Informatics 1

Lecture 10 DPLL & Watched Literals

Michael Fourman

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...Edgar to trace the gradual...
...m illiteracy to literacy.

Literal (li·tĕrăl), *a.* and *sb.* Fo[rms]: ...teral, (5, 6 lyt(t)urall, 6 lyt(t)ar-... lit(t)erall, 4– **literal.** [a. OF. lit... ...teral); ad. L. *litterālis*, f. *littera* LETTE[R]...

A. *adj.*

1. Of or pertaining to letters of the a... of the nature of letters, alphabetical; †... by letters, written. † Of a verse = A...

*c*1475 *Partenay* 6605 And so hau... entent, With litterall carectes fo... *Ess. Poesie* (Arb.) 63 Be Lit... pairt of zour lyne, sall ry... ...ne rynnis vpon F... ...amden) 15 Wh... ...ment of...

Once upon a time …

a Queen ruled a Kingdom. Her capital city was surrounded by seven mountains. To encourage outdoor exercise she announced a grand prize.

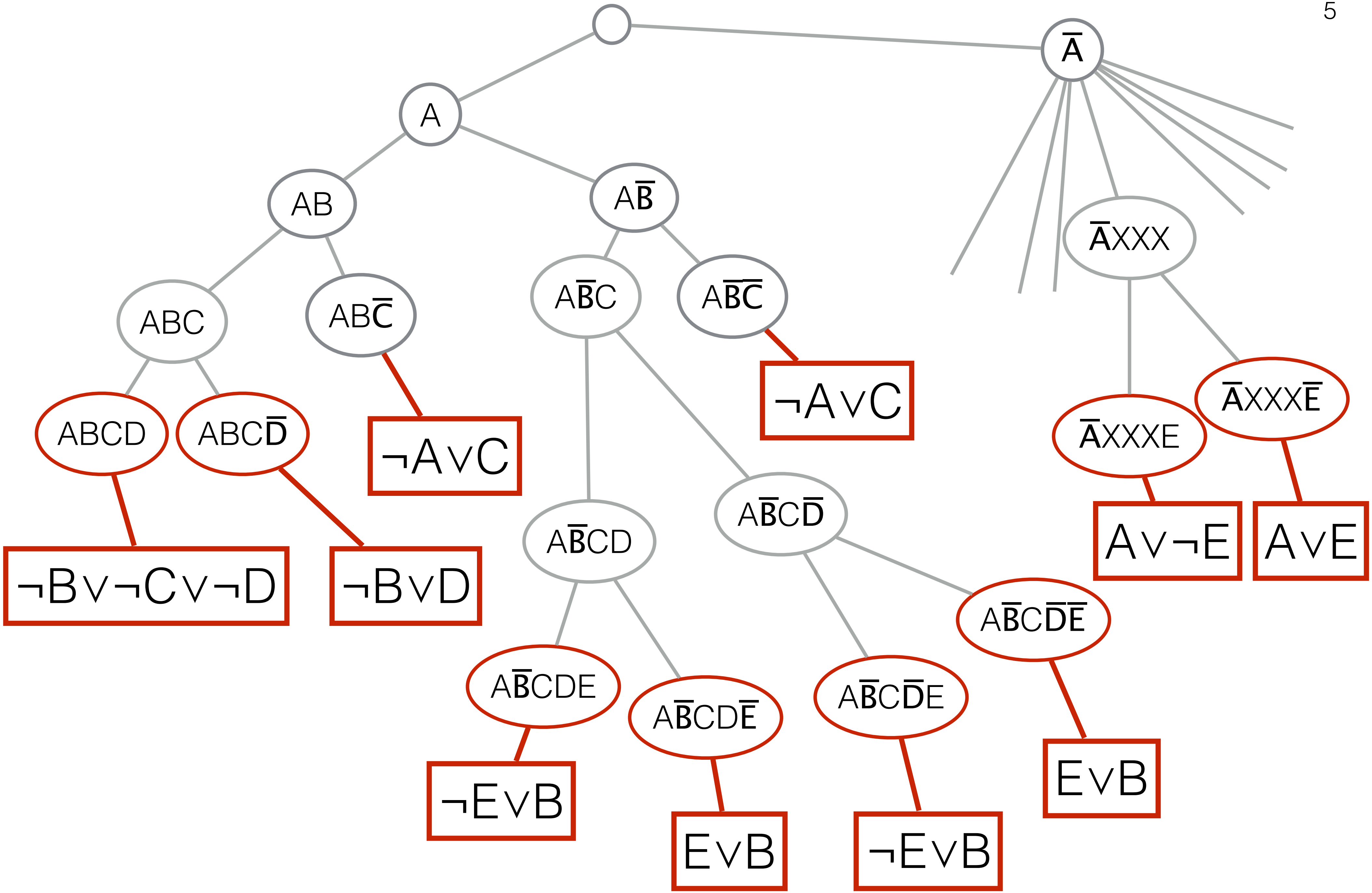
From time to time, a courtier would visit one of the peaks, and carve a special sign, never the same peak twice, never more than one peak a day (and normally none) until every peak had been marked.

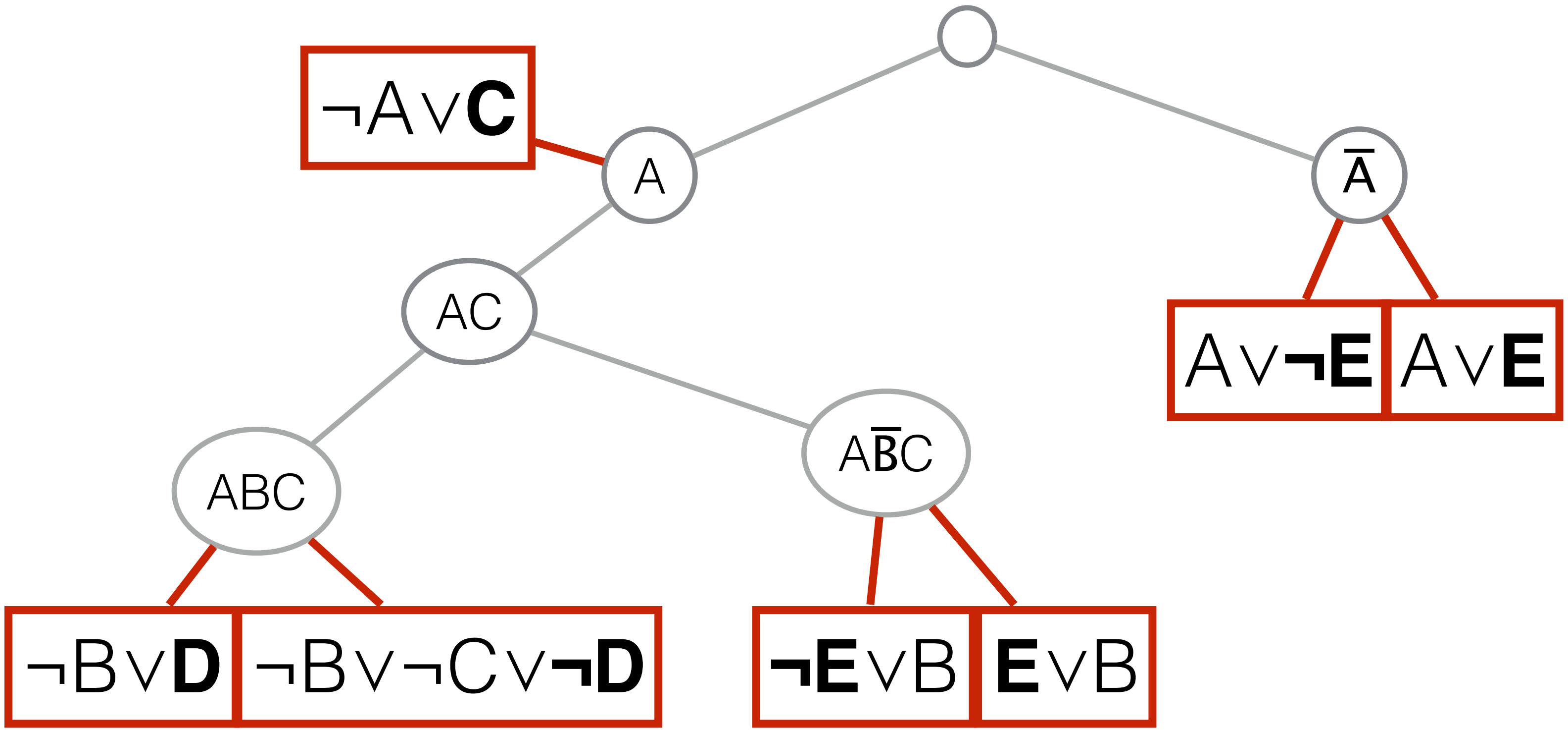
The first person to report the completion of all seven peaks would get the grand prize (a golden egg). To discourage cheating, anyone falsely reporting before the final peak was signed would be beheaded.

It is possible, but arduous, to visit all seven peaks in one day. A trip to the top of any one of the peaks is an easy walk. Explain how two good friends might plan to win the prize, without too much exertion. Assume that one, but not both can spend the night on a mountain, if necessary, but your answer should minimise the number of nights.

Davis Putnam Logemann Loveland (DPLL)

```
function DPLL(Φ)
  if Φ is a consistent set of literals
     then return true;
  if Φ contains an empty clause
     then return false;
  for every unit clause l in Φ
    Φ ← unit-propagate(l, Φ);
  l ← choose-literal(Φ);
  return DPLL(Φ ∪ {l}) or DPLL(Φ ∪ {not(l)});
```


$\bar{A}$
A
AB
$A\bar{B}$
ABC
$AB\bar{C}$
$A\bar{B}C$
$A\bar{B}\bar{C}$
$\bar{A}XXX$
ABCD
$ABC\bar{D}$
$\neg A \vee C$
$\neg A \vee C$
$\bar{A}XXXE$
$\bar{A}XXX\bar{E}$
$\neg B \vee \neg C \vee \neg D$
$\neg B \vee D$
$A\bar{B}CD$
$A\bar{B}C\bar{D}$
$A \vee \neg E$
$A \vee E$
$A\bar{B}CDE$
$A\bar{B}CD\bar{E}$
$A\bar{B}C\bar{D}E$
$A\bar{B}C\bar{D}\bar{E}$
$\neg E \vee B$
$E \vee B$
$\neg E \vee B$
$E \vee B$


¬A∨**C**
A
Ā
AC
A∨**¬E**
A∨**E**
ABC
AB̄C
¬B∨**D**
¬B∨¬C∨**¬D**
¬E∨B
E∨B

Davis Putnam Logemann Loveland (DPLL)

```
DPLL Φ =
  Φ = {} ||
  ( {} ∉ Φ &&
      let y = choose-literal Φ in
          DPLL(unit-propagate (Φ ,y))
        ||
          DPLL(unit-propagate (Φ ,¬y))
   )
```

Davis Putnam Logemann Loveland (DPLL)

$$\text{unit-collapse}(\Phi, x) = \{ C \setminus \{\neg x\} \mid C \in \Phi \wedge \neg x \in C\} \cup \{ C \in \Phi \mid \neg x \notin C \wedge x \notin C\}$$

$$\Big\{ \{\neg A, C\}, \{\neg B, D\}, \{\neg E, B\}, \{\neg E, A\}, \{A, E\}, \{E, B\}, \{\neg B, \neg C, \neg D\} \Big\}$$

$$\Big\downarrow A$$

$$\Big\{ \{{\scriptstyle \neg A,}\, \mathbf{C}\}, \{\neg B, D\}, \{\neg E, B\}, {\scriptstyle \{\neg E, A\}}, {\scriptstyle \{A, E\}}, \{E, B\}, \{\neg B, \neg C, \neg D\} \Big\}$$

Davis Putnam Logemann Loveland (DPLL)

unit-propagate(Φ, x) =
    let Φ′ = unit-collapse(Φ, x) in
    if ∃ y. {y} ∈ Φ′ then unit-propagate(Φ′, y)
                                    else Φ′

{ {¬A,C}, {¬B,D}, {¬E,B}, {¬E,A}, {A,E}, {E,B},{¬B, ¬C, ¬D} }

A ↓   ¬A ↘

{ {¬A,C}, {¬B,D}, {¬E,B}, {¬E,A}, {A,E}, {E,B},{¬B, ¬C, ¬D} }   . . .

C ↓

{ {¬A,C}, {¬B,D}, {¬E,B}, {¬E,A}, {A,E}, {E,B},{¬B, ¬C, ¬D} }

↓ B   ¬B ↓

{ {¬A,C}, {¬B,D}, {¬E,B}, {¬E,A}, {A,E}, {E,B},{¬B, ¬C, ¬D} }   { {¬A,C}, {¬B,D}, {¬E,B}, {¬E,A}, {A,E}, {E,B},{¬B, ¬C, ¬D} }

Davis Putnam Logemann Loveland (DPLL)

DPLL(Φ, V) =

if V ⊨ Φ return true;

if {} ∈ Φ return false;

if V, Φ ⊨ L return DPLL(Φ, V^L)

L ← choose-literal(Φ)

return DPLL(Φ, V^L}) || DPLL(Φ, V^¬L);

Naive search

$$
\begin{aligned}
&S(\Phi, V) = \\
&\qquad \forall\, C \in \Phi.\ V \nvDash \neg C\ \&\& \\
&\quad (\ \ \forall\, C \in \Phi.\ V \vDash C\ \| \\
&\qquad\qquad \exists\, L.\ \big(S(\Phi, V \wedge L)\ \|\ S(\Phi, V \wedge \neg L)))
\end{aligned}
$$

Davis Putnam Logemann Loveland (DPLL)

$$
\begin{aligned}
&\text{DPLL}(\Phi, V) = \\
&\qquad \forall C \in \Phi.\ V \not\models \neg C\ \&\& \\
&\quad (\ \ \forall C \in \Phi.\ V \models C\ \| \\
&\qquad \exists C \in \Phi, L.\ \ V, C \models L\ \&\&\ \text{DPLL}(\Phi, V \wedge L) \\
&\qquad\qquad\qquad \| \\
&\qquad \exists L.\ \big(\text{DPLL}(\Phi, V \wedge L)\ \|\ \text{DPLL}(\Phi, V \wedge \neg L)))
\end{aligned}
$$

Once upon a time …

The friends share a list of mountains they have seen signed.
They each choose an unsigned mountain to watch.

Each day, each friend climbs one of the watched mountains.
If one finds his mountain has been signed she visits other, unwatched and previously unsigned mountains, until she finds one that is still unsigned, which she then watches.

As soon as there is only one unsigned mountain left to watch, they take turns spending the night on that mountain until it is signed and they claim the prize.

As long as there are two or more unsigned mountains, they are watching two of them. As soon as the penultimate mountain is signed they can take turns keeping vigil on the final peak.

1. $V \nvDash \neg C$ unless V contradicts every literal in C
2. $V \vDash C$ iff V establishes some literal in C
3. $V, C \vDash L$ iff V contradicts every literal but L in C

As V gets longer it establishes and contradicts more literals. We watch literals, as long as there are two uncontradicted literals we watch two.

If one of our watched literals is contradicted, we try to find another . If we fail, then 3 is true; we claim our prize, and try to establish L; other clauses may try to establish other literals, at which point we may discover a contradiction, or continue.

If the search backtracks, V gets shorter, both of our two watched literals are again uncontradicted.