Informatics 1

Lecture 12 Inference

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Is this a valid argument?

- Assumptions:

  If the races are fixed or the gambling houses are crooked, then the tourist trade will decline.
  If the tourist trade declines then the police force will be happy.
  The police force is never happy.

- Conclusion:

  The races are not fixed

Assumptions: If the races are fixed or the gambling houses are crooked, then the tourist trade will decline.
If the tourist trade declines then the police force will be happy.
The police force is never happy.

Conclusion: The races are not fixed.

$$\dfrac{\dfrac{(\mathrm{RF} \lor \mathrm{GC}) \to \mathrm{TT} \qquad \dfrac{\mathrm{TT} \to \mathrm{PH} \quad \neg \mathrm{PH}}{\neg \mathrm{TT}}}{\neg(\mathrm{RF} \lor \mathrm{GC})}}{\dfrac{\neg \mathrm{RF} \land \neg \mathrm{GC}}{\neg \mathrm{RF}}}$$

$$\mathrm{RF} \lor \mathrm{VC} \to \mathrm{TTD},\ \mathrm{TTD} \to \mathrm{PH},\ \neg \mathrm{PH} \vdash \neg \mathrm{RF}$$

$$\frac{X \rightarrow Y \quad \neg Y}{\neg X} \; \textit{modus tollendo tollens}$$

$$\frac{A \to B \quad \neg B}{\neg A} \; \textit{modus tollendo tollens}$$

$$\frac{\neg A \quad A \lor B}{B} \; \textit{modus tollendo ponens}$$

$$\frac{A \quad \neg(A \land B)}{\neg B} \; \textit{modus ponendo tollens}$$

$$\frac{A \quad A \to B}{B} \; \textit{modus ponendo ponens}$$

Some valid inferences

$$\frac{A \to B \quad \neg B}{\neg A} \; \textit{modus tollendo tollens}$$

$$\frac{\neg A \quad A \lor B}{B} \; \textit{modus tollendo ponens}$$

$$\frac{A \quad \neg(A \land B)}{\neg B} \; \textit{modus ponendo tollens}$$

$$\frac{A \quad A \to B}{B} \; \textit{modus ponendo ponens}$$

$$\frac{\neg A \lor B \quad \neg B}{\neg A} \; \textit{modus tollendo tollens}$$

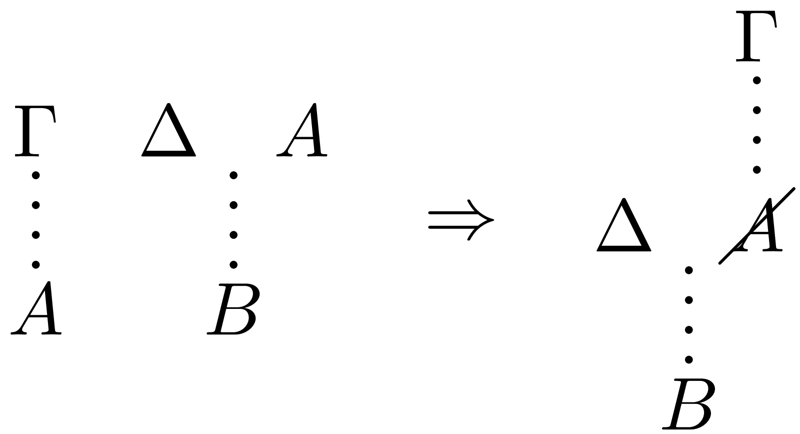
$$\frac{\neg A \quad A \lor B}{B} \; \textit{modus tollendo ponens}$$

$$\frac{A \quad \neg A \lor \neg B}{\neg B} \; \textit{modus ponendo tollens}$$

$$\frac{A \quad \neg A \lor B}{B} \; \textit{modus ponendo ponens}$$

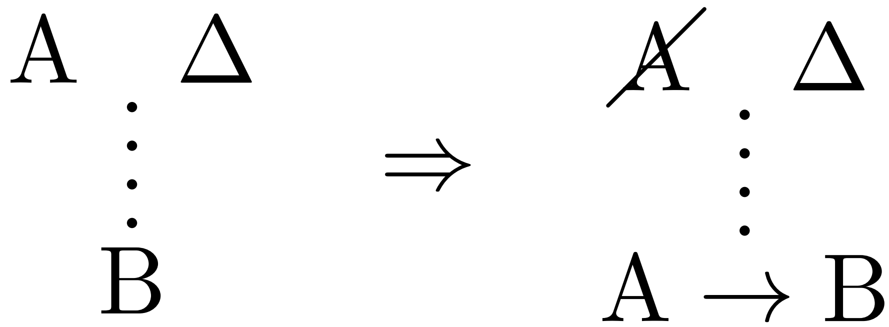
A rule of inference

$$\frac{\Gamma \vdash A \qquad \Delta, A \vdash B}{\Gamma, \Delta \vdash B}\ Cut$$



Another rule of inference

$$\frac{\Delta, A \vdash B}{\Delta \vdash A \to B} \; (\to^{+})$$



More rules

$$\frac{}{\mathcal{A}, X \vdash X}\ (I)$$

$$\frac{\mathcal{A} \vdash X \quad \mathcal{A} \vdash Y}{\mathcal{A} \vdash X \wedge Y}\ (\wedge) \qquad \frac{\mathcal{A}, X \vdash Z \quad \mathcal{A}, Y \vdash Z}{\mathcal{A}, X \vee Y \vdash Z}\ (\vee) \qquad \frac{\mathcal{A}, X \vdash Y}{\mathcal{A} \vdash X \rightarrow Y}\ (\rightarrow)$$

A simple proof

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \to (B \to C) \vdash A \to (B \to C)}\,(I)}{A \to (B \to C) A \vdash B \to C}\,(\to^{-})}{A \to (B \to C), A, B \vdash C}\,(\to^{-})}{A \to (B \to C), B \vdash A \to C}\,(\to^{+})}{A \to (B \to C) \vdash B \to (A \to C)}\,(\to^{+})$$

More rules

$$\overline{\mathcal{A}, X \vdash X} \ (I)$$

$$\frac{\mathcal{A} \vdash X \quad \mathcal{A} \vdash Y}{\mathcal{A} \vdash X \wedge Y} \ (\wedge) \qquad \frac{\mathcal{A}, X \vdash Z \quad \mathcal{A}, Y \vdash Z}{\mathcal{A}, X \vee Y \vdash Z} \ (\vee) \qquad \frac{\mathcal{A}, X \vdash Y}{\mathcal{A} \vdash X \to Y} \ (\to)$$

Can we prove $X \wedge Y \vdash X \vee Y$?

Another Proof

$$\dfrac{\dfrac{}{A \wedge B \vdash A \wedge B}\,(I)}{A \wedge B \vdash A}\,(\wedge^{-}) \qquad \dfrac{\dfrac{}{A \vee B \vdash A \vee B}\,(I)}{A \vdash A \vee B}\,(\vee^{-})$$

$$\dfrac{A \wedge B \vdash A \qquad A \vdash A \vee B}{A \wedge B \vdash A \vee B}\,Cut$$

Gentzen's rules (1)

$$\frac{}{\Gamma, A \vdash \Delta, A} \ (I)$$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \ (\wedge L1) \qquad \frac{\Gamma \vdash A, \Delta}{\Gamma \vdash A \vee B, \Delta} \ (\vee R1)$$

$$\frac{\Gamma, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \ (\wedge L2) \qquad \frac{\Gamma \vdash B, \Delta}{\Gamma \vdash A \vee B, \Delta} \ (\vee R2)$$

A proof

$$\dfrac{\dfrac{\overline{B \vdash B}\ (I)}{A \wedge B \vdash B}\ (\wedge L2)}{A \wedge B \vdash A \vee B}\ (\vee R2)$$

A rule

$$\frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta} \ (\to R)$$

A valuation is a counterexample to the top line iff it is a counterexample to the bottom line

Another rule

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \to B \vdash \Delta} \ (\to L)$$

A valuation is a counterexample to the bottom line
iff it is a counterexample to
at least one of the entailments on the top line

$$\frac{}{\Gamma, A \vdash \Delta, A}\ (I)$$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ (\wedge L1) \qquad \frac{\Gamma \vdash A, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R1)$$

$$\frac{\Gamma, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ (\wedge L2) \qquad \frac{\Gamma \vdash B, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R2)$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ (\wedge R)$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \to B \vdash \Delta}\ (\to L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta}\ (\to R)$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ (\neg R)$$