# Informatics 1

## Computation and Logic

### CNF DNF and quantifiers

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# Boolean Algebra

| | | |
|---|---|---|
| $x \vee (y \vee z) = (x \vee y) \vee z$ | $x \wedge (y \wedge z) = (x \wedge y) \wedge z$ | associative |
| $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ | $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ | distributive |
| $x \vee y = y \vee x$ | $x \wedge y = y \wedge x$ | commutative |
| $x \vee 0 = x$ | $x \wedge 1 = x$ | identity |
| $x \vee 1 = 1$ | $x \wedge 0 = 0$ | annihilation |
| $x \vee x = x$ | $x \wedge x = x$ | idempotent |
| $x \vee \neg x = 1$ | $\neg x \wedge x = 0$ | complements |
| $x \vee (x \wedge y) = x$ | $x \wedge (x \vee y) = x$ | absorbtion |
| $\neg(x \vee y) = \neg x \wedge \neg y$ | $\neg(x \wedge y) = \neg x \vee \neg y$ | de Morgan |
| $\neg\neg x = x$ | $x \to y = \neg x \leftarrow \neg y$ | |

# an algebraic proof

To show that,

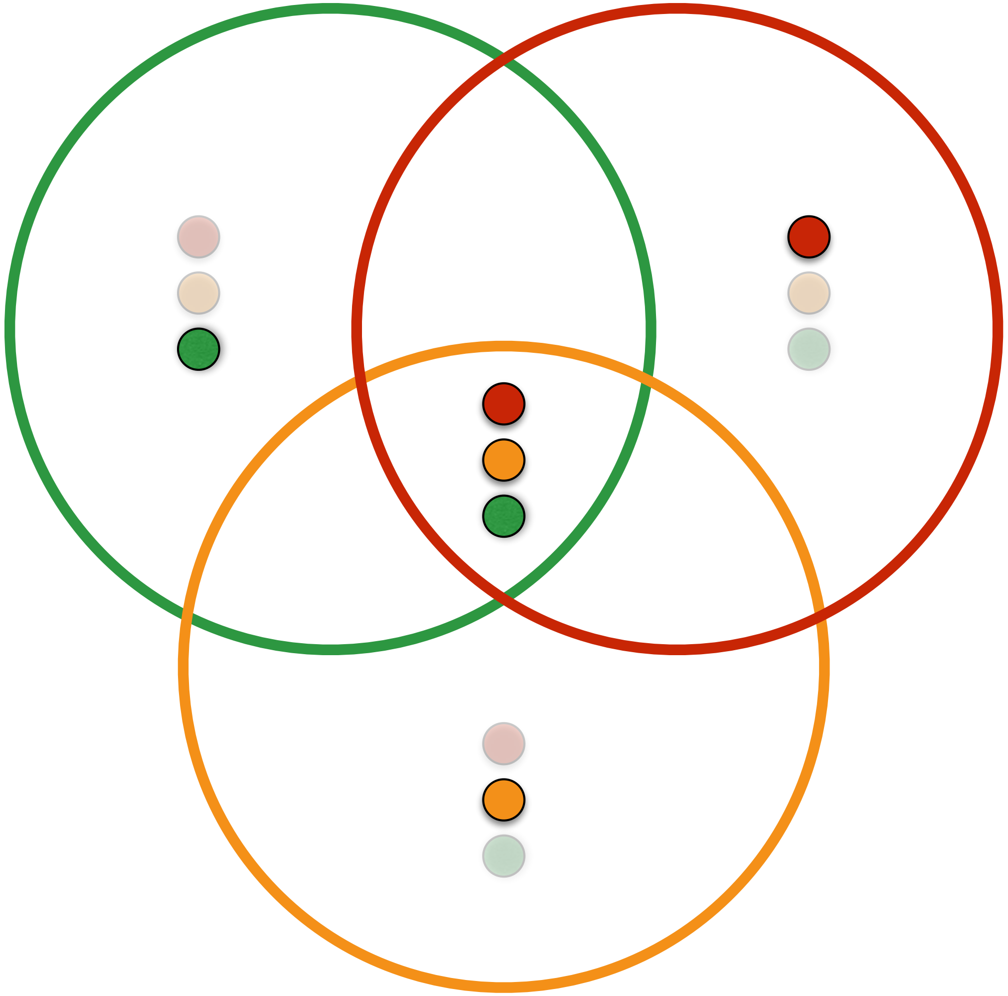
$$(x \leftrightarrow y) \leftrightarrow z = (x \oplus y) \oplus z \qquad \text{Equations used}$$

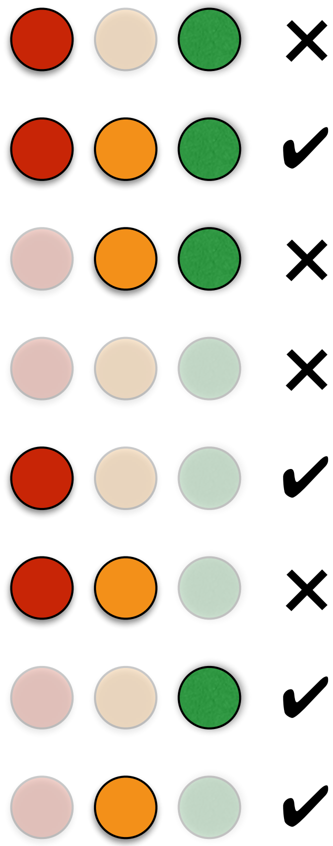
we use equations:

$$\begin{aligned}(x \leftrightarrow y) \leftrightarrow z &= \neg(x \leftrightarrow y) \leftrightarrow \neg z && (a \leftrightarrow b = \neg a \leftrightarrow \neg b)\\ &= (x \oplus y) \leftrightarrow \neg z && (\neg(a \leftrightarrow b) = a \oplus b)\\ &= (x \oplus y) \oplus z && (a \leftrightarrow \neg b = a \oplus b)\end{aligned}$$

$$\{x \mid G(x) \leftrightarrow R(x) \leftrightarrow A(x)\}$$

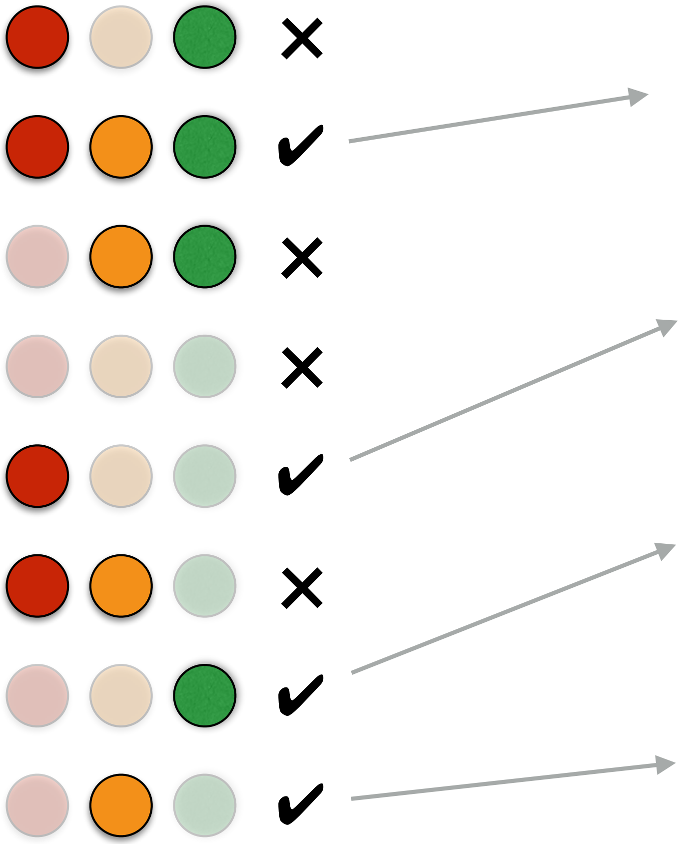
The meaning of an expression is the set of states in which it is true.





$$\{x \mid G(x) \leftrightarrow R(x) \leftrightarrow A(x)\}$$

# Disjunctive Normal Form (DNF)



$$R(x) \wedge A(x) \wedge G(x)$$

$$\vee$$

$$R(x) \wedge \neg A(x) \wedge \neg G(x)$$

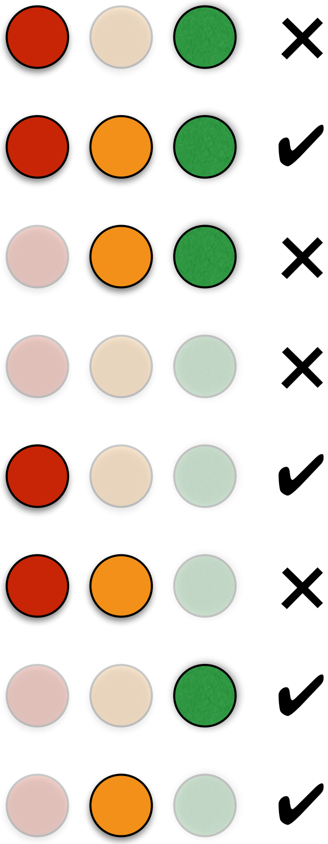
$$\vee$$

$$\neg R(x) \wedge \neg A(x) \wedge G(x)$$

$$\vee$$

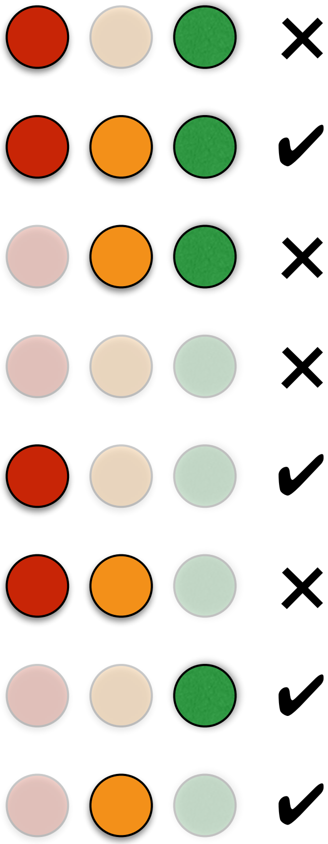
$$\neg R(x) \wedge A(x) \wedge \neg G(x)$$

$$\{x \mid G(x) \leftrightarrow R(x) \leftrightarrow A(x)\}$$



$$\neg\Big( R(x) \wedge \neg A(x) \wedge G(x) \\ \vee \\ \neg R(x) \wedge A(x) \wedge G(x) \\ \vee \\ \neg R(x) \wedge \neg A(x) \wedge \neg G(x) \\ \vee \\ R(x) \wedge A(x) \wedge \neg G(x) \Big)$$

$$\{x \mid G(x) \leftrightarrow R(x) \leftrightarrow A(x)\}$$



$$\neg\big(R(x) \wedge \neg A(x) \wedge G(x)\big)$$

$$\wedge$$

$$\neg\big(\neg R(x) \wedge A(x) \wedge G(x)\big)$$

$$\wedge$$

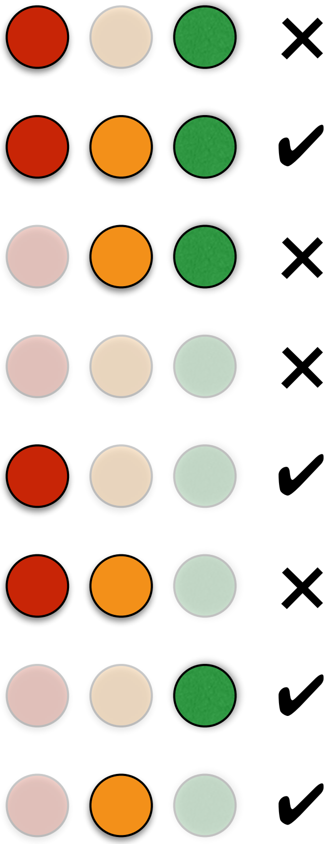
$$\neg\big(\neg R(x) \wedge \neg A(x) \wedge \neg G(x)\big)$$

$$\wedge$$

$$\neg\big(R(x) \wedge A(x) \wedge \neg G(x)\big)$$

$$\{x \mid G(x) \leftrightarrow R(x) \leftrightarrow A(x)\}$$

## Conjunctive Normal Form (CNF)



$$
\begin{aligned}
&(\neg R(x) \vee A(x) \vee \neg G(x)) \\
&\wedge \\
&(R(x) \vee \neg A(x) \vee \neg G(x)) \\
&\wedge \\
&(R(x) \vee A(x) \vee G(x)) \\
&\wedge \\
&(\neg R(x) \vee \neg A(x) \vee G(x))
\end{aligned}
$$

# Is this a valid argument?

- Assumptions:
  If I am clever then I will pass
  If I will pass then I am clever,
  Either I am clever or I will pass

- Conclusion:
  I am clever and I will pass

# Is this a valid argument?

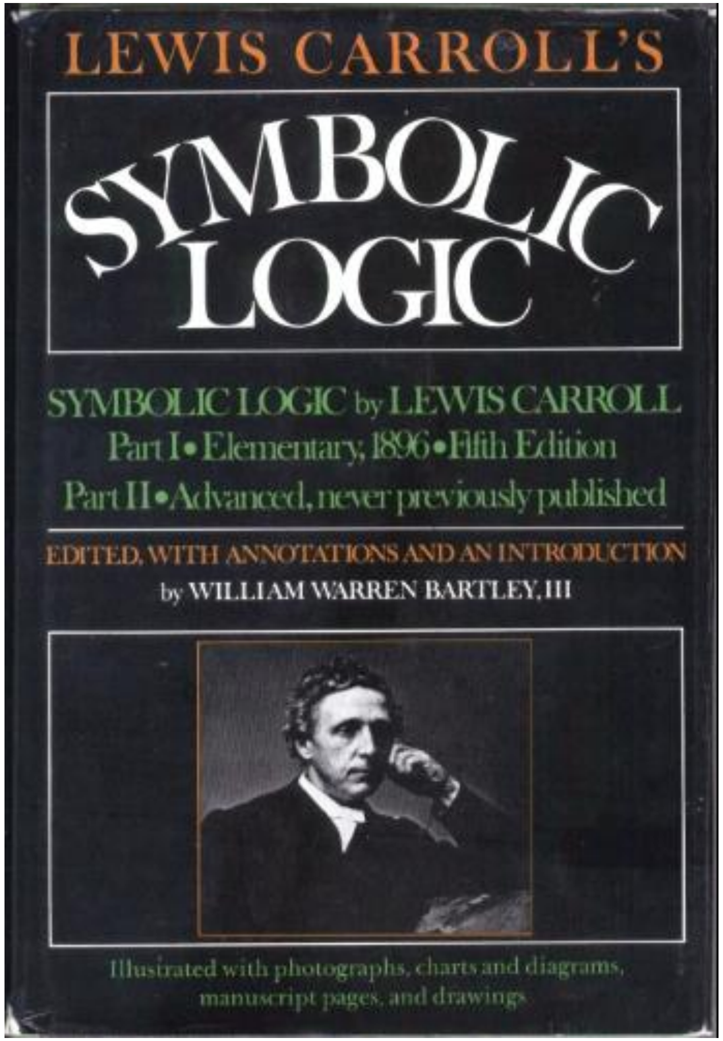
- Assumptions:

  If the races are fixed or the gambling houses are crooked, then the tourist trade will decline.
  If the tourist trade declines then the police force will be happy.
  The police force is never happy.

- Conclusion:

  The races are not fixed

$$\dfrac{\dfrac{\dfrac{(\mathrm{RF} \lor \mathrm{GC}) \to \mathrm{TT} \qquad \dfrac{\mathrm{TT} \to \mathrm{PH} \qquad \neg \mathrm{PH}}{\neg \mathrm{TT}}}{\neg(\mathrm{RF} \lor \mathrm{GC})}}{\neg \mathrm{RF} \land \neg \mathrm{GC}}}{\neg \mathrm{RF}}$$




A Syllogism worked out.

That story of yours, about your once meeting the sea-serpent, always sets me off yawning;
I never yawn, unless when I'm listening to something totally devoid of interest.

http://www.gutenberg.org/ebooks/28696

82. Some of these shops are not crowded;
    No crowded shops are comfortable.

83. Prudent travelers carry plenty of small change;
    Imprudent travelers lose their luggage.

84. Some geraniums are red;
    All these flowers are red.

85. None of my cousins are just;
    All judges are just.

No crowded shops are comfortable.

$$\mathrm{Crowded}(s) \rightarrow \neg\mathrm{Comfortable}(s)$$

# No crowded shops are comfortable.

The expression $\qquad \text{Crowded}(s) \to \neg\text{Comfortable}(s)$

means $\qquad \big\{s \mid \text{Crowded}(s) \to \neg\text{Comfortable}(s)\big\}$

To make the universal statement that *all* crowded shops are uncomfortable,

we write, $\qquad \forall s.\, \text{Crowded}(s) \to \neg\text{Comfortable}(s)$

which means, $\qquad \big\{s \mid \text{Crowded}(s) \to \neg\text{Comfortable}(s)\big\} = S,$

where $S$ is the set of all shops.

# No crowded shops are comfortable.

To make the existential statement that *some* crowded shops are comfortab
we introduce a third expression:

$$\text{we write,} \qquad \exists s.\, \big(\mathrm{Crowded}(s) \wedge \mathrm{Comfortable}(s)\big),$$

$$\text{which means,} \qquad \big\{s \mid \mathrm{Crowded}(s) \wedge \mathrm{Comfortable}(s)\big\} \neq \emptyset\,,$$

where $\emptyset$ is the empty set.

Exercise 2.5

# Crowded is not Comfortable vs Crowded but Comfortable

$$\text{If,}\quad \text{CnC} = \big\{ s \mid \text{Crowded}(s) \to \neg\text{Comfortable}(s) \big\}\,,$$
$$\text{If,}\quad \text{CbC} = \big\{ s \mid \text{Crowded}(s) \wedge \text{Comfortable}(s) \big\}\,,$$

$$\text{then,}\quad \forall s.\,\big(\text{Crowded}(s) \to \neg\text{Comfortable}(s)\big) \qquad \text{means, CnC} = S\,,$$

$$\text{and,}\quad \exists s.\,\big(\text{Crowded}(s) \wedge \text{Comfortable}(s)\big) \qquad \text{means, CbC} \neq \emptyset.$$
$$\text{So,}\quad \neg\exists s.\,\big(\text{Crowded}(s) \wedge \text{Comfortable}(s)\big) \qquad \text{means, CbC} = \emptyset.$$

Check that $\text{CnC} = S$ iff $\text{CbC} = \emptyset$ .