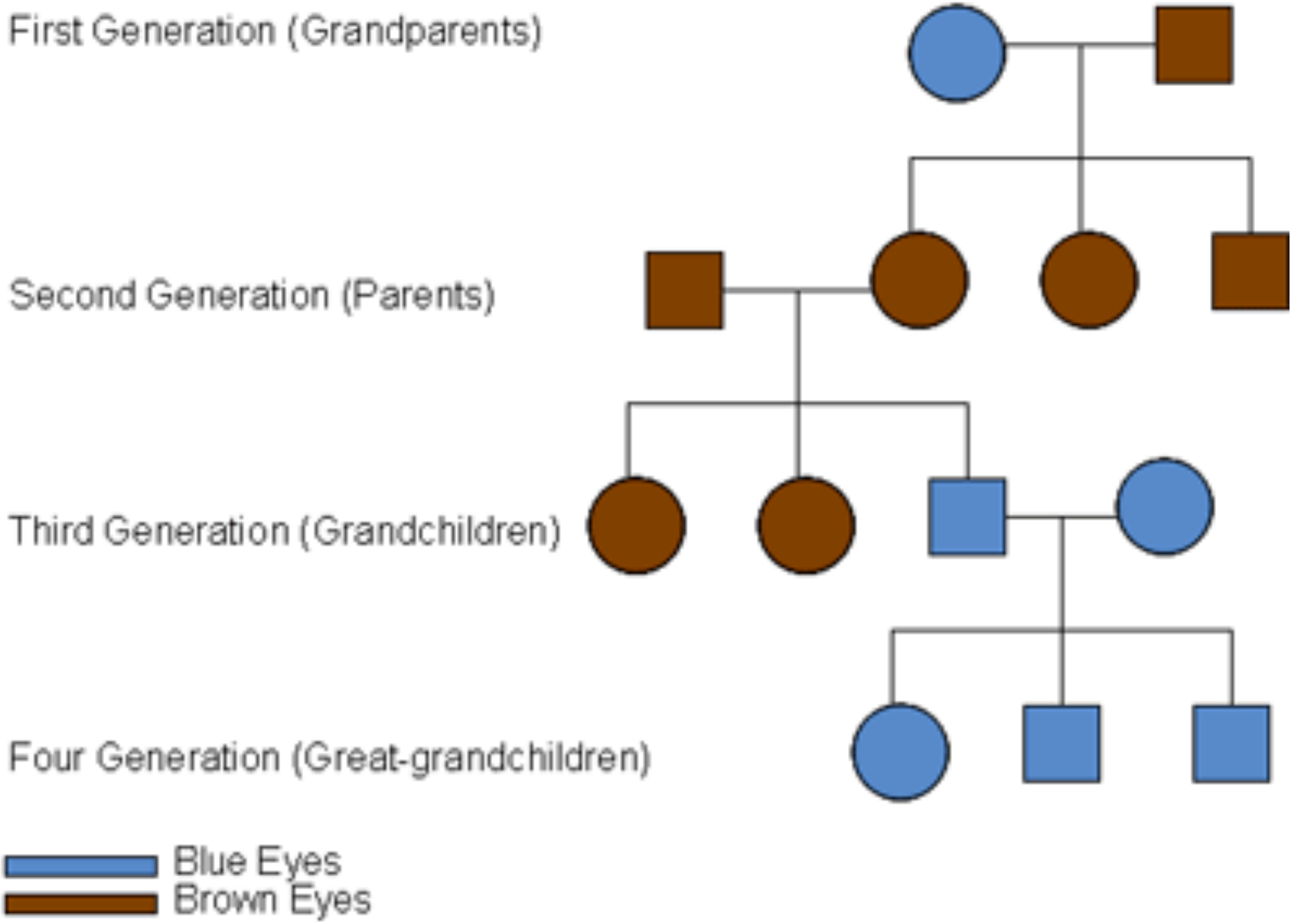
where $\emptyset = \{\}$ is the empty set.

So, to state that *no* crowded shops are comfortable,

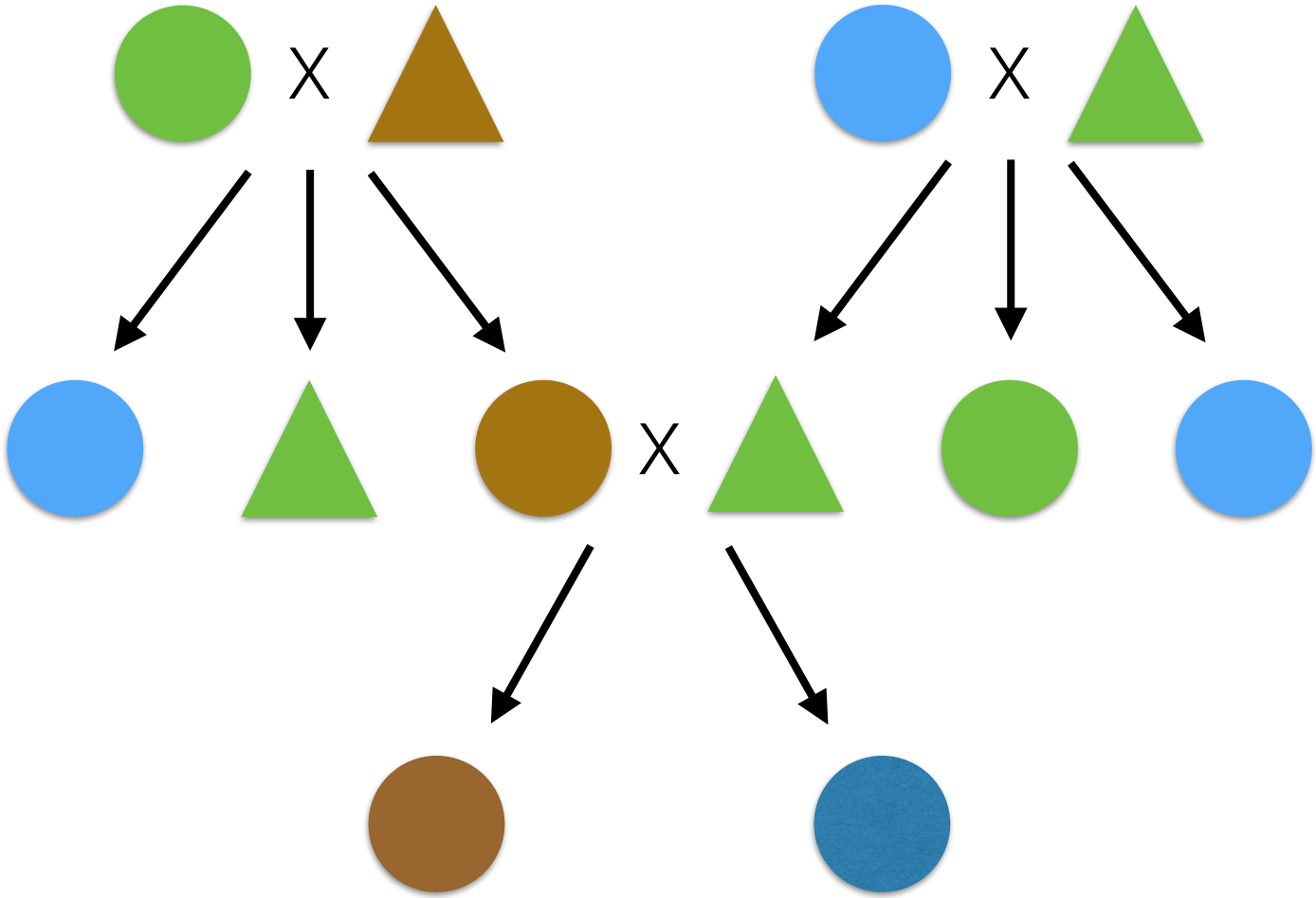
we write,	$\neg\exists s.\,\big(\text{Crowded}(s) \wedge \text{Comfortable}(s)\big)$
which means,	$\big\{s \mid \text{Crowded}(s) \wedge \text{Comfortable}(s)\big\} = \emptyset$

Relational Structures




First Generation (Grandparents)
Second Generation (Parents)
Third Generation (Grandchildren)
Four Generation (Great-grandchildren)
Blue Eyes
Brown Eyes

Relational Structures



properties and relations

$F(x)$

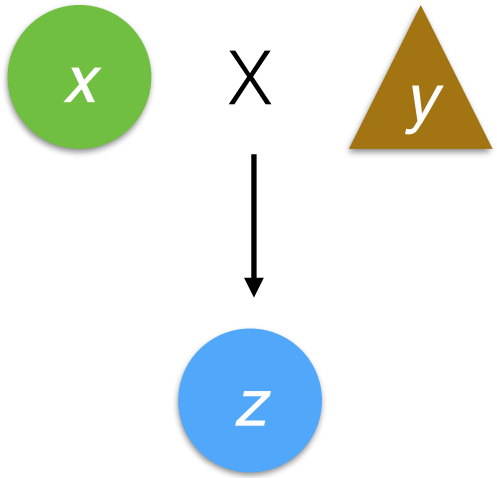
$M(y)$

$F(z)$

$\text{green}(x)$

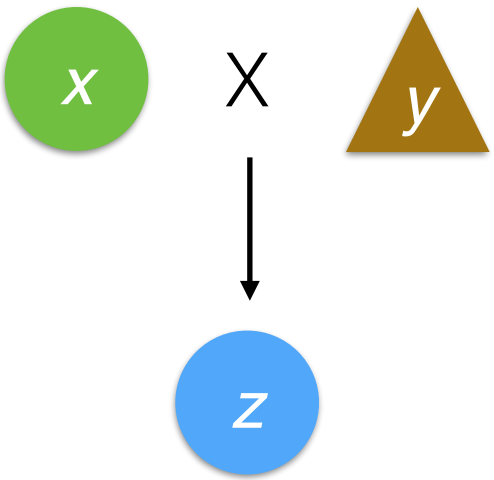
$\text{brown}(y)$

$\text{blue}(z)$



$C(x, y, z)$

properties and relations

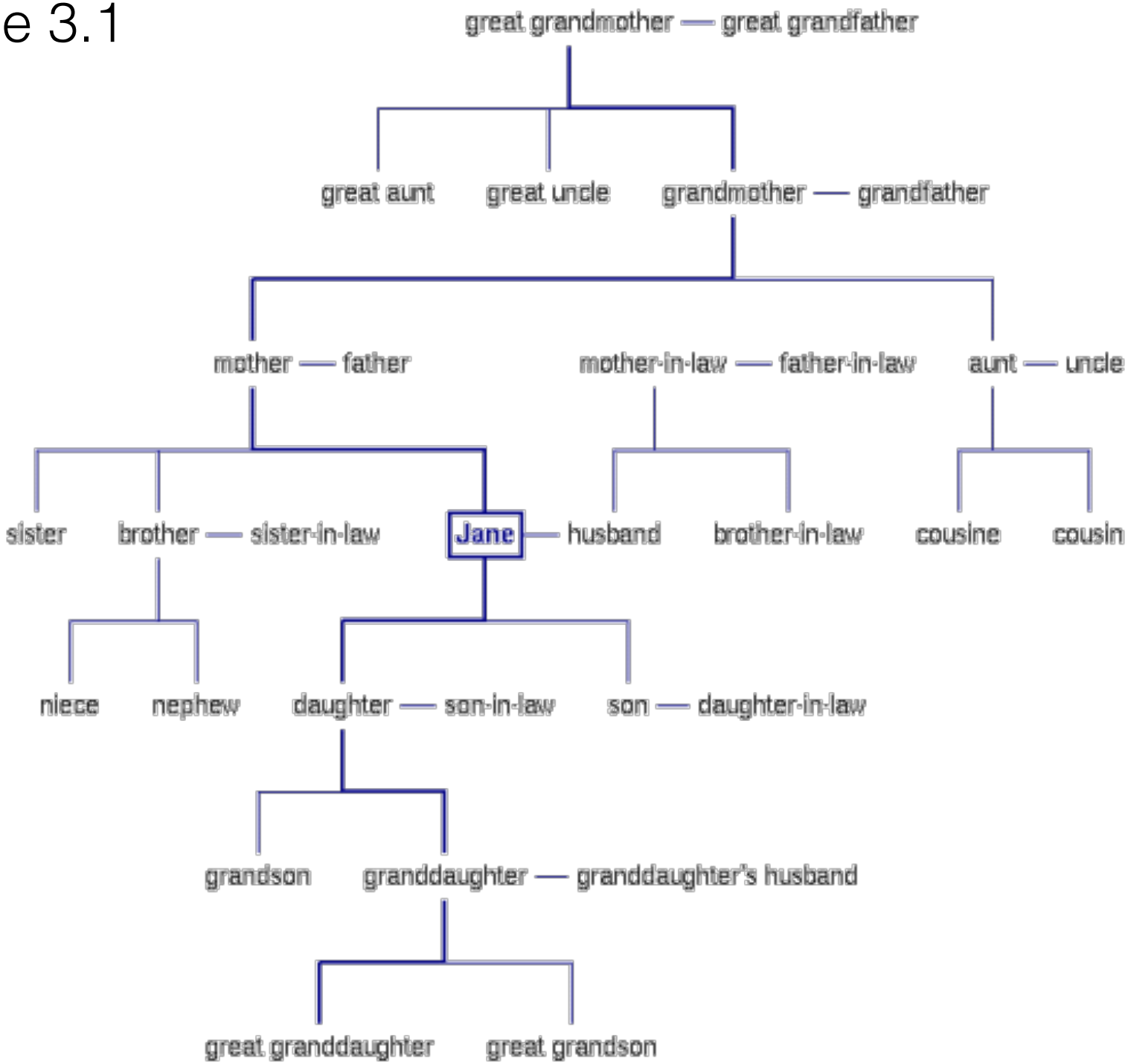


$$C(x, y, z) \rightarrow F(x) \wedge M(y)$$

$$\begin{aligned}
\text{father}(z, f) &\equiv \exists m.\, C(m, f, z) \\
\text{mother}(z, m) &\equiv \exists f.\, C(m, f, z)
\end{aligned}$$

$$\begin{aligned}
\text{parent}(z, p) &\equiv \exists m.\, C(m, p, z) \vee \exists f.\, C(p, f, z) \\
&\equiv \exists s.\, \big(C(s, p, z) \vee C(p, s, z)\big)
\end{aligned}$$

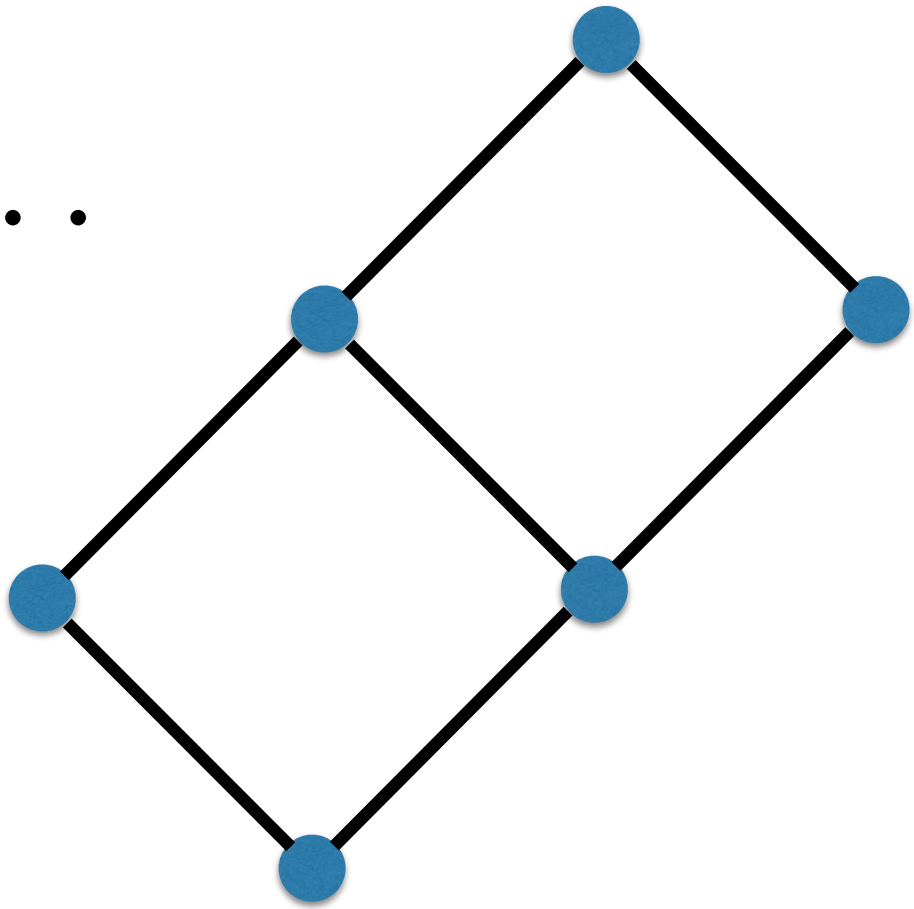
Exercise 3.1


great grandmother — great grandfather
great aunt
great uncle
grandmother — grandfather
mother — father
mother-in-law — father-in-law
aunt — uncle
sister
brother — sister-in-law
Jane
husband
brother-in-law
cousine
cousin
niece
nephew
daughter — son-in-law
son — daughter-in-law
grandson
granddaughter — granddaughter's husband
great granddaughter
great grandson

Graphs

nodes $x, y, z, \ldots$

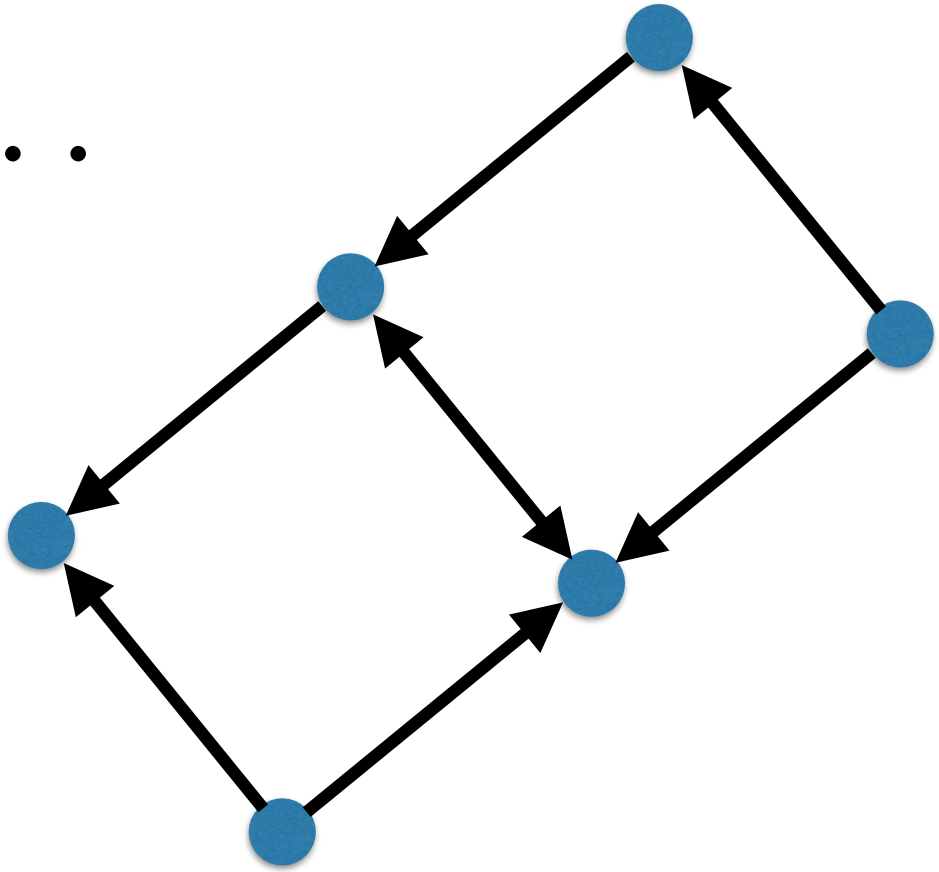
$$E(x, y)$$

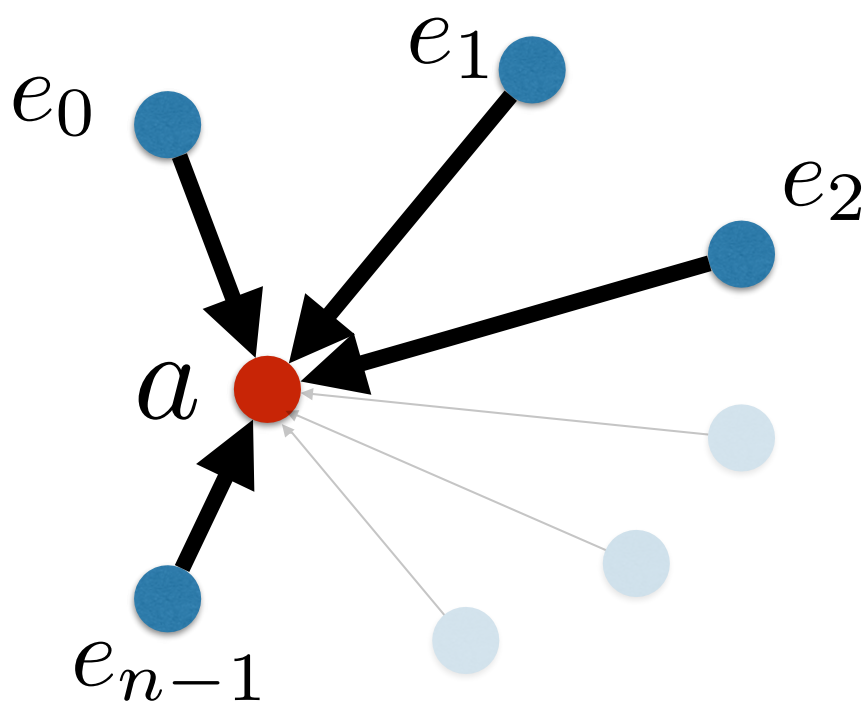


Directed Graphs

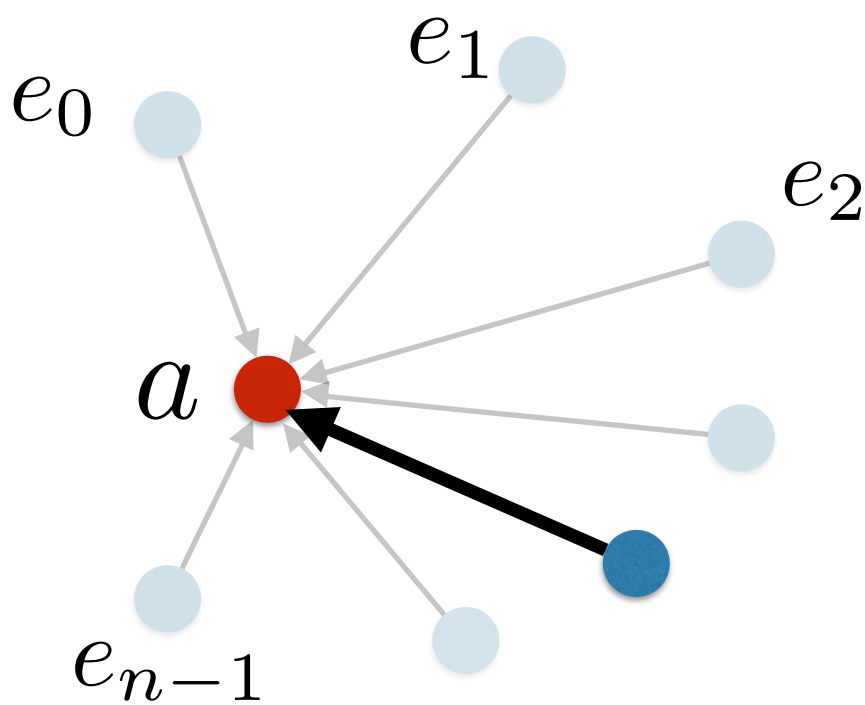
nodes $x, y, z, \ldots$

$$L(x, y)$$



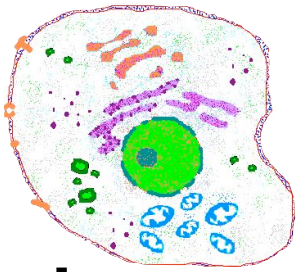


$$\forall x \neq a.\, L(x, a) \equiv \ \forall x.\, x \neq a \rightarrow L(x, a)$$
$$\equiv L(e_0, a) \wedge L(e_1, a) \wedge L(e_2, a) \wedge \ldots L(e_{n-1}, a)$$



$$\exists x \neq a.\, L(x, a) \equiv \ \exists x.\, x \neq a \wedge L(x, a)$$
$$\equiv L(e_0, a) \vee L(e_1, a) \vee L(e_2, a) \vee \ldots L(e_{n-1}, a)$$

Relational Data



A **table** of **data**. Each **entry** in a **cell** is a **datum**

Books

isbn	title	author	publD	pages
029785593X	From Nature To Plate	Tom Kitchin	7642	272
955904609	Cookbook	Martin Wishart	3556	256
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...

Relational Data

Rows represent the things we're interested in.

We call each row a **record**.

Books

isbn	title	author	publD	pages
029785593X	From Nature To Plate	Tom Kitchin	7642	272
955904609	Cookbook	Martin Wishart	3556	256
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...

A **table** of similar records is called a **relation**.

Relational Data

Columns represent properties or attributes.

Each of these is a **field**.

Books

isbn	title	author	publD	pages
029785593X	From Nature To Plate	Tom Kitchin	7642	272
955904609	Cookbook	Martin Wishart	3556	256
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...

Each record in the relation has the same format.

Relational data

Publishers

ID	name	address
7642	Weidenfeld & Nicolson	London
3556	Mr Max Publishing	Edinburgh
...	...	...

Books

isbn	title	author	publD	pages
029785593X	From Nature To Plate	Tom Kitchin	7642	272
955904609	Cookbook	Martin Wishart	3556	256
...	...	...	...	...

A typical database has many relations.

An ID or **key** field uniquely identifies a record.

A query

Publishers

ID	name	address
7642	Weidenfeld & Nicolson	London
3556	Mr Max Publishing	Edinburgh

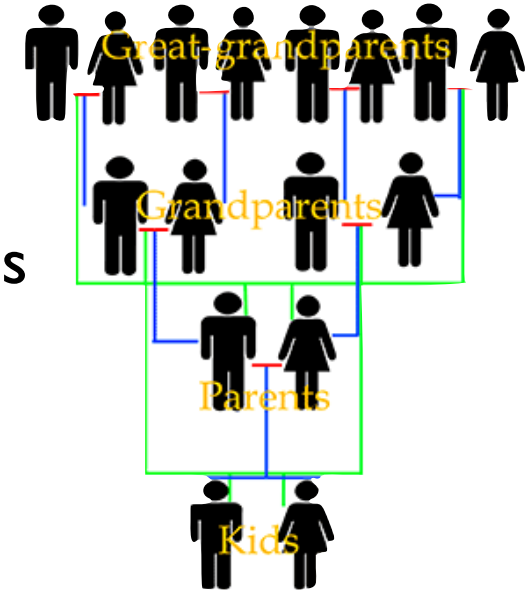
Books

isbn	title	author	pubID	pages
029785593X	From Nature To Plate	Tom Kitchin	7642	272
955904609	Cookbook	Martin Wishart	3556	256
...	...	...	...	...

$$\{\texttt{name} \mid$$
$$\exists\ \texttt{isbn}, \texttt{title}, \texttt{pubID}, \texttt{pages}, \texttt{ID}, \texttt{address}.$$
$$\texttt{pubID} = \texttt{ID}$$
$$\wedge\ \texttt{Books}(\texttt{isbn}, \texttt{title}, \texttt{"Tom Kitchin"}, \texttt{pubID}, \texttt{pages})$$
$$\wedge\ \texttt{Publishers}(\texttt{ID}, \texttt{name}, \texttt{address})\}$$

Relational data

- field
  - a property or attribute
- record
  - values for each field, for a given item
- relation or table
  - set of records, representing a set of items
- key
  - a field that uniquely identifies an item



propositional methods

- first-order logic gives an expressive language for talking about structures
- for each fixed finite structure we can translate any first-order sentence into a propositional combination of atomic propositions

(A?B:C)