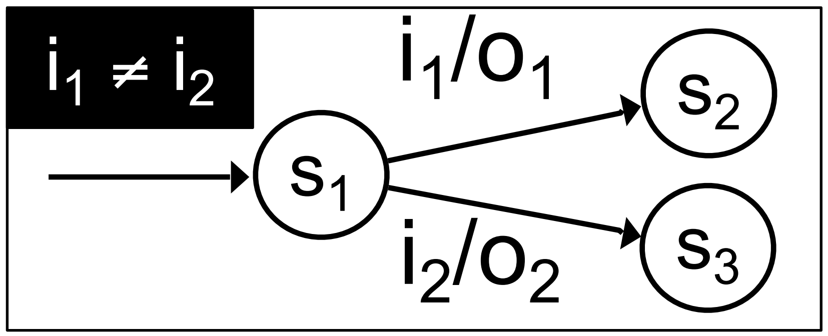
Languages and Automata

cl

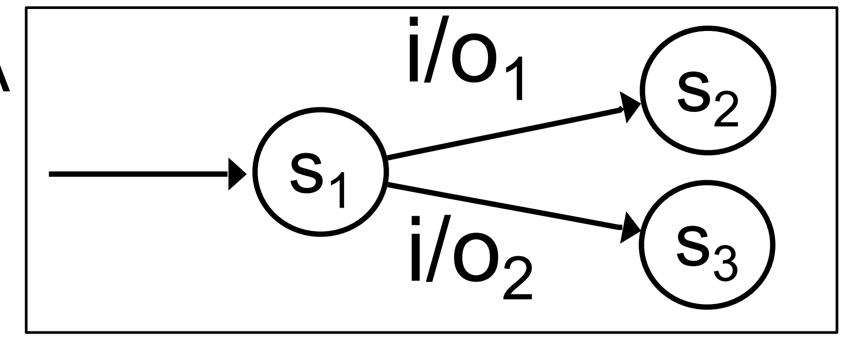
- deterministic machines
- languages and machines
- the Boolean algebra of languages
- non-determinism

Determinism

DFA



NFA

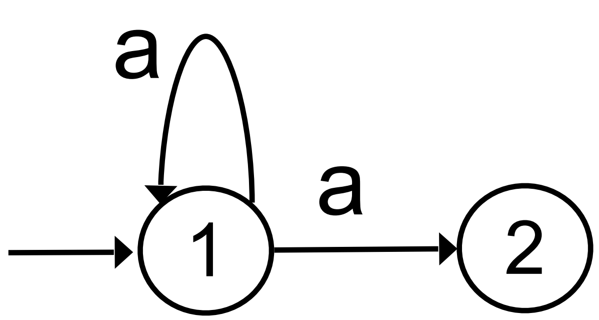


- In a deterministic machine (DFA), all states have no more than one transition leaving the state for each input symbol.
- In a non-deterministic machine (NFA), each state may have any number of transitions leaving to different successor states for the same input symbol.
- Sometimes NFA are easier to define.
- Can always convert from a NFA to a deterministic DFA.

Determinism and Traces

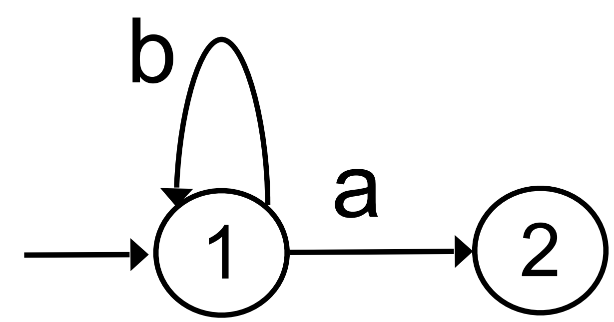
A FSM, M, is deterministic if for every string $x \in \Sigma^*$ there is at most one trace for x in M (where Σ^* is the set of all strings in alphabet of M)

non-deterministic (choice)



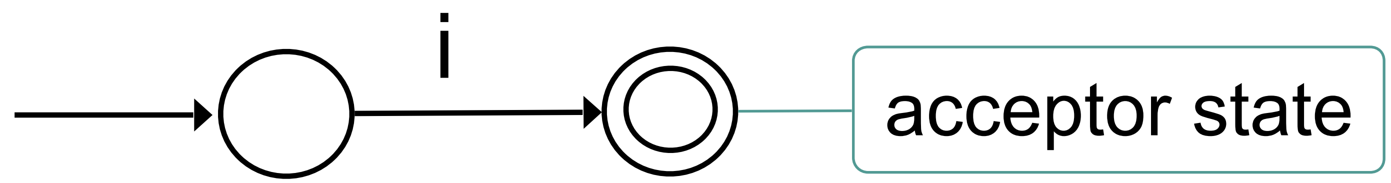
Sequence	Trace
aa	[1,a,1,a,2]
aa	[1,a,1,a,1]

deterministic (no choice)



Sequence	Trace
ba	[1,b,1,a,2]
bb	[1,b,1,b,1]

The language accepted by a machine



Input sequence is accepted if there is a trace from the initial state to an acceptor state.

Language of the FSM is the set of sequences it accepts.

$$L(M) \subseteq \Sigma^*$$

Two machines are **equivalent** if they define the same language.

Regular languages

Language of the FSM is the set of sequences it accepts.

$$L(M) \subseteq \Sigma^*$$

We say $A \subseteq \Sigma^*$ is **regular** iff

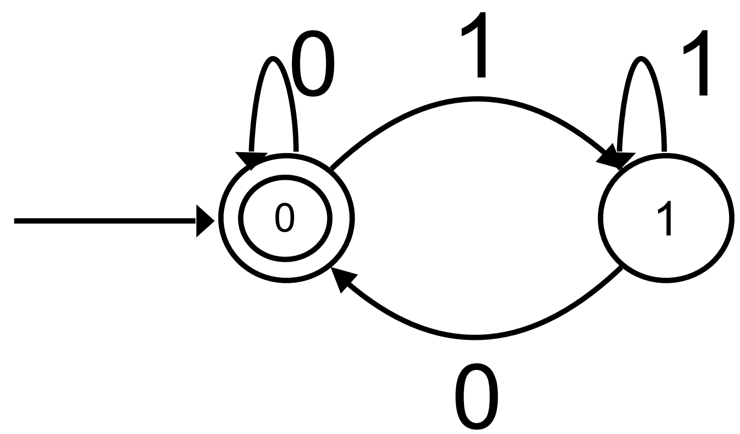
$$A = L(M)$$

for some machine M.

A language is regular iff it is the language accepted by some FSM

Two examples

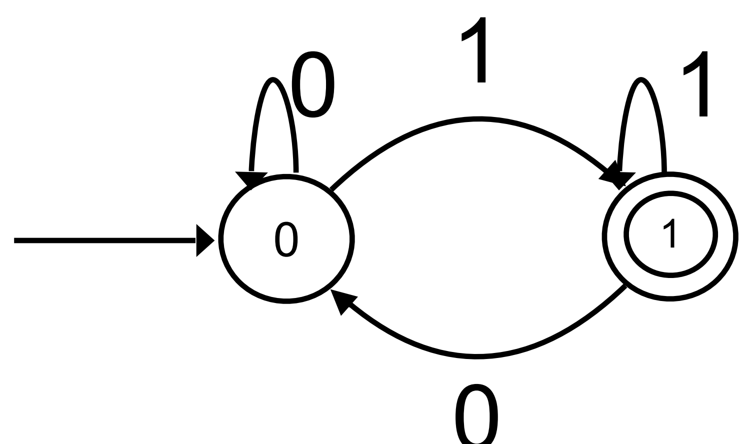
	×2	×2 + 1
	0	**1**
0	0	1
1	0	1



Even binary numbers

Input sequence is accepted if it ends with a zero.

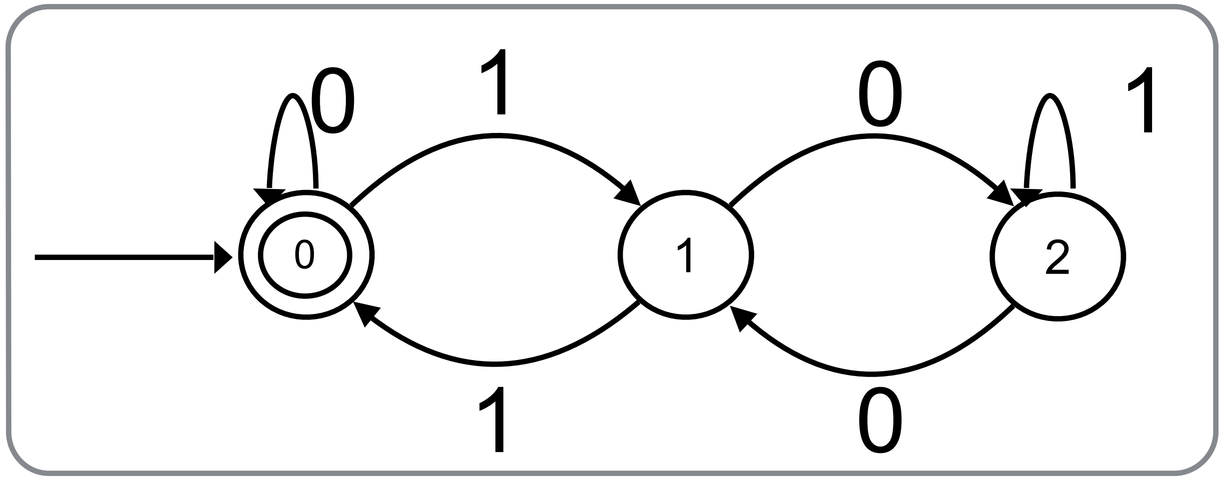
	×2	×2 + 1
	0	**1**
0	0	1
1	0	1



Odd binary numbers

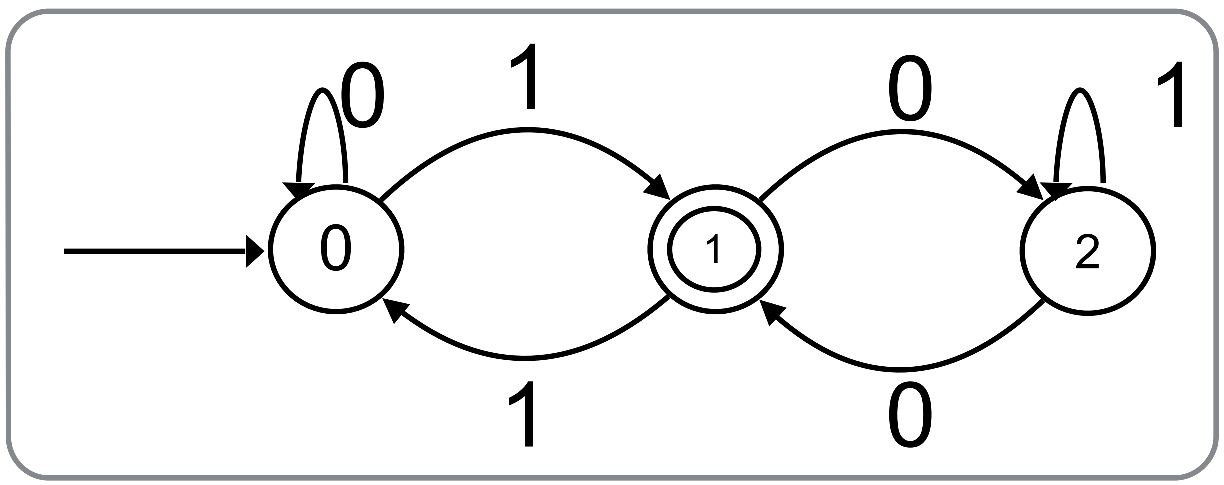
Input sequence is accepted if it ends with a one.

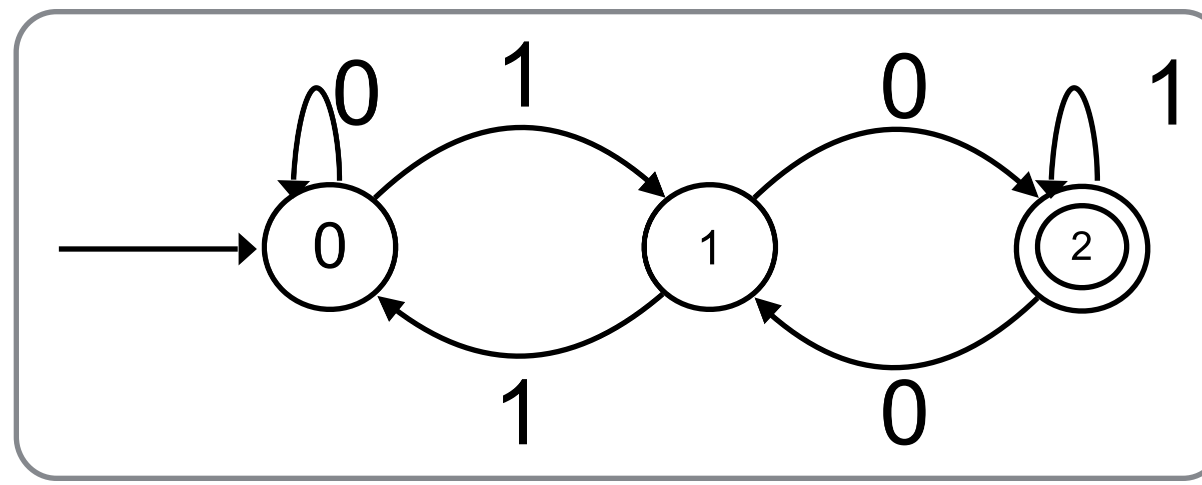
Three examples



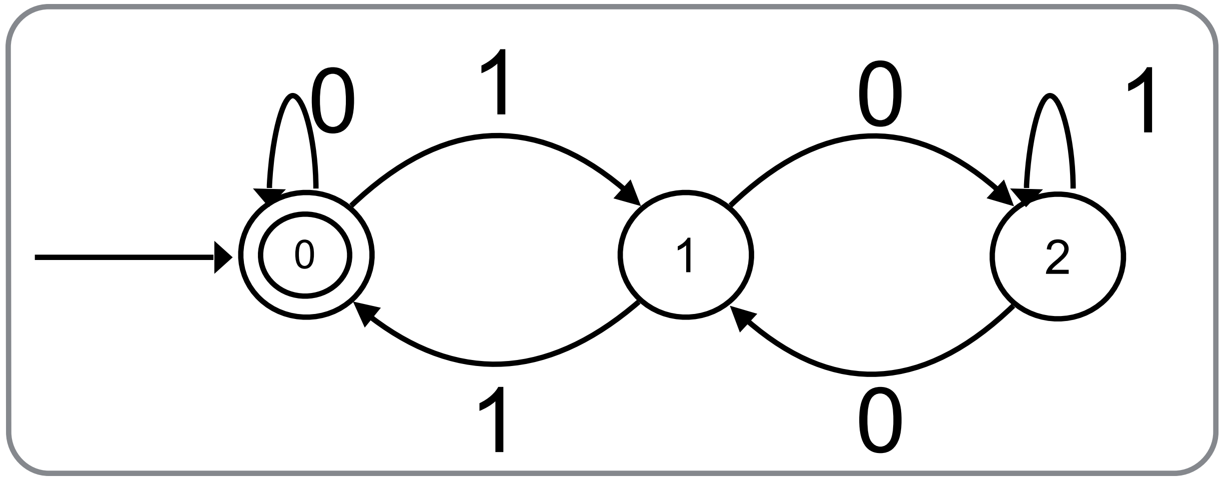
Which binary numbers are accepted?

	×2	×2 + 1
mod 3	**0**	**1**
0	0	1
1	2	0
2	1	2

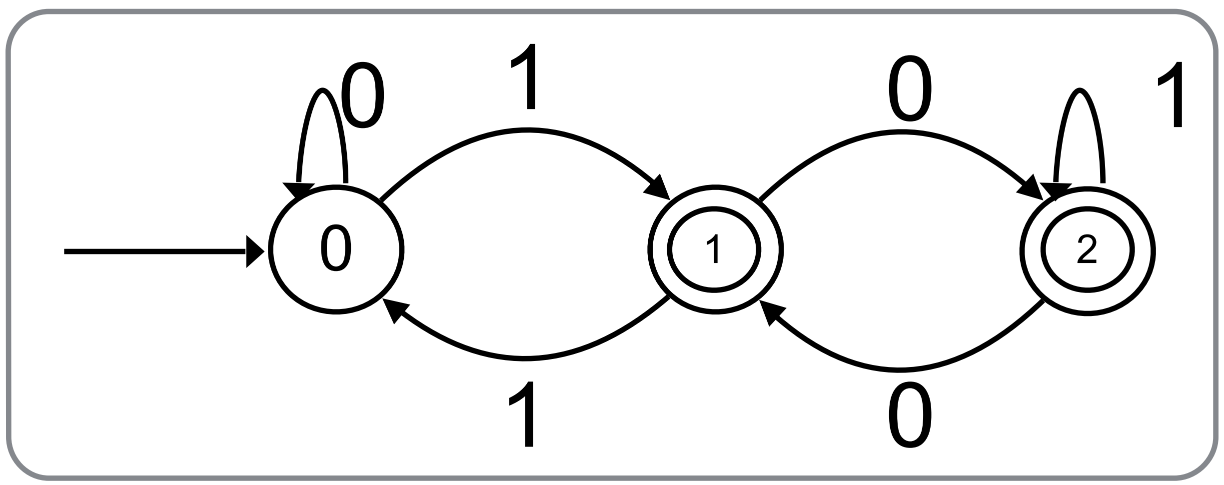




By three or not by three?

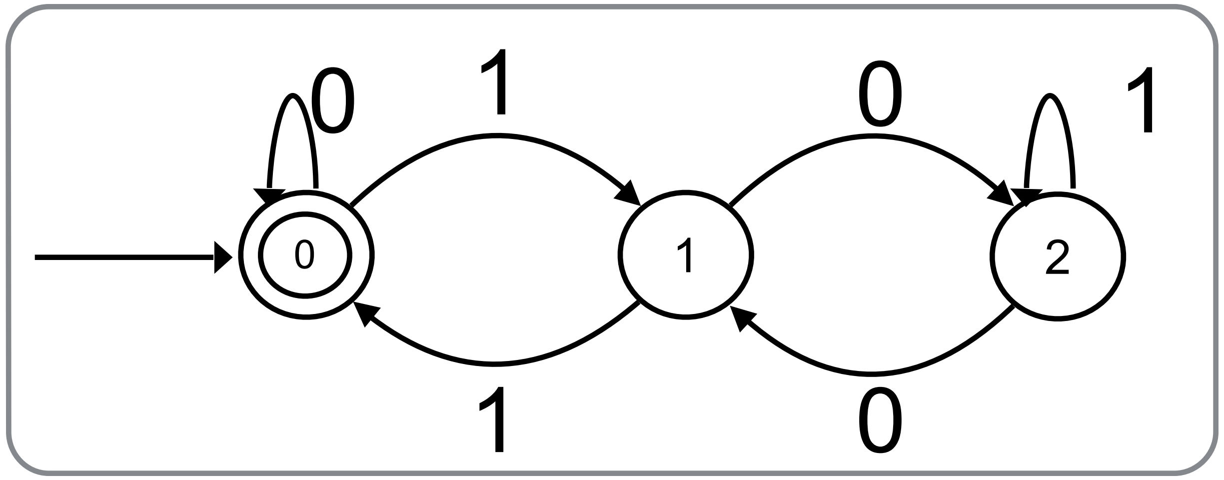


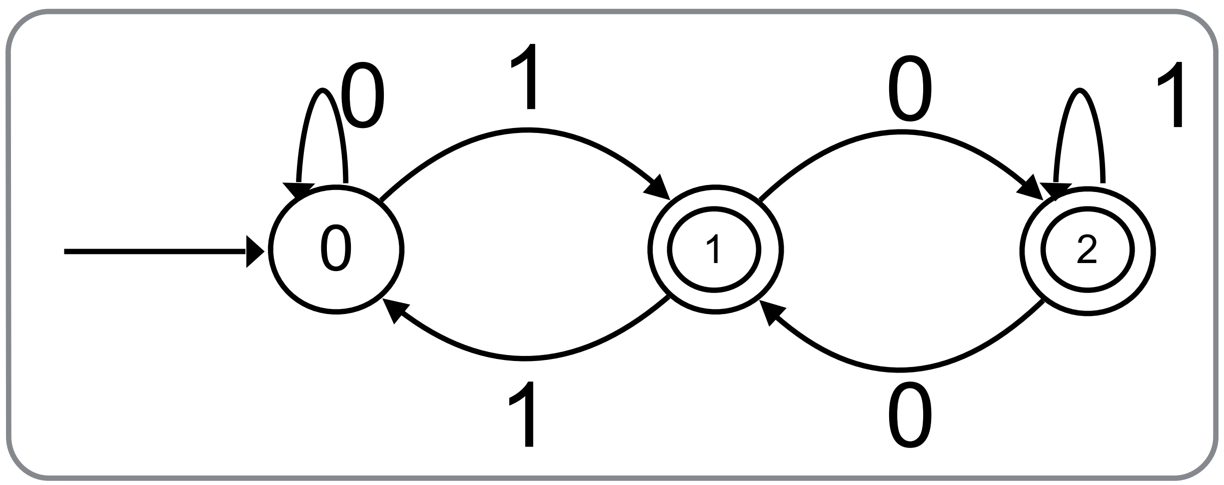
divisible by three



not
divisible by three

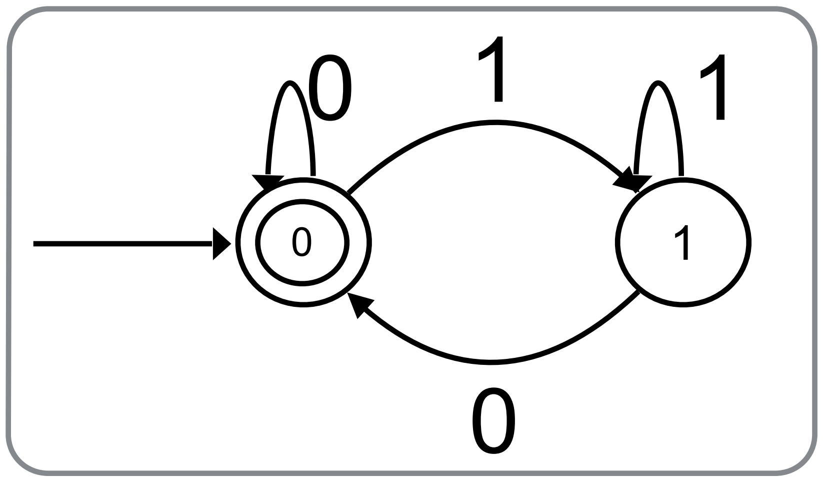
The complement of a regular language is regular

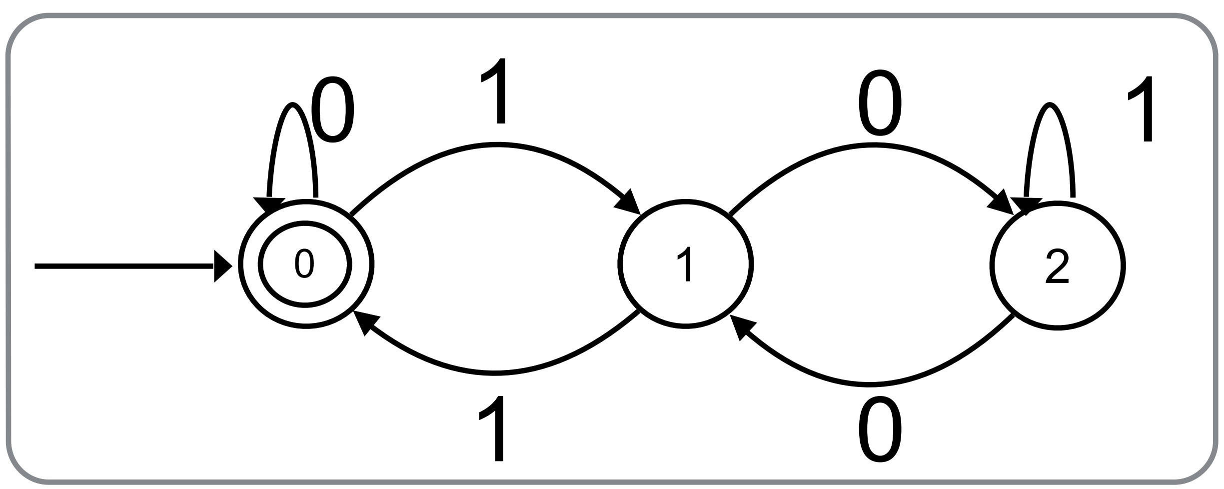




If $A \subseteq \Sigma^*$ is recognised by M then $\overline{\mathbf{A}} = \Sigma^* \setminus A$ is recognised by $\overline{\mathbf{M}}$ where $\overline{\mathbf{M}}$ and **M** are identical except that the accepting states of $\overline{\mathbf{M}}$ are the non-accepting states of **M** and vice-versa

The intersection of two regular languages is regular





divisible by 6

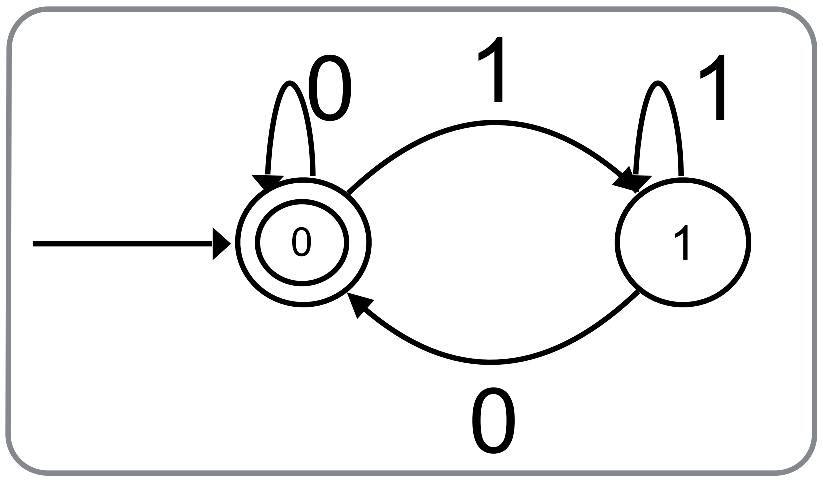
≣

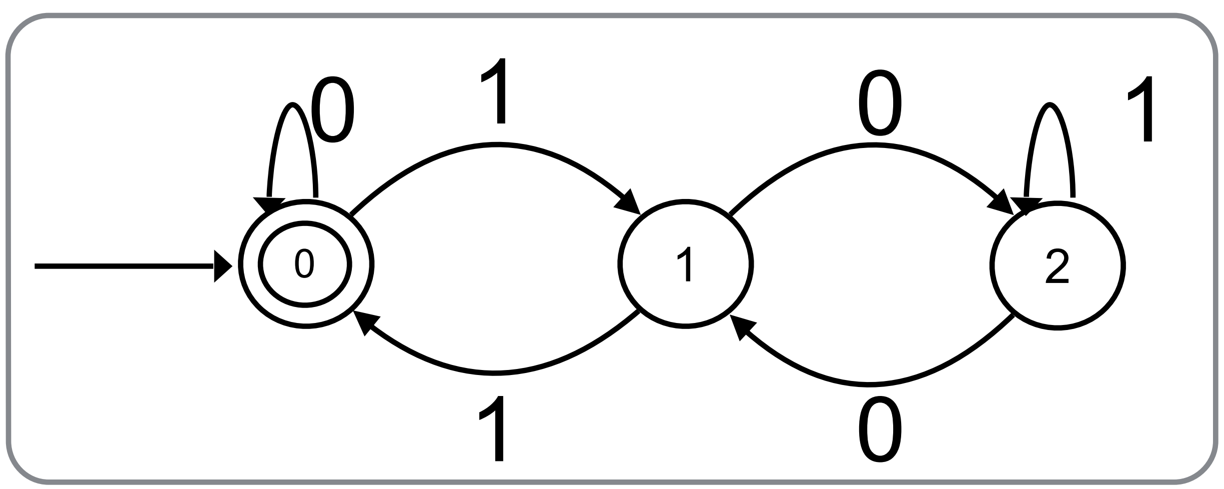
divisible by 2

and

divisible by 3

The intersection of two regular languages is regular



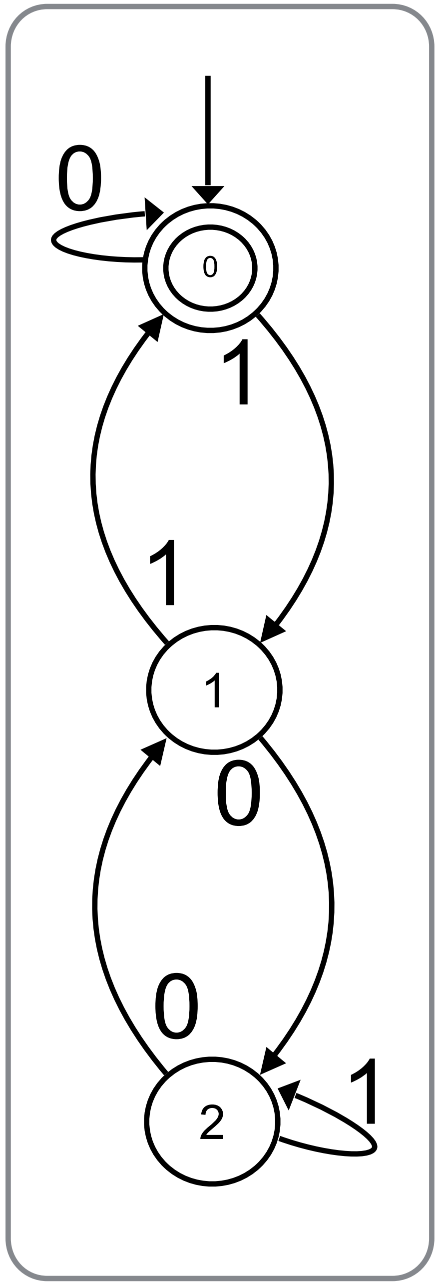


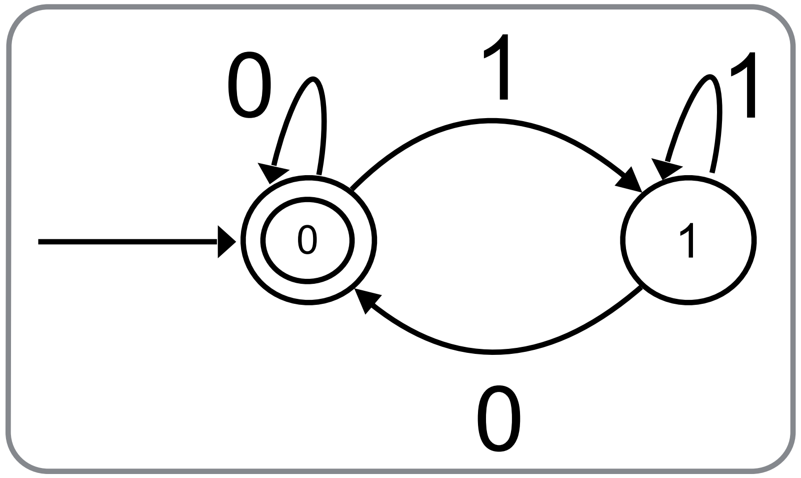
Run both machines in parallel?

Build one machine that simulates two machines running in parallel!

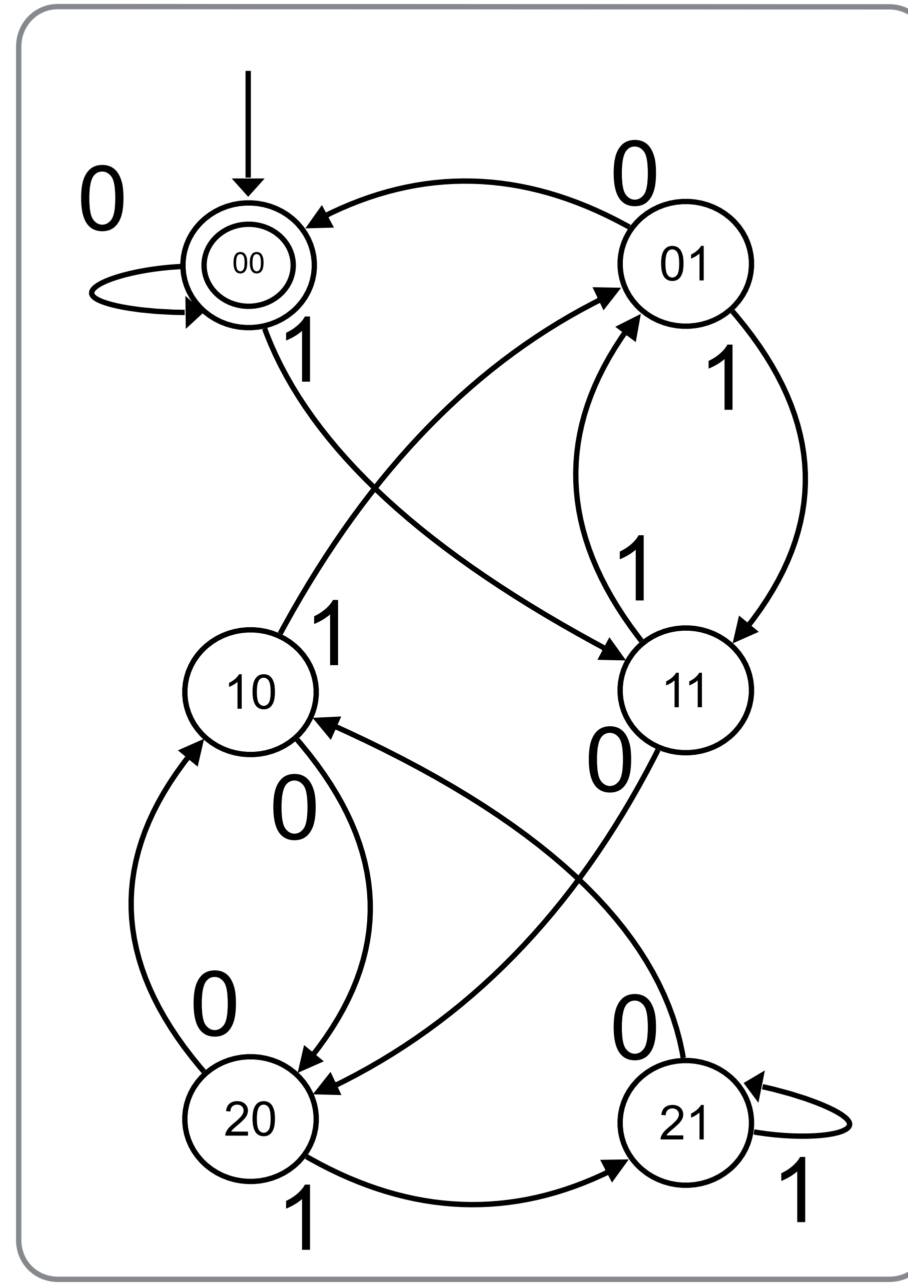
Keep track of the state of each machine.

The intersection of two regular languages is regular

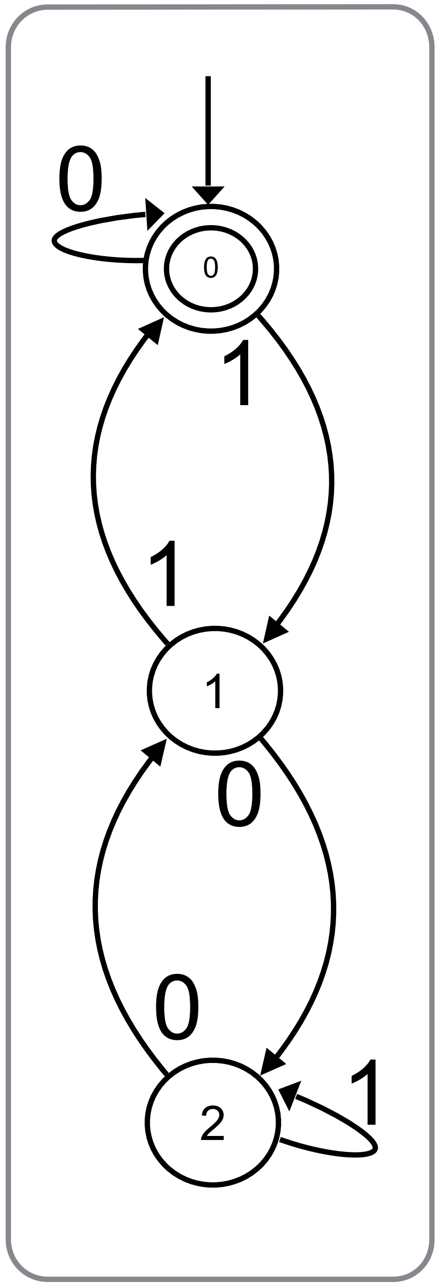


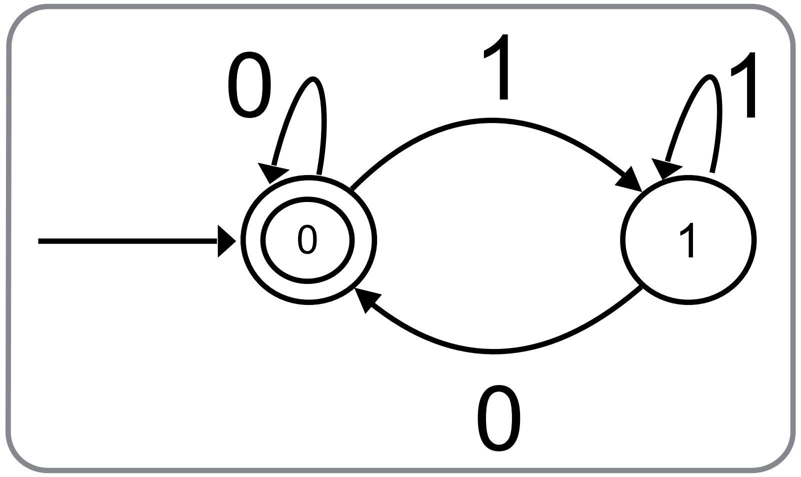


mod 3	mod 2	0: 3	0: 2	1: 3	1: 2
0	0	0	0	1	1
1	0	2	0	0	1
2	0	1	0	2	1
0	1	0	0	1	1
1	1	2	0	0	1
2	1	1	0	2	1

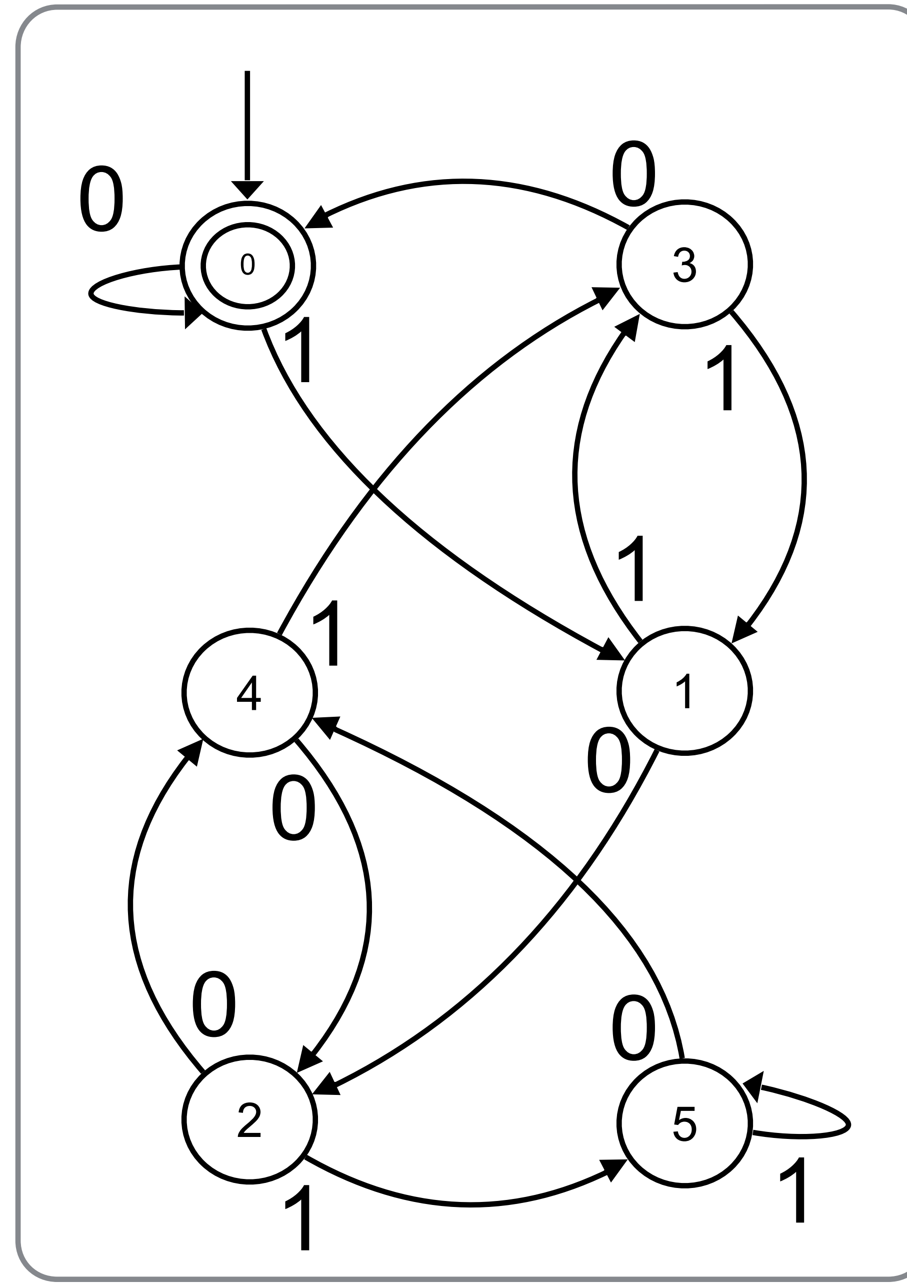


The intersection of two regular languages is regular

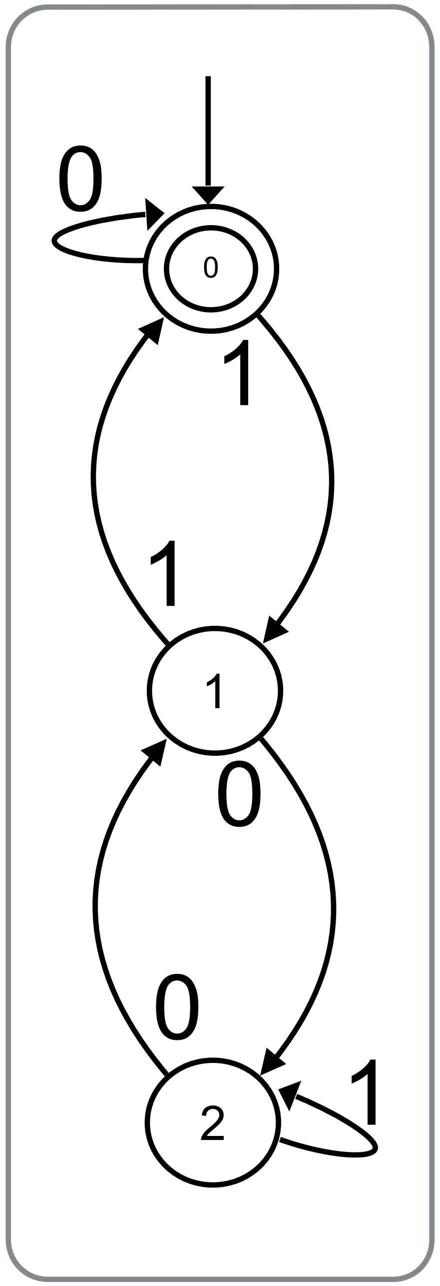


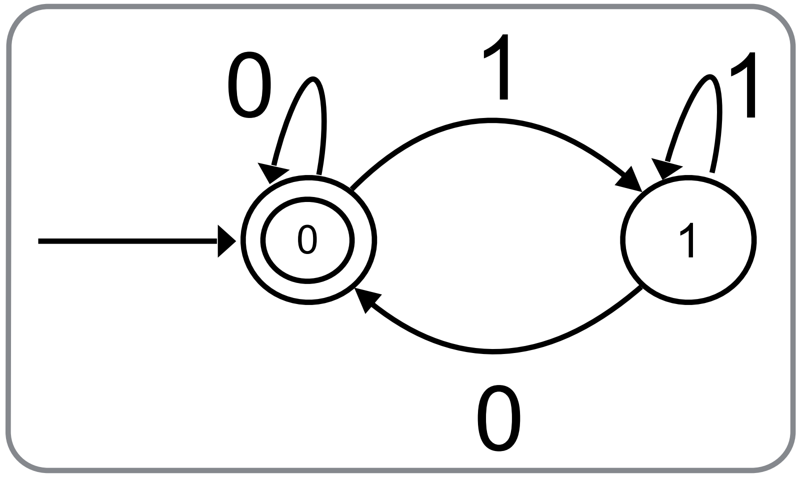


mod 3	mod 2	mod 6	0			1		
			3	2	6	3	2	6
0	0	0	0	0	0	1	1	1
1	0	4	2	0	2	0	1	3
2	0	2	1	0	4	2	1	5
0	1	3	0	0	0	1	1	1
1	1	1	2	0	2	0	1	3
2	1	5	1	0	4	2	1	5

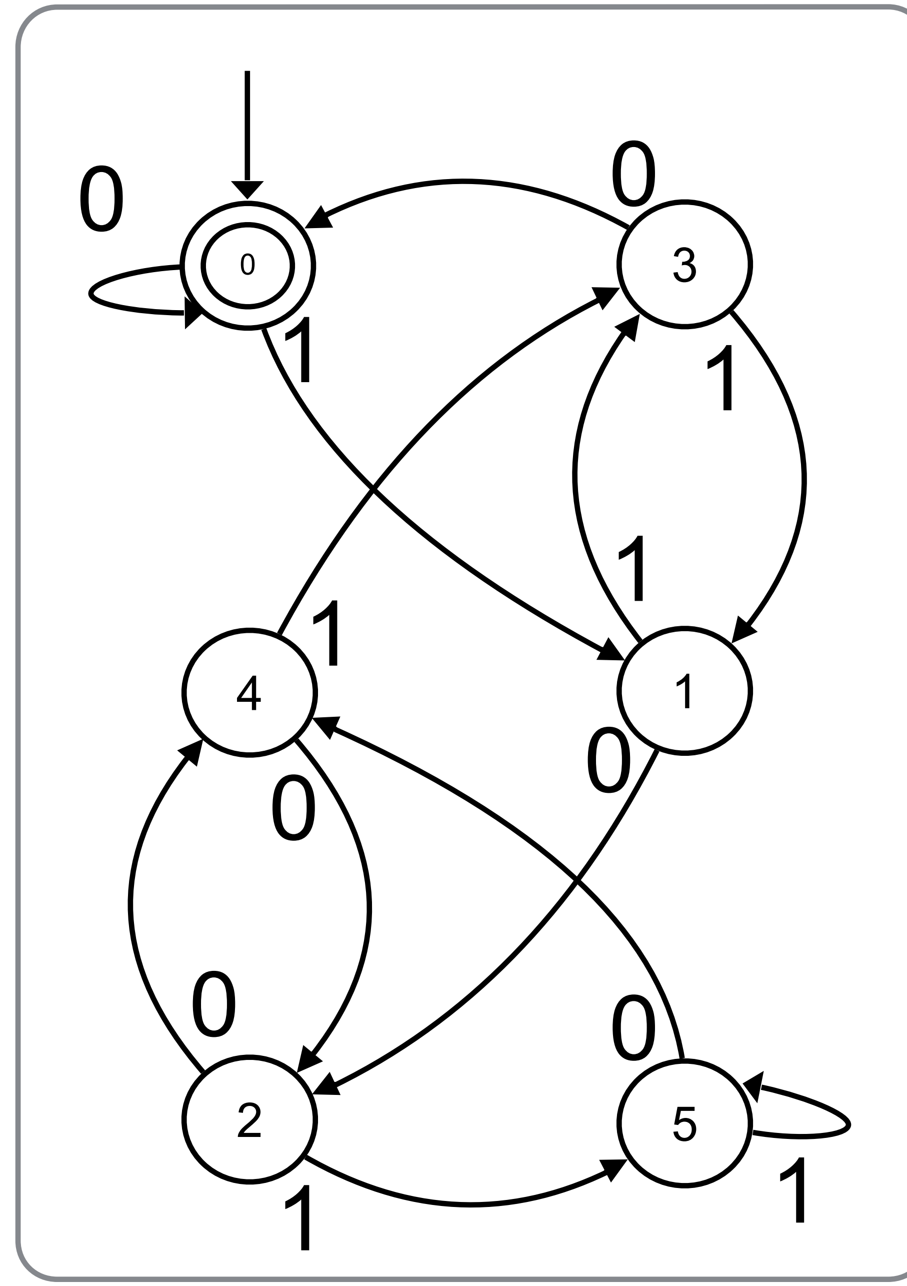


The intersection of two regular languages is regular





mod 3	mod 2	mod 6	0: 3	0: 2	0: 6	1: 3	1: 2	1: 6
0	0	0	0	0	0	1	1	1
1	1	1	2	0	2	0	1	3
2	0	2	1	0	4	2	1	5
0	1	3	0	0	0	1	1	1
1	0	4	2	0	2	0	1	3
2	1	5	1	0	4	2	1	5



Determinism

Can always convert to an equivalent DFA for which every state has exactly one transition leaving the state for each input symbol.

For this machine there is exactly one trace for each input string

- Proof

  Add a new "black hole" state, ●

  For every pair (s, a) for which there is no state t with a transition T(s, a, t), add a transition T(s, a, ●).

  This includes a transition T(●, a, ●) for each $a \in \Sigma$. You cannot escape from the black hole.

  The black hole ● is not an accepting state.

This machine accepts the same language as the original.